Orbital Mechanics

AA236A

3 November 2008

STANFORD ENGINEERING

SSDL
Space and Systems Development Laboratory

Announcements

- Nice job on the first checkout
- Rover 2 checkout
  - Mon. 17 November, 3:45 pm
  - GPS, SD Card, and Color/Rad sensor are only major hardware components left!
- VOTE TOMORROW!!!

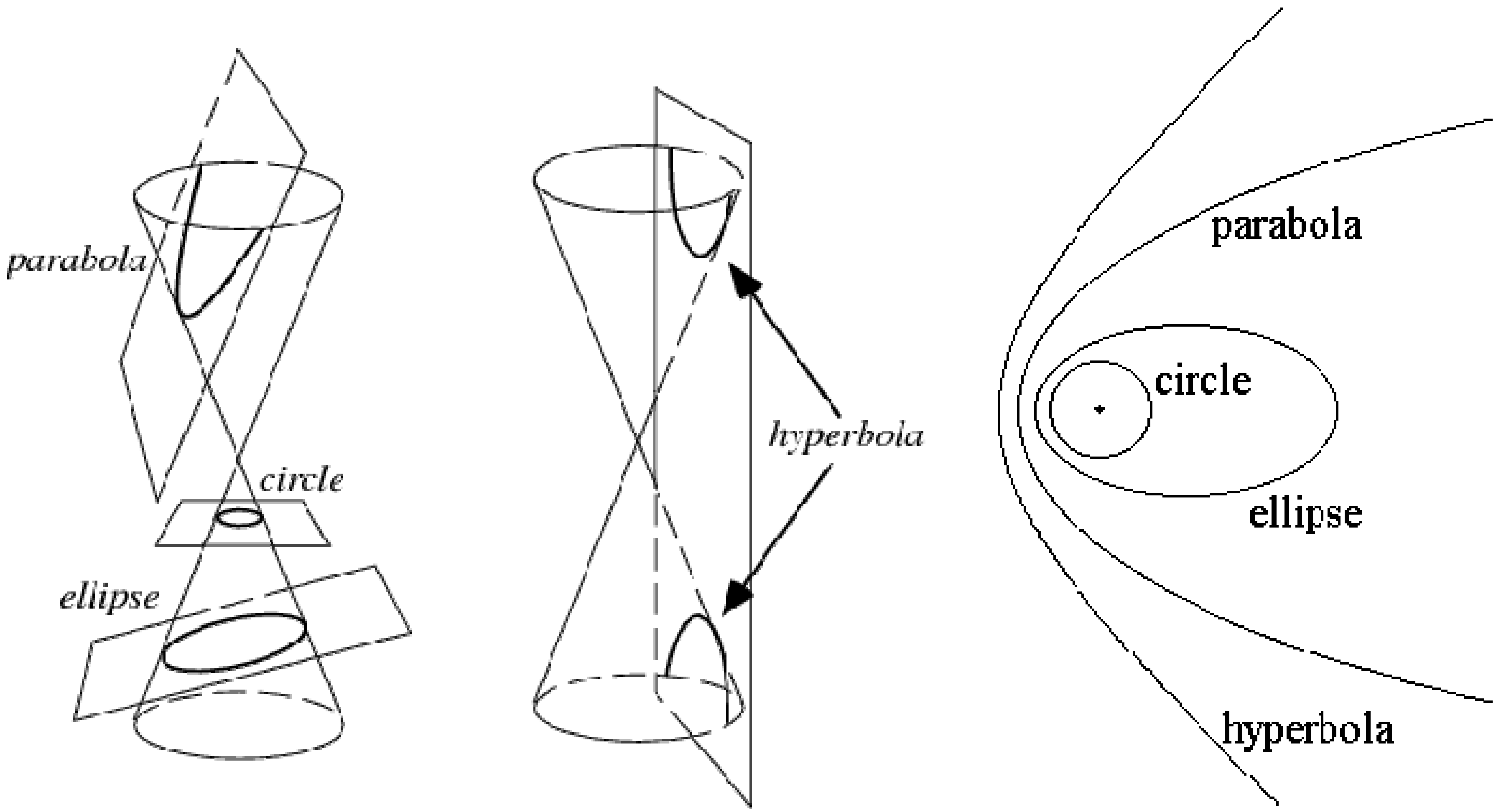
Source Material

- SMAD: Chapters 6-7
- Bate, Mueller, White – "Fundamentals of Astrodynamics"
- AA279 – Space Mechanics

Kepler's Laws

1. The orbit of each planet is an ellipse, with the sun at a focus.
2. The line joining the planet to the sun sweeps out equal areas in equal times.
3. The square of the period of a planet is proportional to the cube of its mean distance from the sun.

Conic Sections



Orbit Geometry

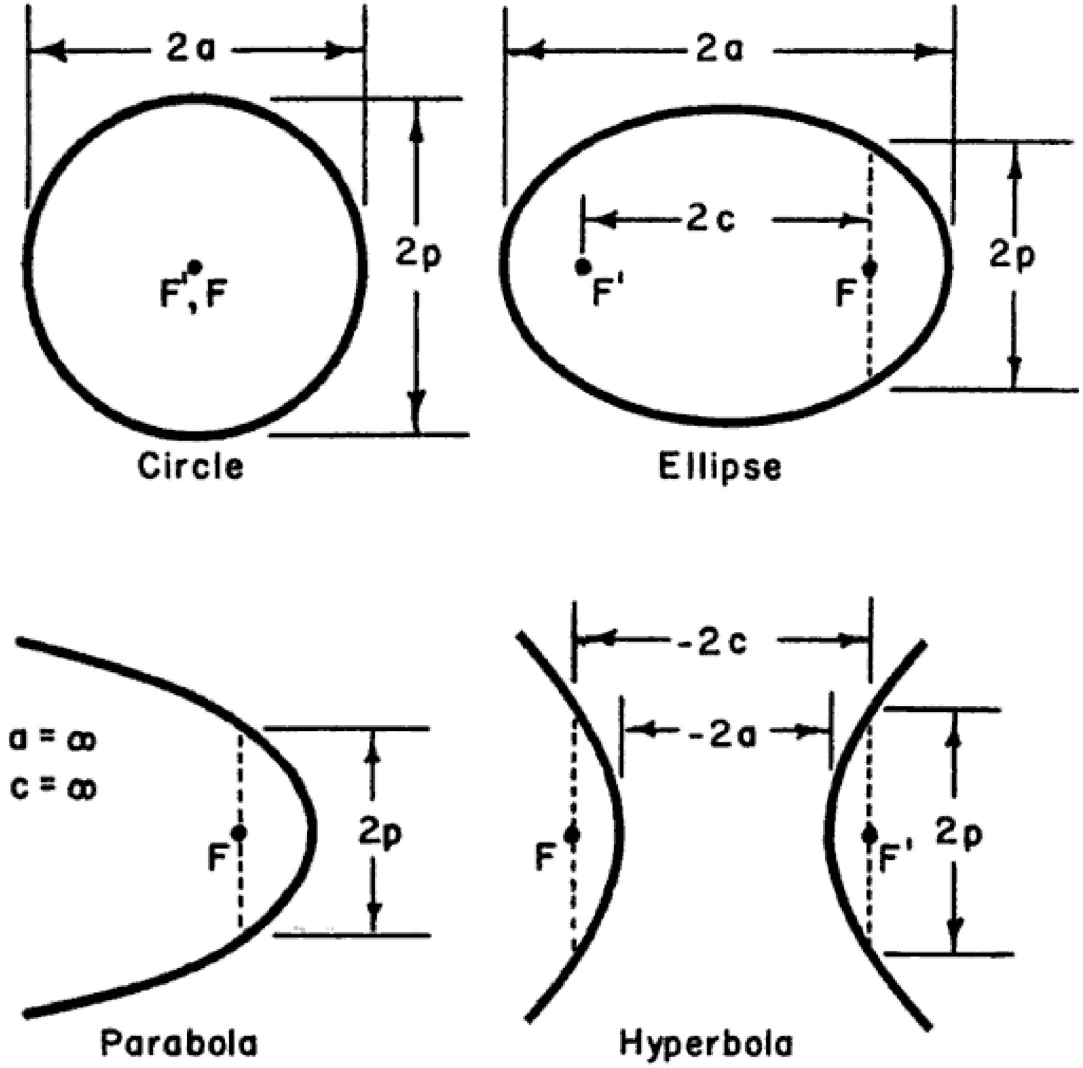


Figure 1.5-3 Geometrical dimensions common to all conic sections

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Elliptical Orbit Geometry

General:	Periapsis	Apoapsis
Sun:	Perihelion	Aphelion
Earth:	Perigee	Apogee
Moon:	Perilune	Apolune

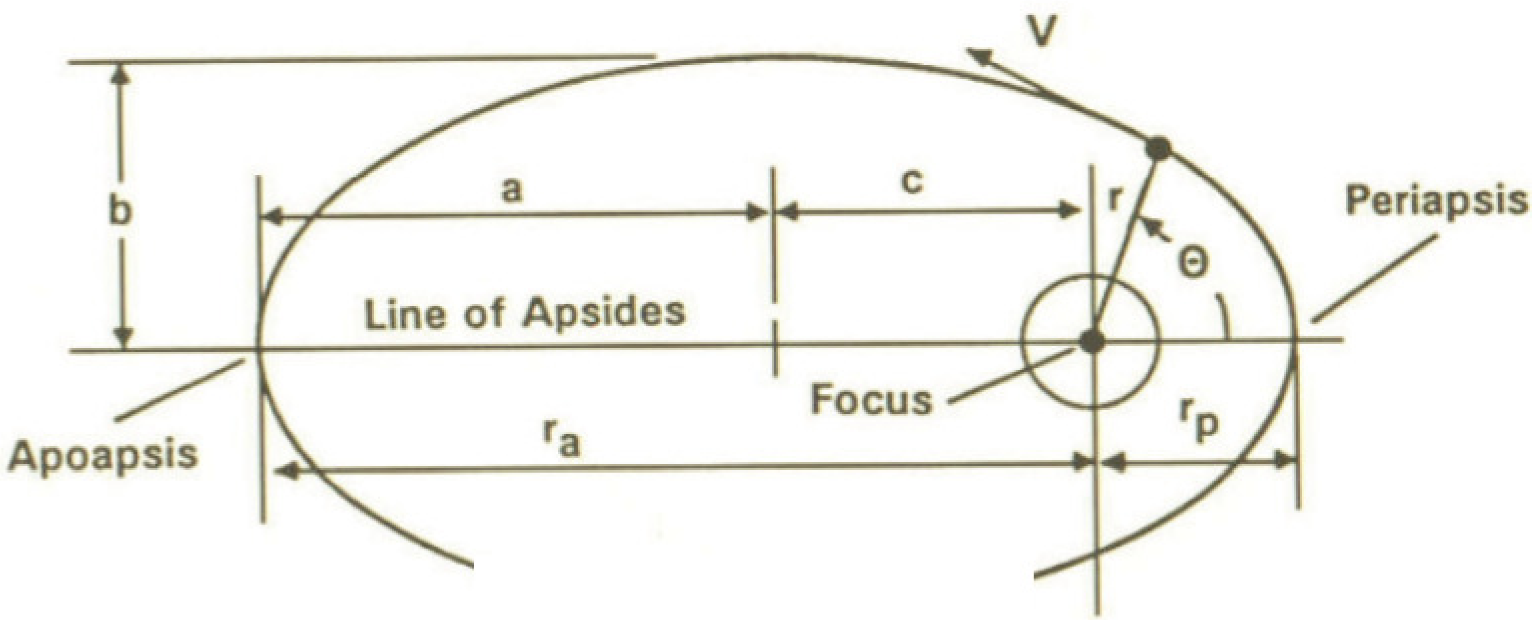


Fig. 2.3 Elliptical orbit.

