Dr. Alghadhban

Section 1.1: Set Definitions

- A set is a collection of elements.
- $a \in A \equiv$ a is an element of Set A.
  small letter → a; capital letter → A
  $a \notin A \equiv$ a is not an element of A.
- Examples
  $A = \{6, 7, 8, 9\}$
  A = {integers between 5 and 10}
- A set can be countable or uncountable.
- The empty set has no elements
  empty set $\equiv \emptyset$
  $\equiv$ null set
- Finite set $\equiv$ has a finite number of elements
  $\equiv$ countable
  ~~or it is empty~~
  the null set is also finite set
- A is a subset of B $\equiv A \subseteq B$
- If at least one element exists in B which is not in A
  $\Rightarrow$ A is a proper subset of B
  $A \subset B$

- The null set is clearly a subset of all other sets.
- Two sets, A and B are disjoint or mutually exclusive if they have NO common elements.
  See Examples 1.1-1 and Examples 1.1-2
- Universal Set: is the largest set of objects under discussion. Denoted by S.
- For a universal set of N elements, there are $2^N = 64$ possible subsets.

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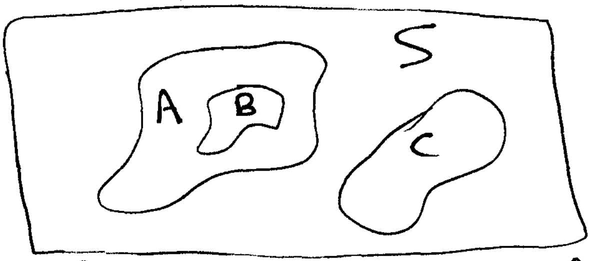
Section 1.2: Set Operations

Venn Diagram:

- Venn Diagram is a geometrical representation of sets.
- The universal set is represented by a rectangle.



$B \subset A$; C is disjoint from both A and B.

Equality and Difference

- Two sets A and B are equal if all elements in A are present in B and vise versa.

we say $A = B$ if $A \subseteq B$ and $B \subseteq A$.

- The difference of two sets A and B is $A - B$

$A - B \equiv$ is the set containing all elements of A that are not present in B.

- Note that

Union and Intersection:

Union: $C = A \cup B$

$\equiv$ the sum of two sets

$\equiv$ all elements of A or B or Both

Intersection:

$$D = A \cap B$$

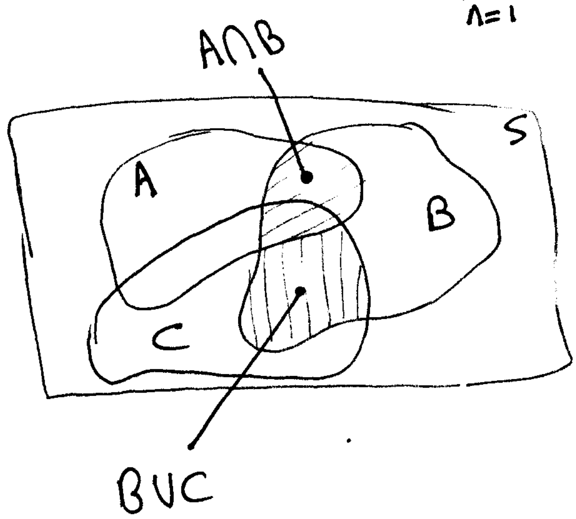
$\equiv$ Product of two Sets

$\equiv$ all elements common to both A and B.

- If A and B are mutually exclusive $\Rightarrow A \cap B = \emptyset$
- For N sets:

$$C = A_1 \cup A_2 \cdots \cup A_N = \bigcup_{n=1}^{N} A_n$$

$$D = A_1 \cap A_2 \cdots \cap A_N = \bigcap_{n=1}^{N} A_n$$



Complement:

- Complement of A $\equiv \bar{A}$
  $\equiv$ is the set of all elements **not** in A.

Thus, $\bar{A} = S - A$

- Note that $\bar{\emptyset} = S$ and $\bar{S} = \emptyset$
  $A \cup \bar{A} = \emptyset$ and $A \cap \bar{A} = \emptyset$

See example 1.2-1

Algebra of Sets

- Commutative law
  $$A \cap B = B \cap A$$
  $$A \cup B = B \cup A$$
- Distributive law
  $$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
  $$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
- associative law
  $$(A \cup B) \cup C = A \cup (B \cup C) = A \cup B \cup C$$
  $$(A \cap B) \cap C = A \cap (B \cap C) = A \cap B \cap C$$

De Morgan's Law

$$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$$
$$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$$

- See example 1-2-2

Duality Principle

Givin an Identity, if we replace unions by intersetions, intersection by unions, S by $\emptyset$, and $\emptyset$ by S $\Longrightarrow$ then the identity is preserved

Example:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

then $$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

- See example 1.2-3

1.3 Probability Definition

Set Theory — relative Frequency

(1) Set Theory:

we define probability using Set theory and fundamental axioms.