Orbit Equations

Two-Body Equation of Motion

$$\ddot{\vec{r}} + \frac{\mu}{r^3}\vec{r} = \vec{a}_{thrust} + \vec{a}_{perturbation}, \quad \mu = GM, \quad G \approx 6.674 * 10^{-11} \frac{Nm^2}{kg^2}$$

Specific Mechanical Energy and Specific Angular Momentum

$$E = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$\vec{h} = \vec{r} \times \vec{v}$$

Orbital Period

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$

Trajectory Equation

$$r = \frac{a\left(1 - e^2\right)}{1 + e\cos\nu}$$

Orbit Type	Eccentricity, e	Energy, E
Circle	e = 0	E < 0
Ellipse	0 < e < 1	E < 0
Parabola	e = 1	E = 0
Hyperbola	e > 1	E > 0

Coordinate Systems

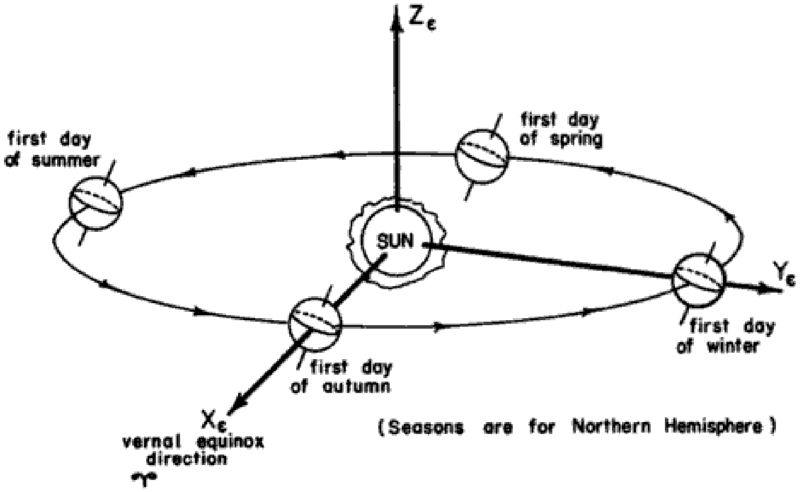


Figure 2.2-1 Heliocentric–ecliptic coordinate system

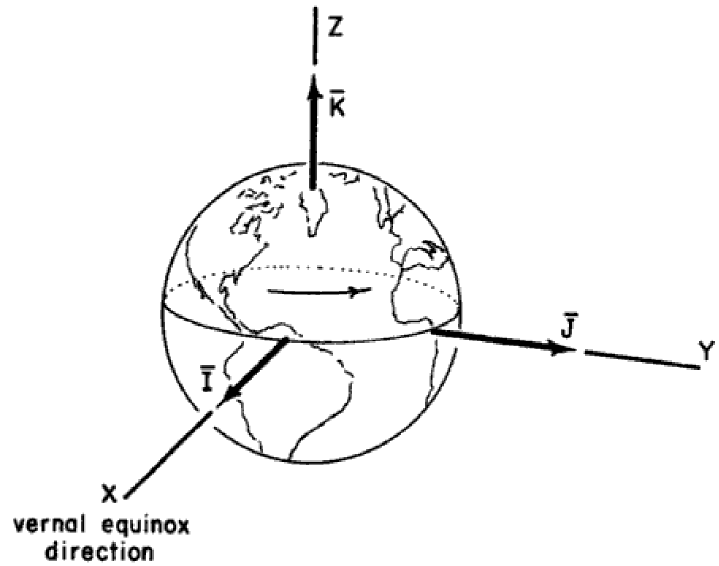


Figure 2.2-2 Geocentric-equatorial coordinate system

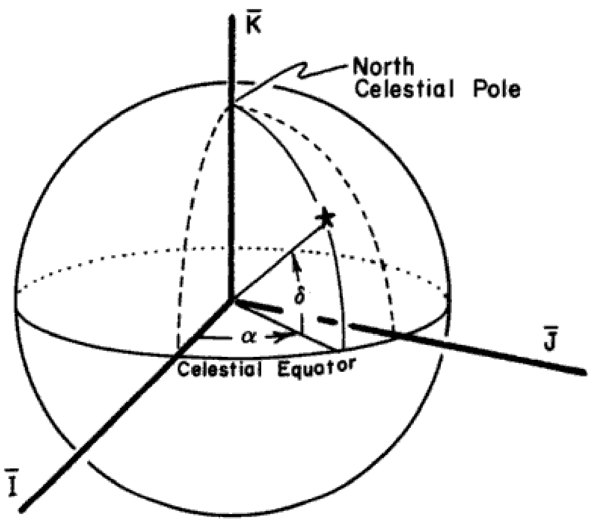


Figure 2.2-3 Right ascension–declination coordinate system

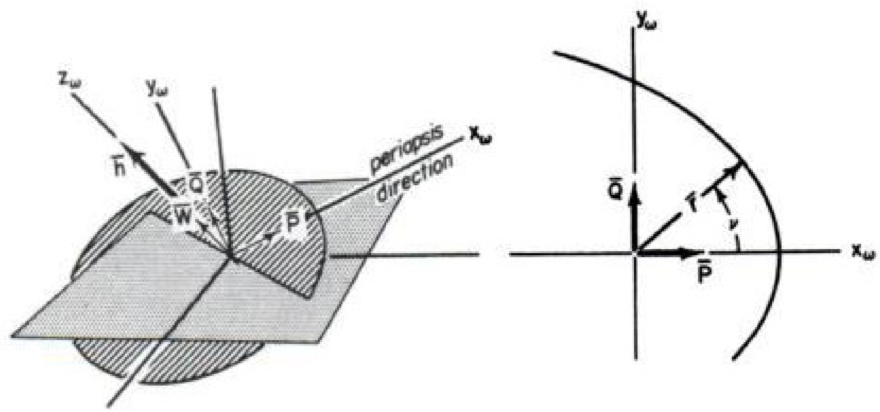


Figure 2.2-4 Perifocal coordinate system

Orbital Parameters

- Orbital or Keplerian Elements

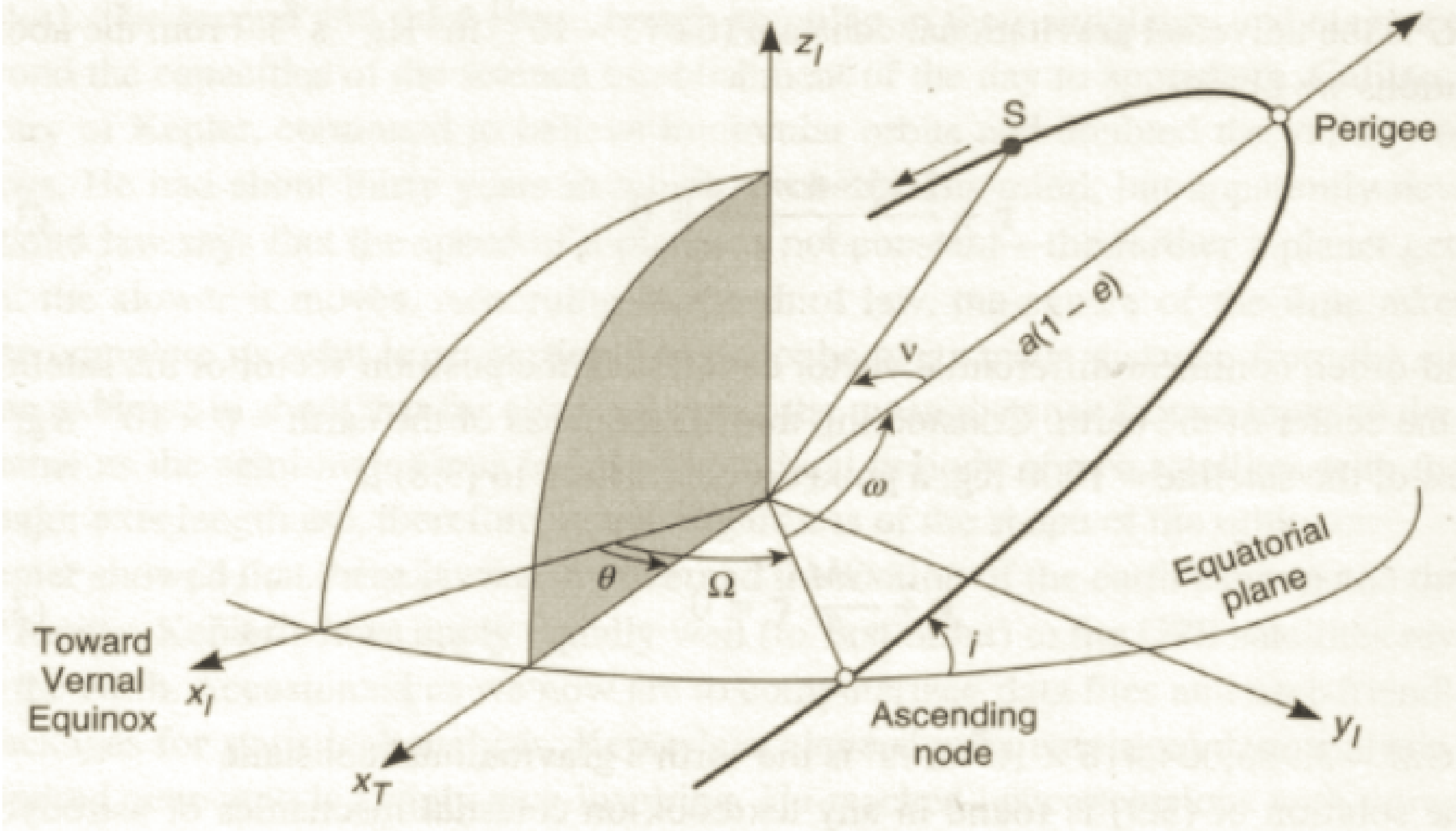


Figure 3.11 Characterization of an ideal orbit and the satellite position by Keplerian elements: {a, e, i, Ω, ω, and ν}.

Orbital Parameters

- *a*, semi-major axis: size of the orbit
- *e*, eccentricity: shape of the orbit
- *i*, inclination: angle between the angular momentum vector and Earth's rotation axis
- *Ω*, longitude of the ascending node: angle between vernal equinox direction and the point where the orbit crosses the equatorial plane in a northerly direction
- *ω*, argument of periapsis: angle between the ascending node and the orbit's periapsis
- *ν*, true anomaly: angle between periapsis and the satellite's current location

Orbital Parameters

- *Π, longitude of periapsis*
- u_0, argument of latitude at epoch
- l_0, true longitude at epoch

$$\Pi = \Omega + \omega$$

$$u_0 = \omega + \nu$$

$$l_0 = \Omega + \omega + \nu$$

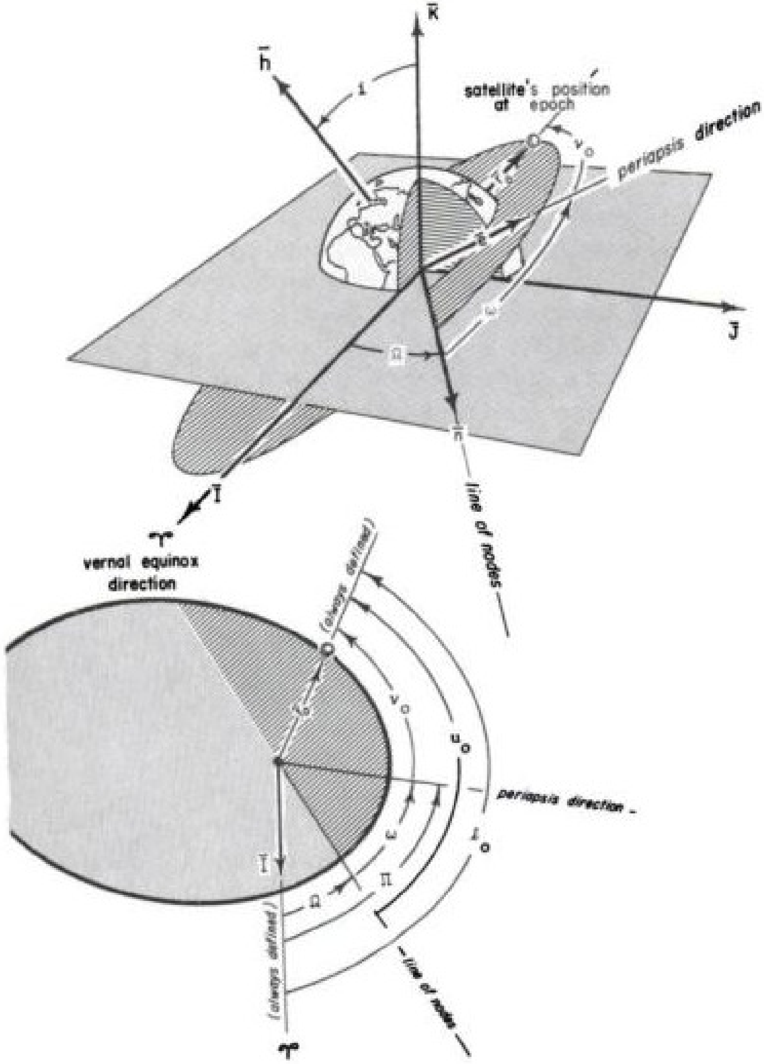


Figure 2.3-1 Orbital elements

Keplerian Elements Formats

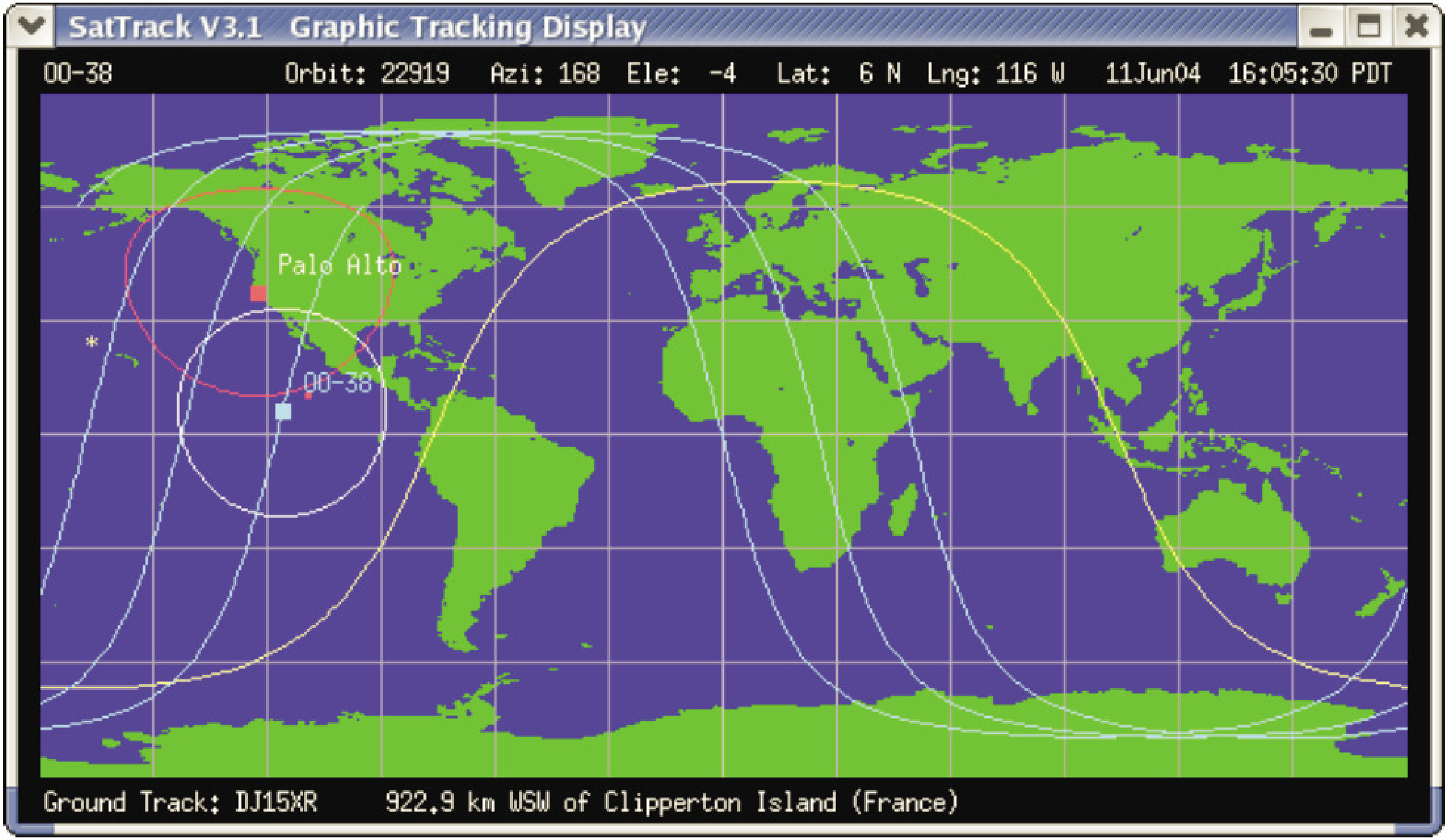
- NASA (2-line) Format

```
ISS
1 25544U 98067A 05286.76865936 .00025787 00000-0 18021-3 0 5452
2 25544 051.6447 013.4294 0001367 098.1184 352.9371 15.74101621394341
```

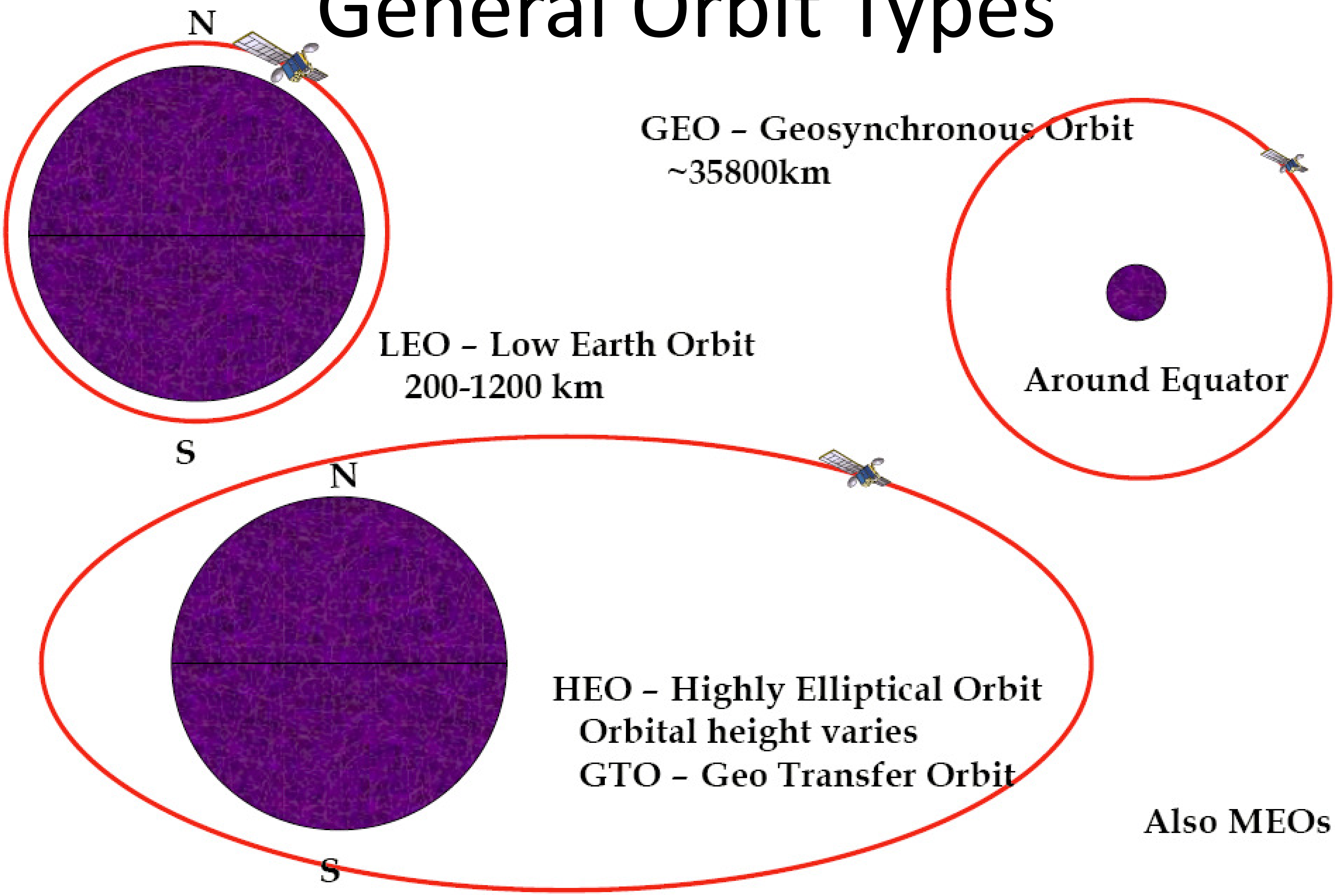
- AMSAT Verbose Format

```
Satellite: ISS
Catalog number: 25544
Epoch time: 05286.76865936
Element set: 545
Inclination: 051.6447 deg
RA of node: 013.4294 deg
Eccentricity: 0.0001367
Arg of perigee: 098.1184 deg
Mean anomaly: 352.9371 deg
Mean motion: 15.74101621 rev/day
Decay rate: 2.5787e-04 rev/day^2
Epoch rev: 39434
Checksum: 320
```