Definitions:

- Experiment (Give Examples)
- Trial: one performance of the experiment
- outcome: the output of the experiment

(S) Sample Space: The set of all possible outcomes in any given experiments
(Give example: Rolling a single die)

Note: Sample space can be discrete or Continuous. It could be finite or infinite
(Give examples: See textbook Page 9)

- Events:
An event is a subset of Sample space.
(Give examples) (See textbook Page 10)

- Probability:
The Probability is a nonnegative number assigned to each event on the sample

- Probability (Continue)
we use the following notation:
$P(A) \equiv$ the probability of event A.

- Probability satisfies three axioms:

1- $P(A) \geq 0$ "nonnegative"

2- $P(S) = 1$ | Probability of all events is one. "Certain Event"

3- $P\left(\bigcup_{n=1}^{N} A_n\right) = \sum_{n=1}^{N} P(A_n)$

if $A_m \cap A_n = \emptyset$ "Mutually Exclusive Events"

* Mathematical Model of Experiments.

A real experiment is defined Mathematically by three things

1. Assignment of Sample space
2. Definition of Events of Interest
3. making Probability assignments to the events such that the axioms

- See example 1.3-2 in Textbook

1.3 Probability Def.

(2) Relative Frequency

$$P(A) = \lim_{n \to \infty} \frac{n_A}{n}$$

successful Event

# of experiments

average number of success

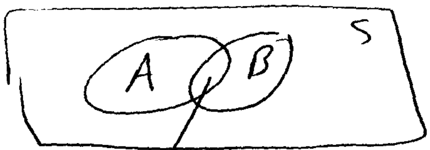
Give example:

- flipping a fair coin several times.
- Probability of drawing a six from a throughly shuffled regular ~~card of~~ deck of 52 cards

- See example 1.3-3

1.4 Joint and Conditional Prob.

Joint Probability



A and B are not mutually exclusive.

What is the probability of the Joint event $A \cap B$?

From the above Venn diagram,

$$\underbrace{P(A \cap B)}_{\text{Joint Prob.}} = P(A) + P(B) - P(A \cup B)$$

Thus, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\leq P(A) + P(B)$$

Equality holds only for mutually exclusive events. i.e "$A \cap B = \emptyset$"

Conditional Probability

Conditional probability of an event A, given B is

Joint Prob.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad P(B) > 0$$

"Prob. of event A may depend on event B

- If A and B are mutually exclusive

$\Rightarrow P(A|B) = 0$ because $A \cap B = \emptyset$

Total Probability

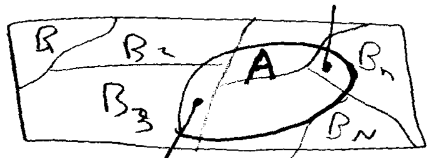
- The total probability of event A defined on a sample space S can be expressed as:

$$P(A) = \sum_{n=1}^{N} P(A|B_n) P(B_n)$$

where $B_m \cap B_n = \emptyset$ for $m \neq n = 1, 2, \cdots, N$

and $\bigcup_{n=1}^{N} B_n = S$

Why?



By Inspection

$$P(A) = \sum_{n=1}^{N} P(A \cap B_n)$$

From conditional Prob. Def:

$$P(A \cap B_n) = P(A|B_n) P(B_n)$$

Thus,

$$P(A) = \sum_{n=1}^{N} P(A|B_n) P(B_n)$$

Bayes' Theorem

Since $P(B_n|A) = \dfrac{P(B_n \cap A)}{P(A)}$

and $P(A/B_n) = \dfrac{P(A \cap B_n)}{P(B_n)}$

Bayes Theorem $\Longrightarrow$ $P(B_n|A) = \dfrac{P(A|B_n)\,P(B_n)}{P(A)}$

and also

$$P(B_n|A) = \frac{P(A|B_n)\,P(B_n)}{\sum_{m=1}^{N} P(A|B_m)\,P(B_m)}$$

* Solve Example 1.4-2 Page 18