Ground Tracks



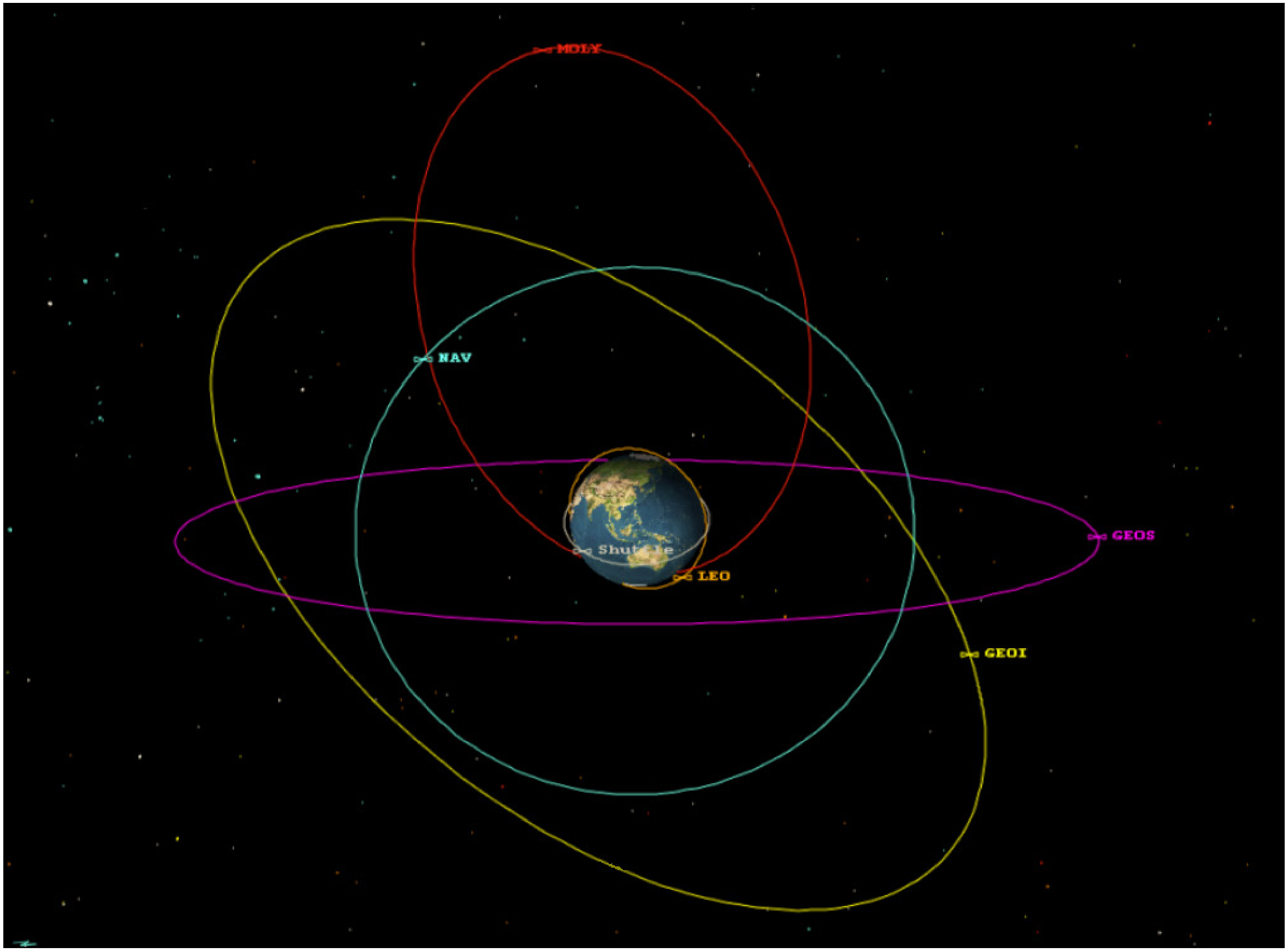
General Orbit Types



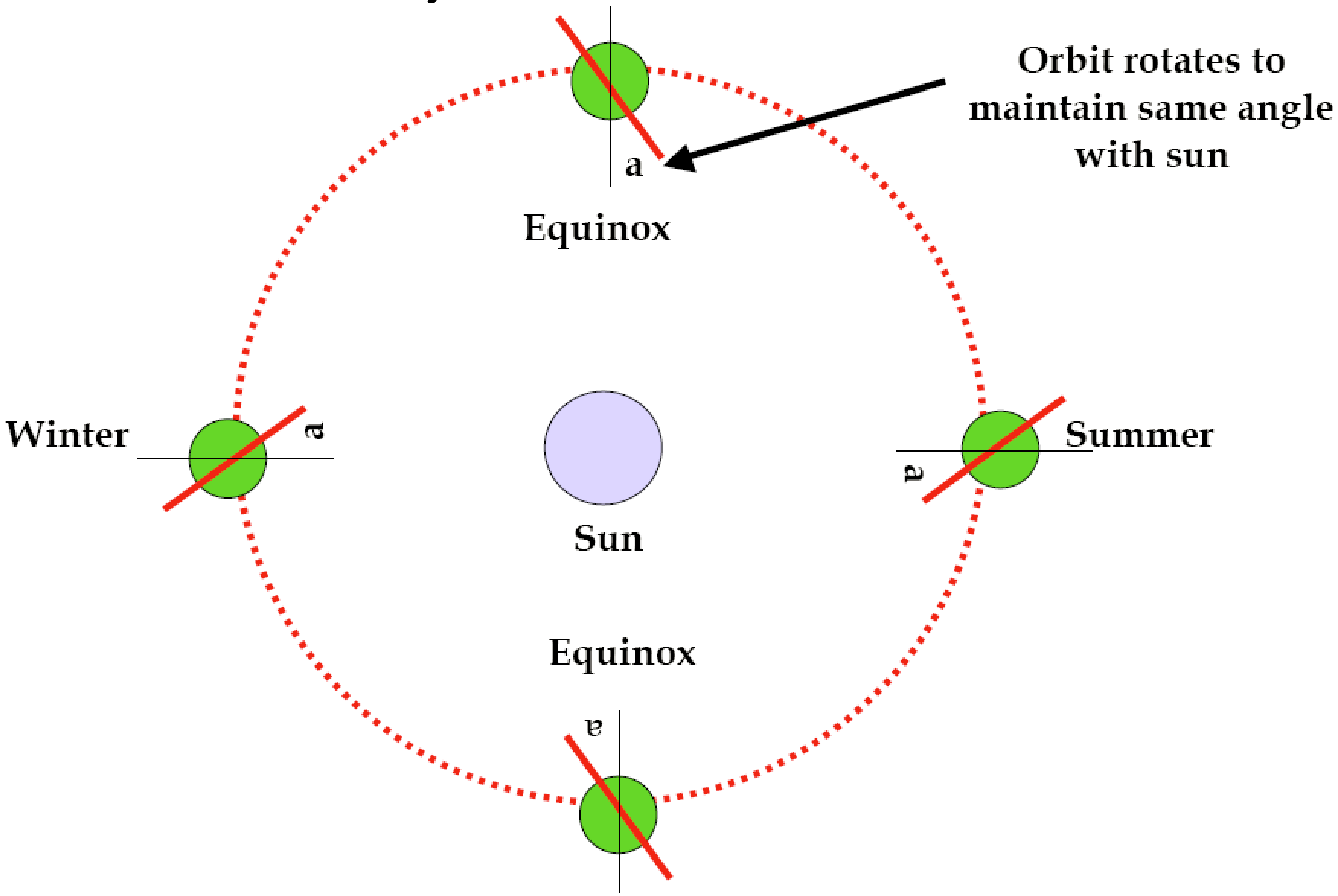
Interesting Orbit Types

- Polar Orbits
  - LEO orbits with high inclination and travel near the poles
- Equatorial Orbits
  - Low inclination and travel near the equator
- Geosynchronous
  - Orbital period matches Earth's sidereal rotation period
  - Returns to the same place in the sky at same time each day
- Geostationary
  - Circular, 0° inclination geosynchronous orbit
- Sun Synchronous
  - Passes over any given point at the same local solar time
  - Approx. constant orientation with respect to the Sun
  - Orbit must precess ~1 degree/day eastward (600-800 km, ~98° inclination)
- Molniya
  - High eccentricity
  - ~12 hr period, 63.4° inclination
- GPS
  - ~12 hr period, 26,600 km
- Lagrange Points
  - Gravitational equilibria

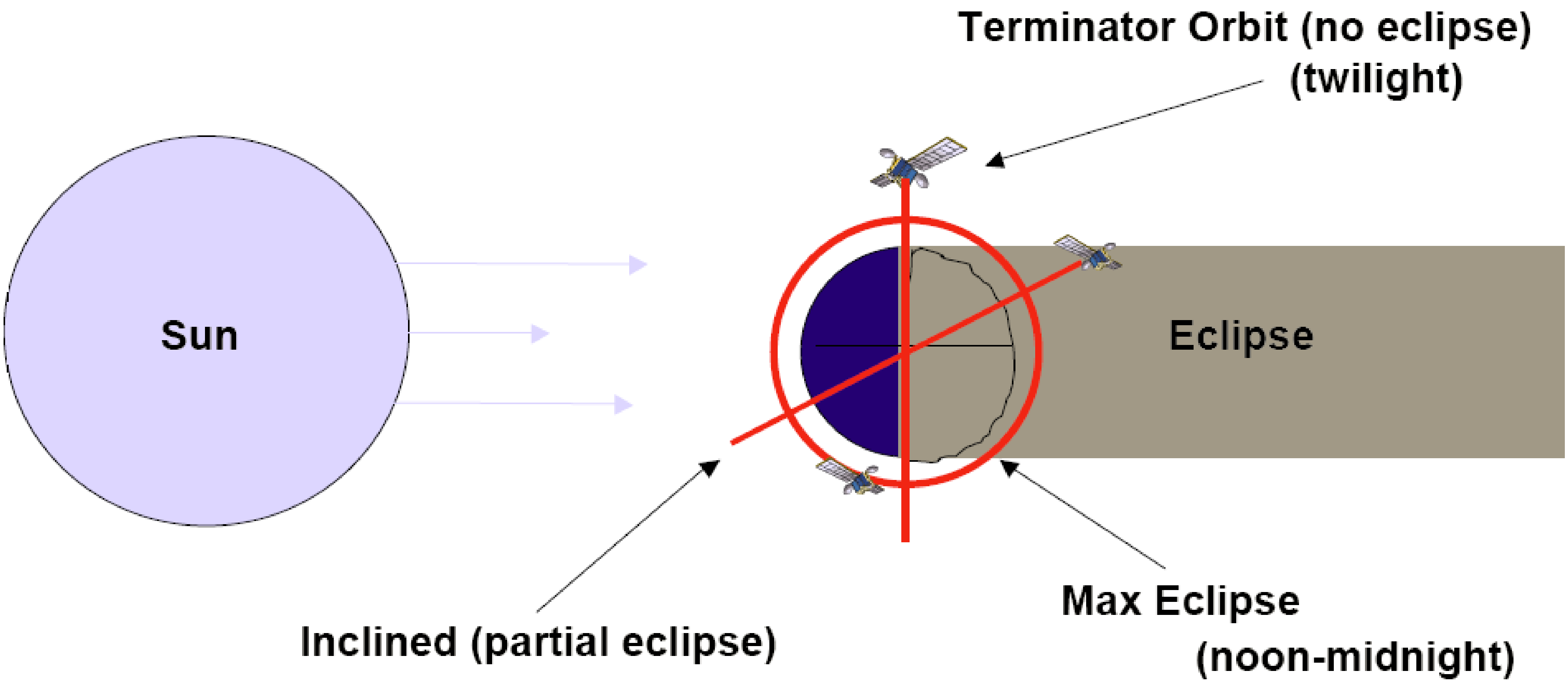
Orbit Types



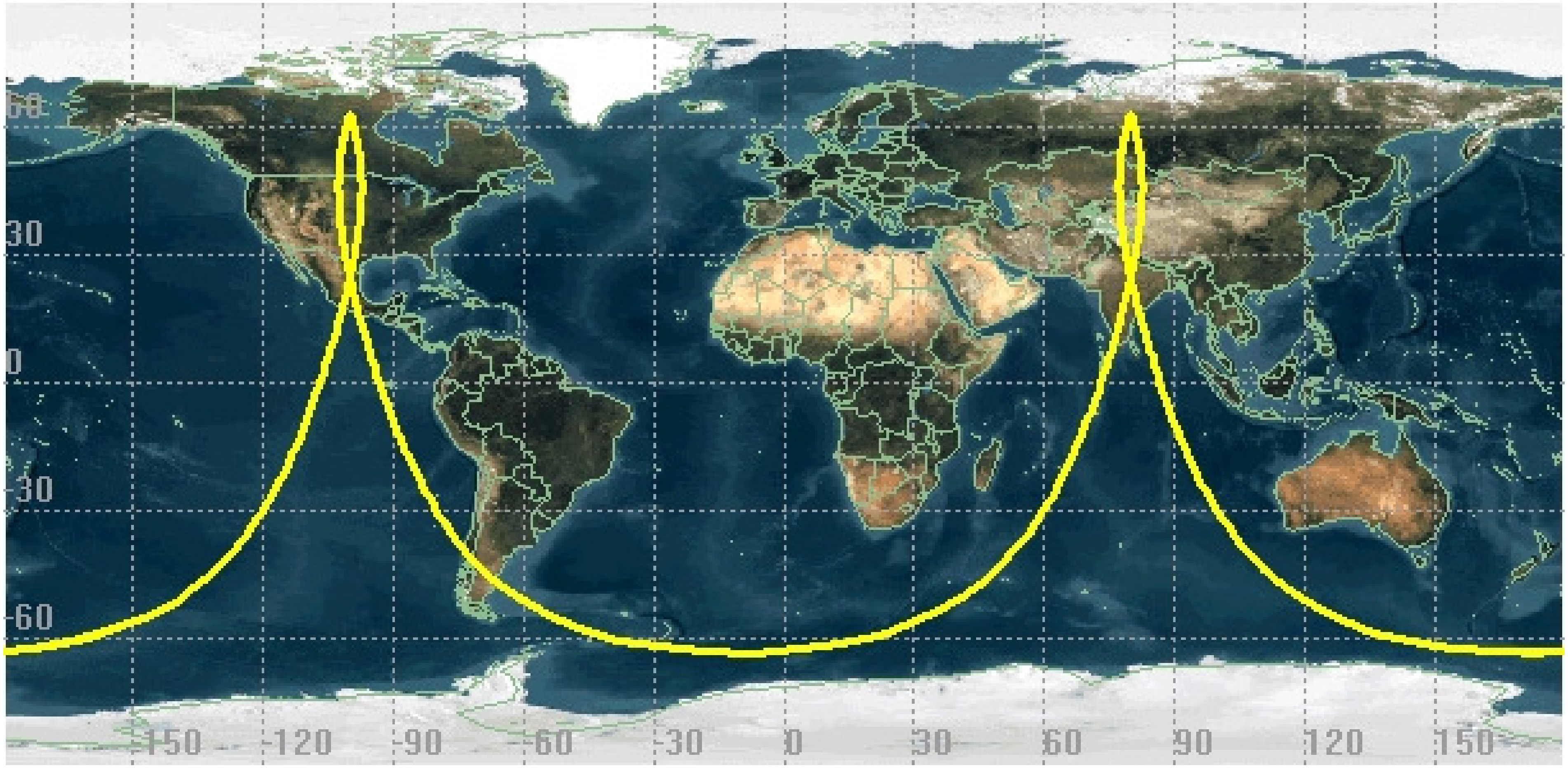
Sun Synchronous Orbit



Orbit with Respect to the Sun



Molniya



GPS Satellites

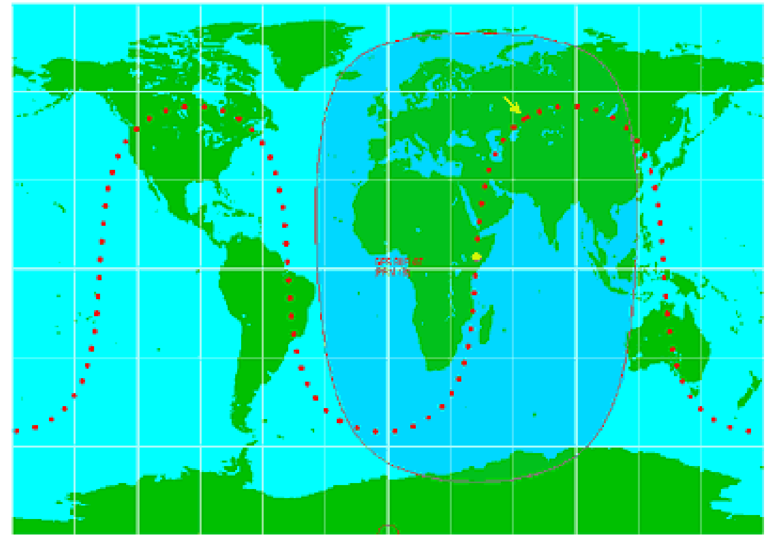
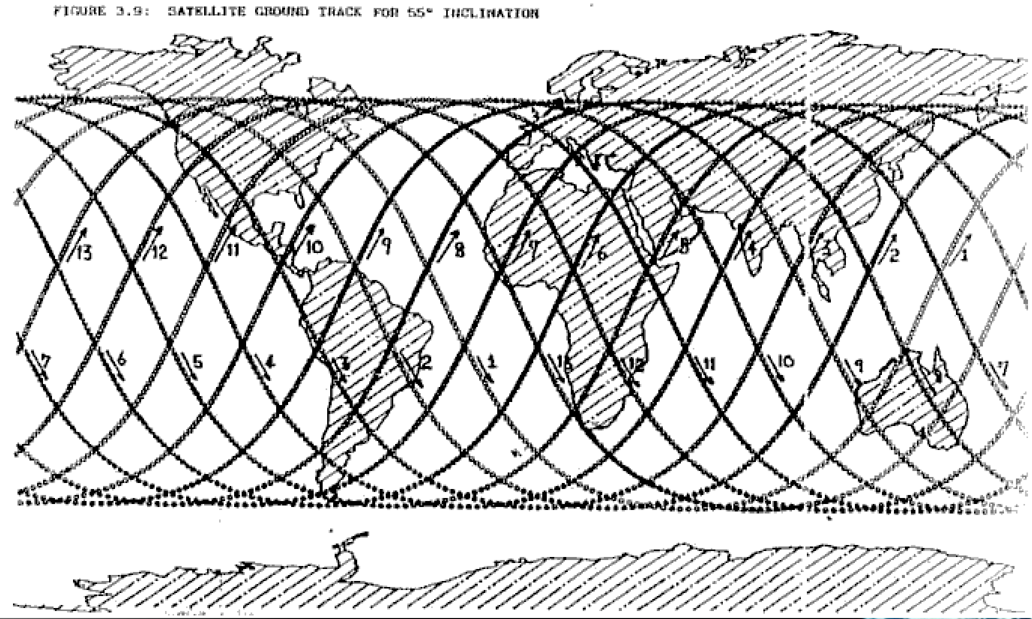
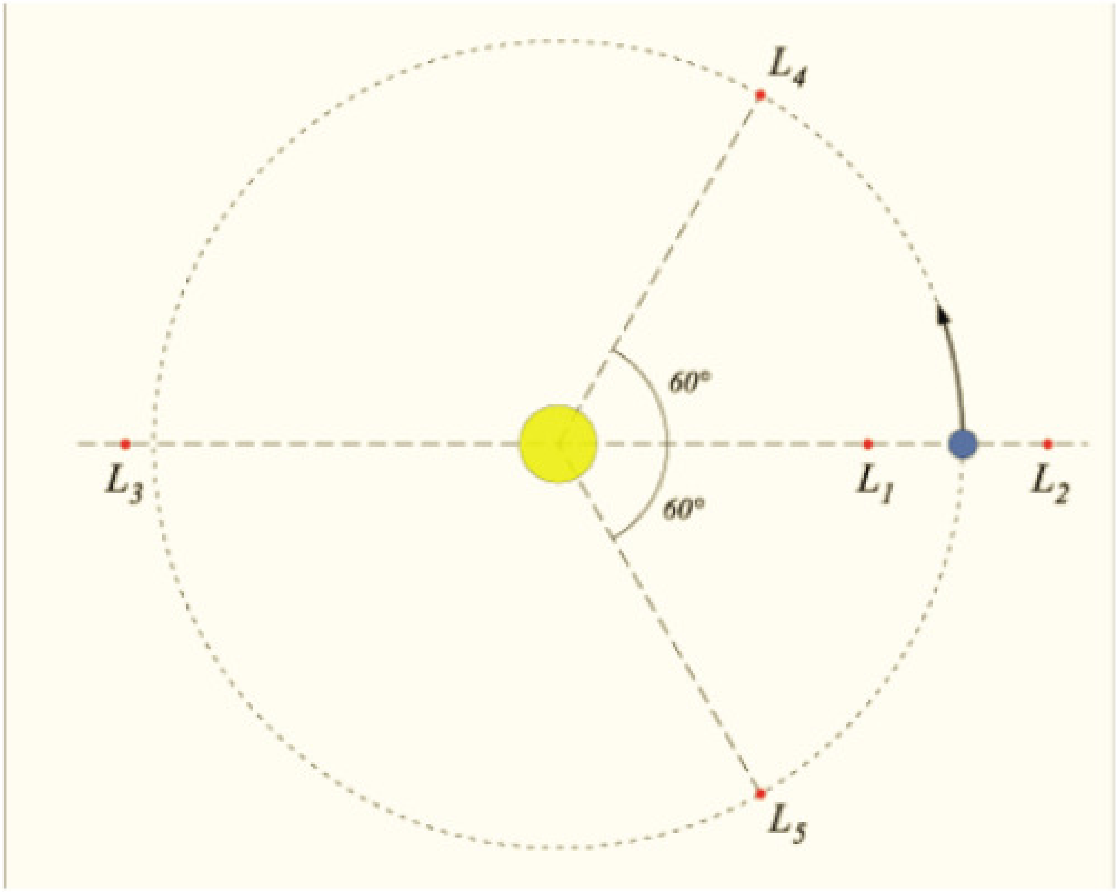


FIGURE 3.9: SATELLITE GROUND TRACK FOR 55° INCLINATION

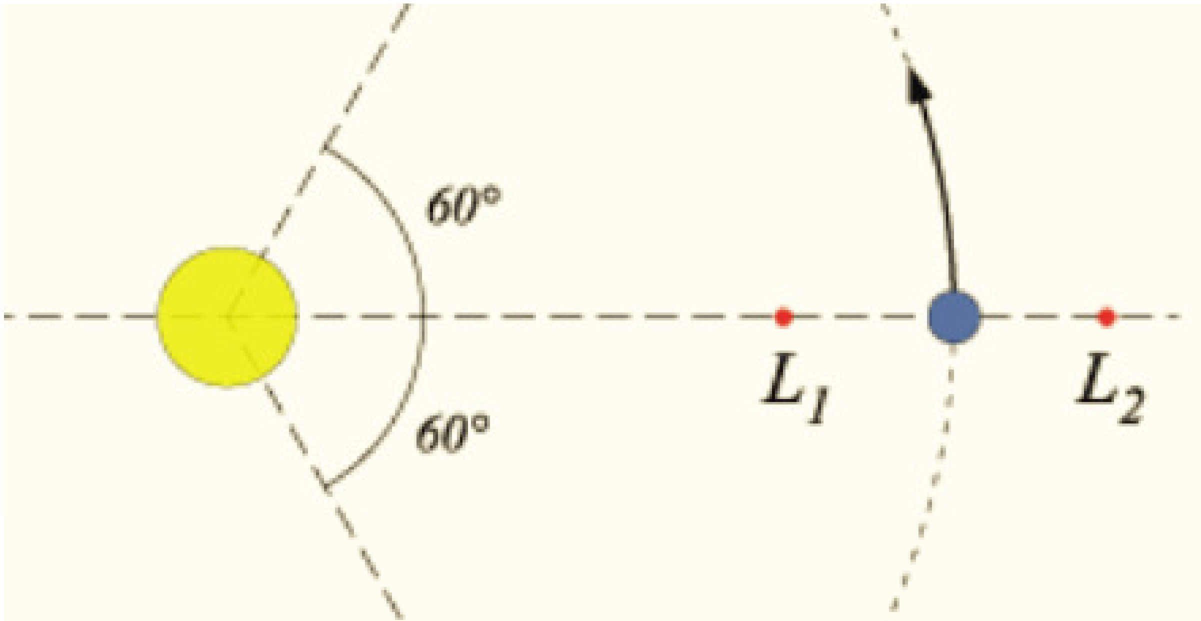


Lagrange Points

- Positions where the gravitational pull of the two large masses precisely equals the centripetal force required to rotate with them

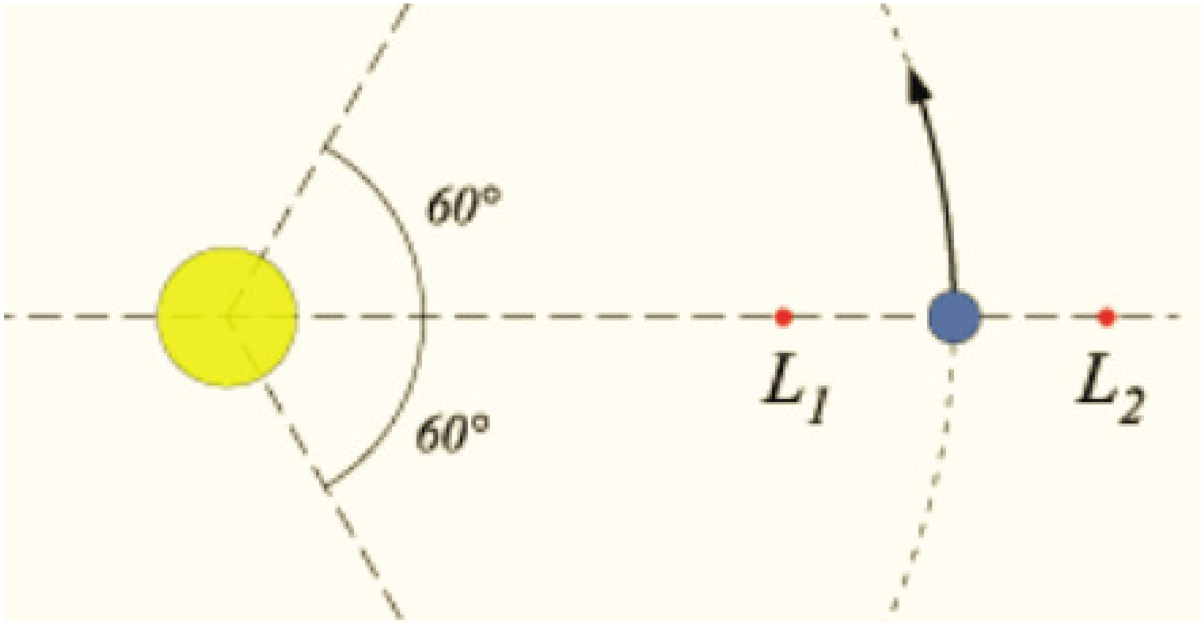


L_1 and Examples



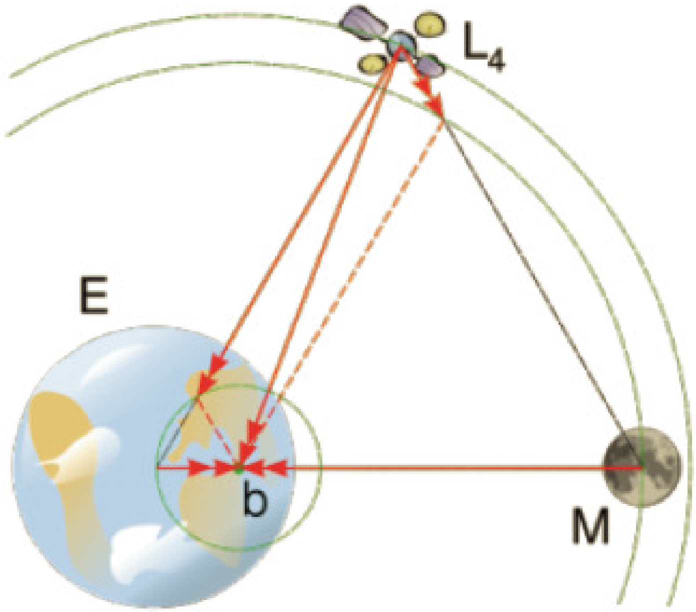
- Objects at L_1 orbit with same period as Earth
- Earth "weakens" the pull of the Sun and slows the orbit down
- Sun-Earth L_1 good for sun observations
  - SOHO: Solar and Heliospheric Observatory
  - ACE: Advanced Composition Explorer

L_2 and Examples



- Objects at L_2 orbit with same period as Earth
- Earth "strengthens" the pull of the Sun and speeds up the orbit
- Sun-Earth L_2 good for telescope missions
  - WMAP: Wilkinson Microwave Anisotropy Probe
  - ACE: Advanced Composition Explorer

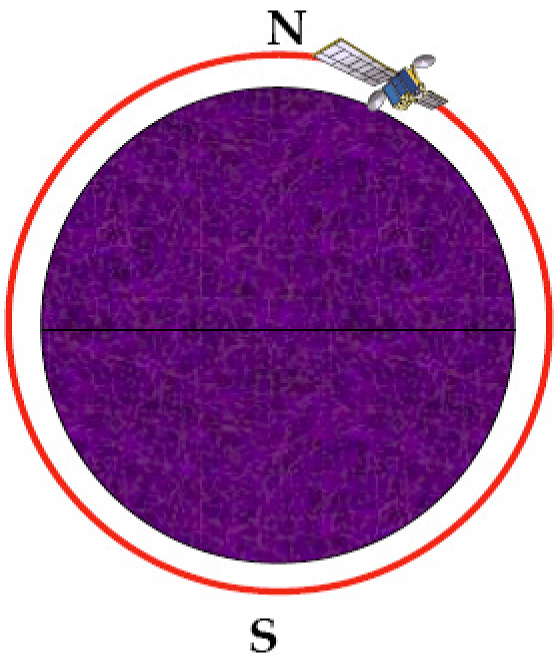
L_4 and L_5 Examples

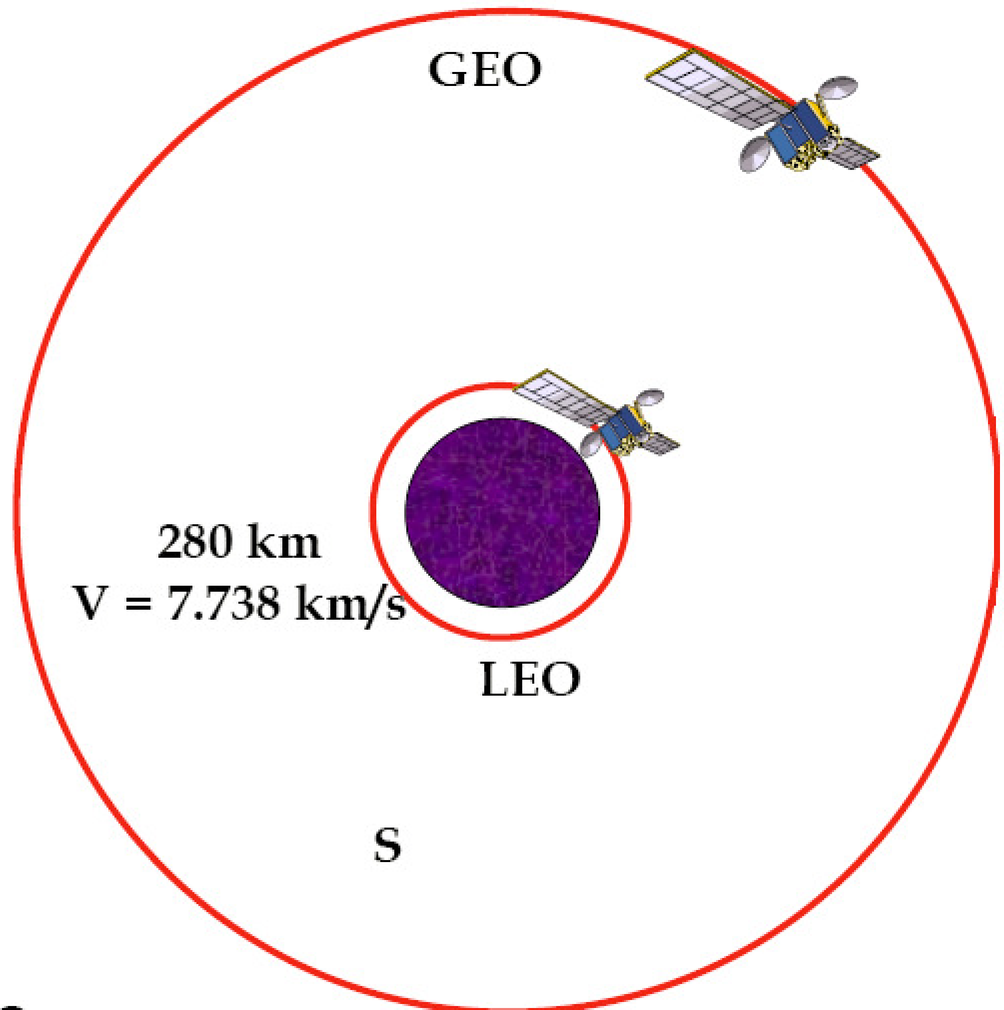


- Also called triangular Lagrange or Trojan points
- Only stable Lagrange Points
- Jupiter, Saturn, and Neptune have objects caught in their L_4 and L_5 points

Orbit Transfer

LEO – Low Earth Orbit
from a shuttle launch – 280 km



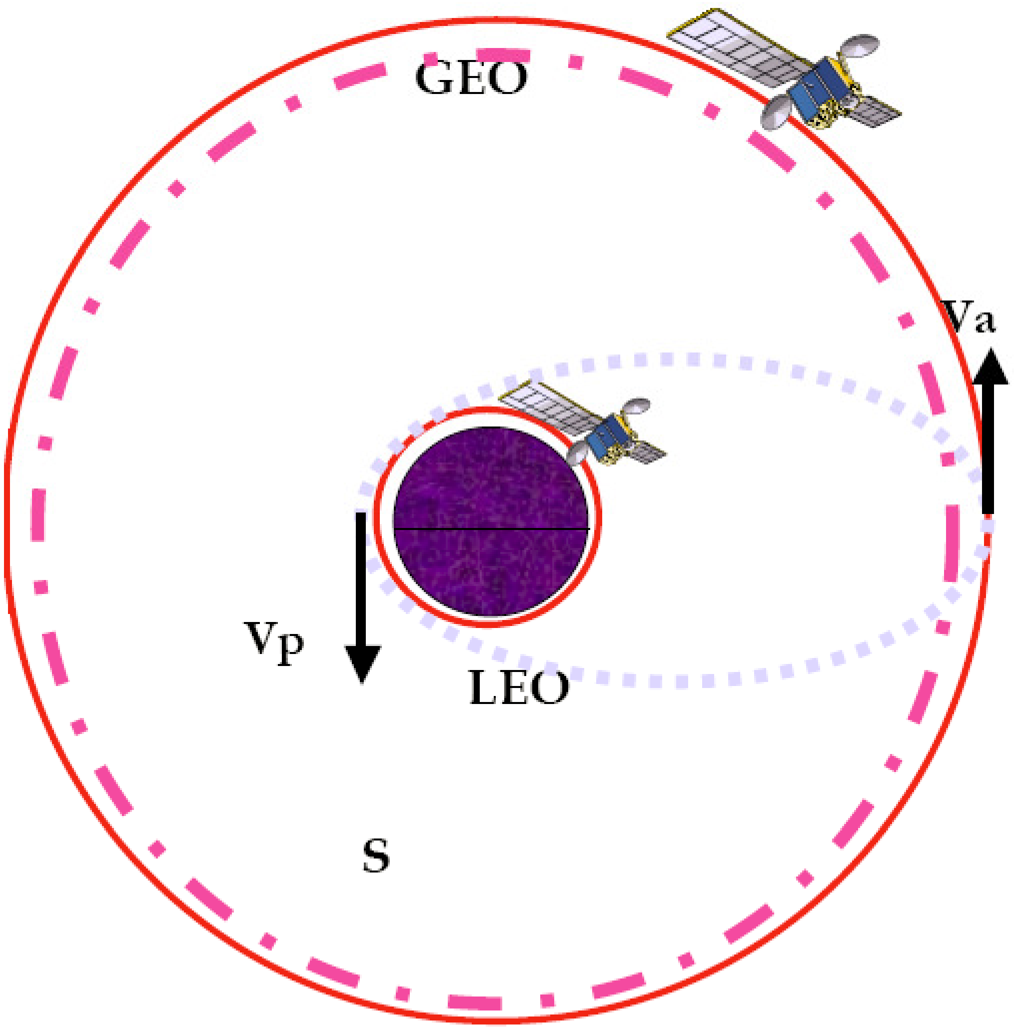


Want to Change Orbit
LEO to GEO

How?

35,786 km
V = 3.0727 km/s

Orbit Transfer - How



1. Change to a GTO (GEO transfer Orbit)

   Want:
   Vp = 10.169 km/s
   Va = 1.606 km/s

   For GTO

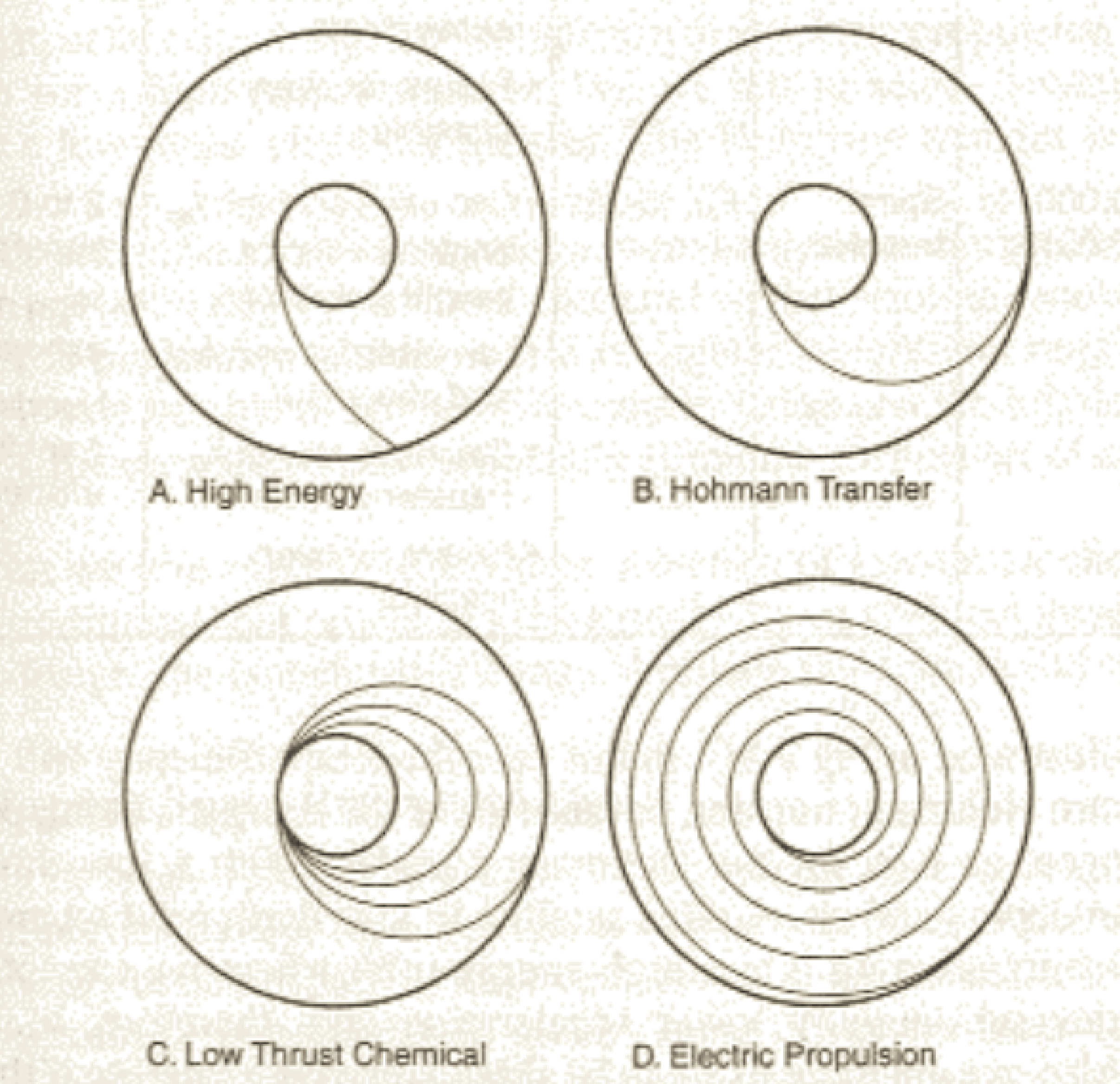
2. Circularize orbit

Need

V = 3.0727 km/s for GEO
Change V = 3.0727-1.606 = 1.4667 km/s

3. Burn at Va to increase V to 3.0727 km/s for circular orbit at GEO

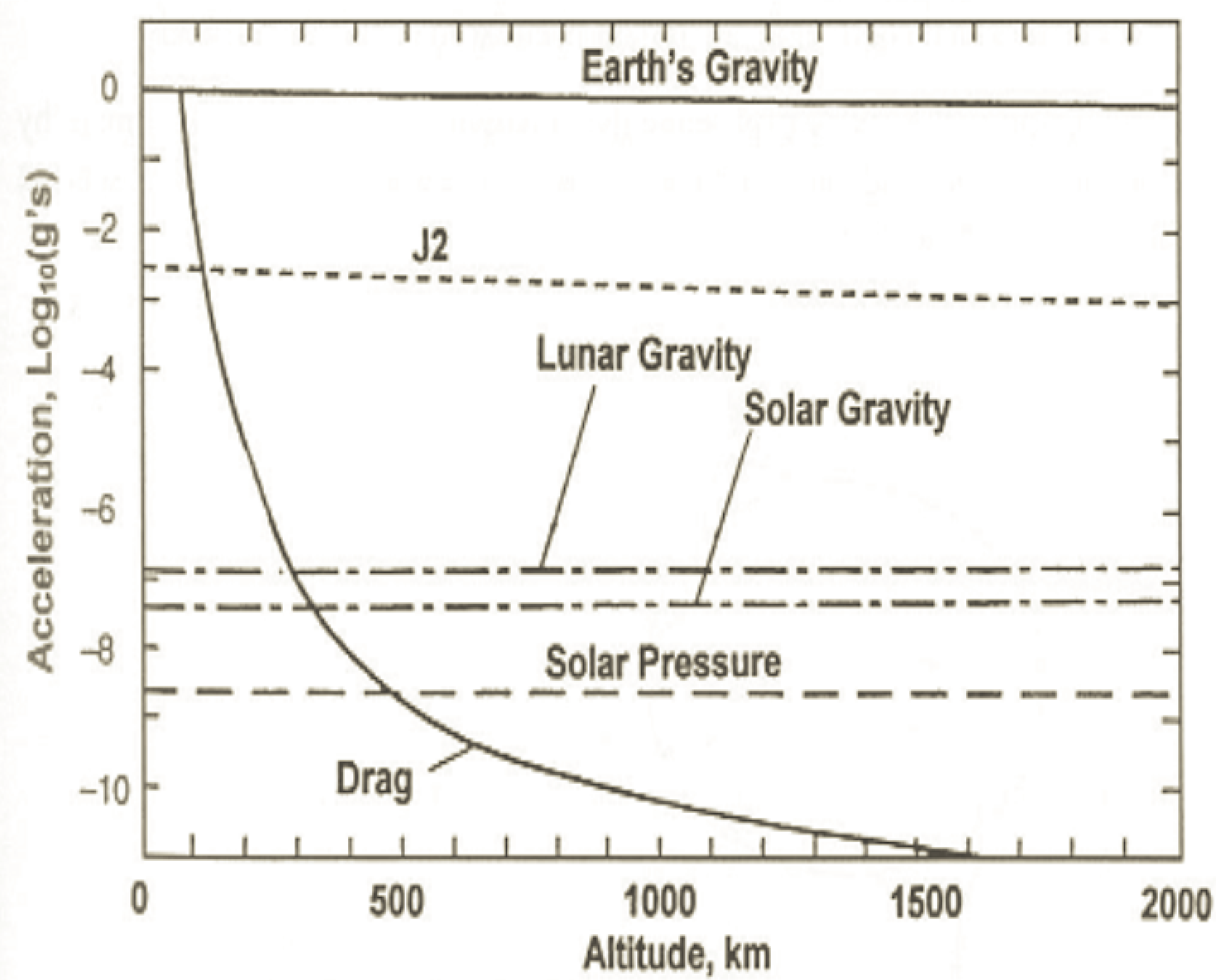
Orbit Transfer Methods



Orbit Perturbations

- Will your satellite stay where you put it?
- Is there anything that will change the satellite's orbit?

On-Orbit Accelerations



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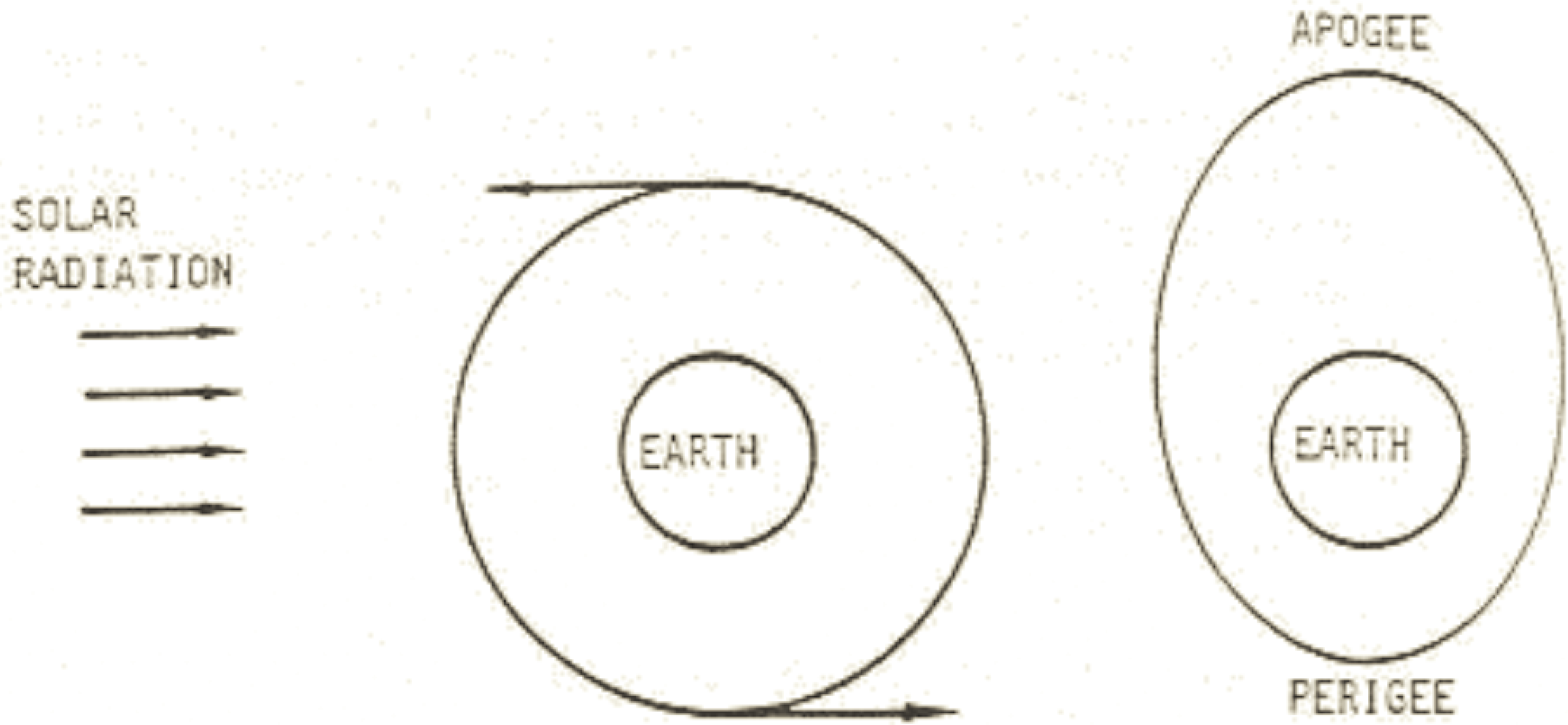
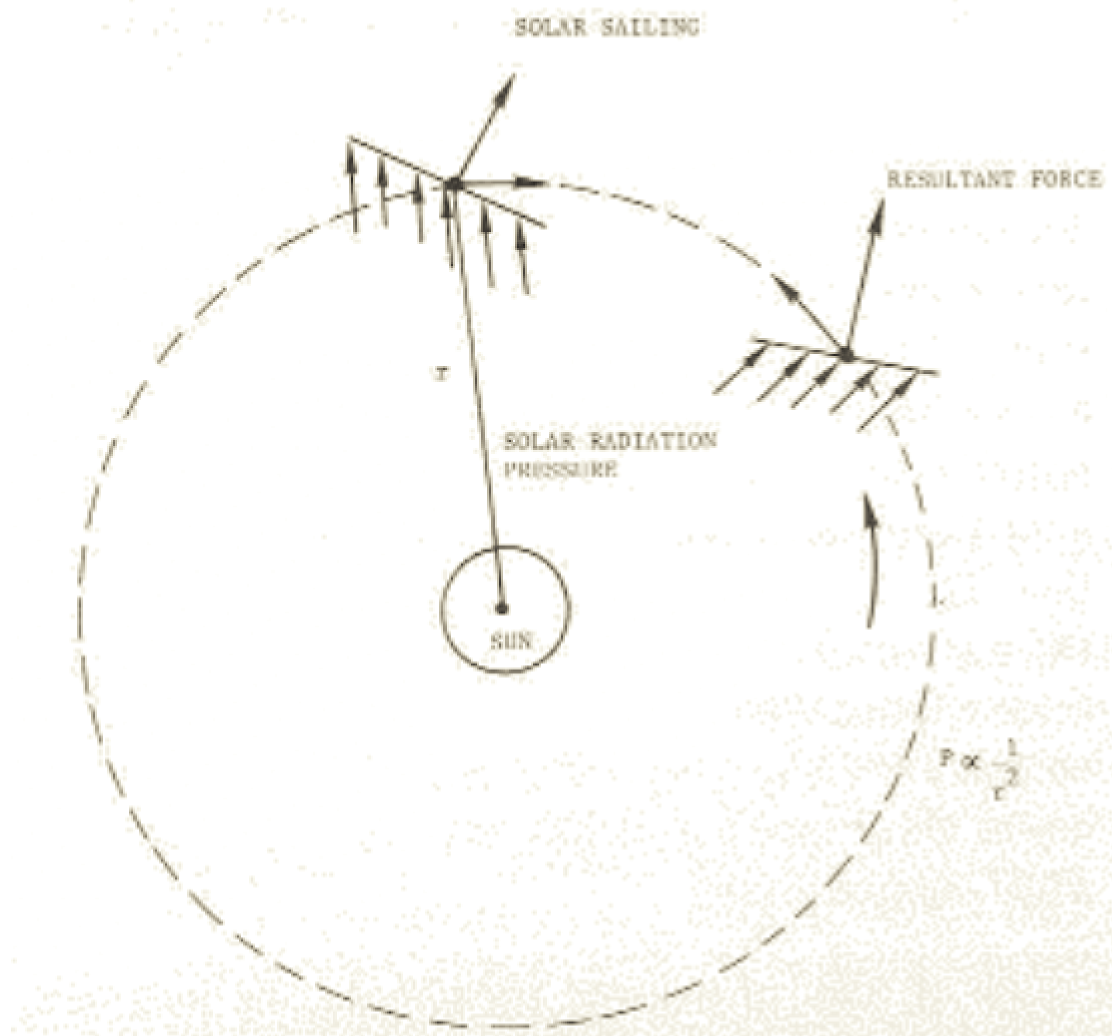
Solar Pressure



Solar Pressure

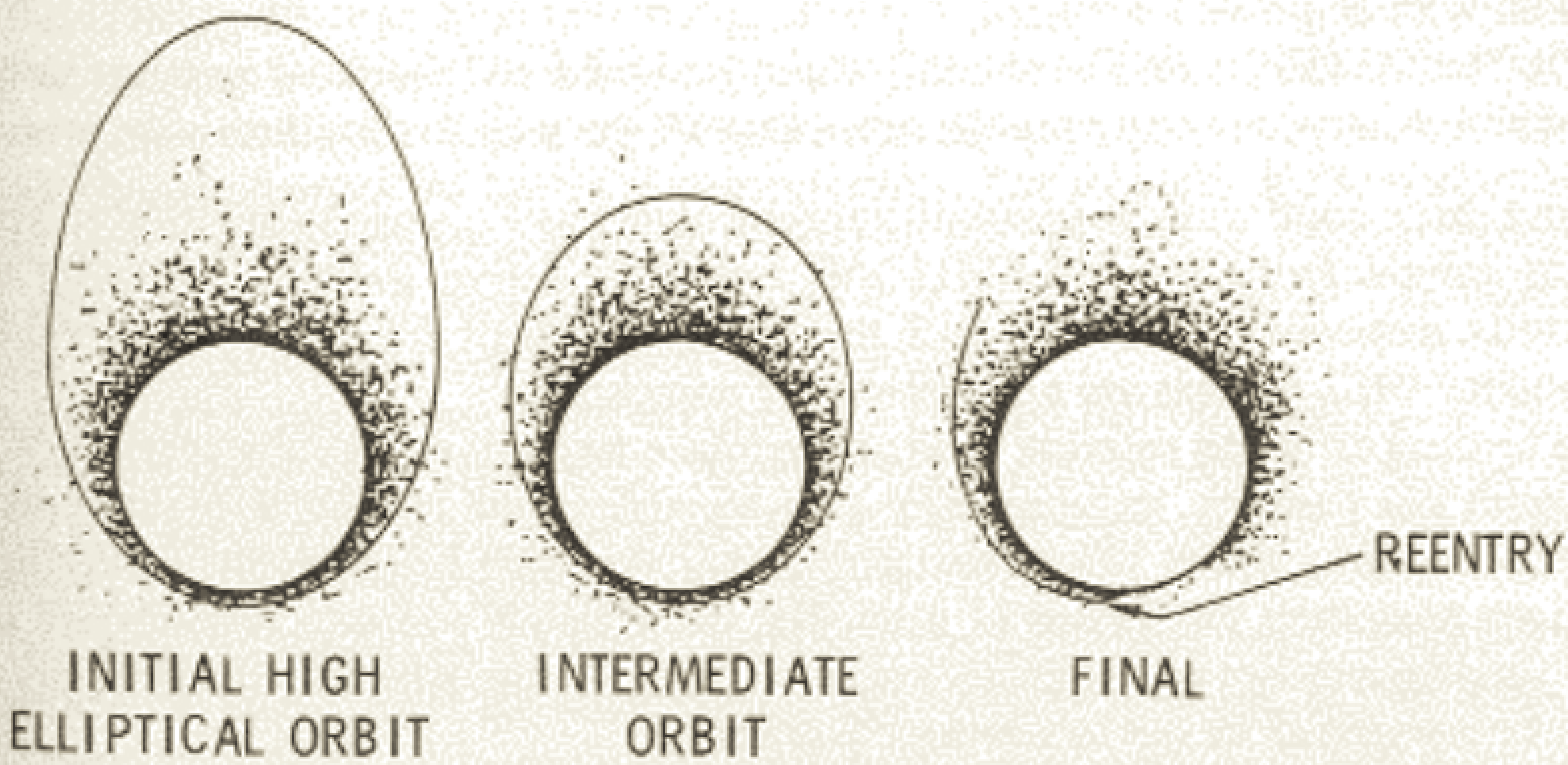
- Solar Sails
  - Sailing on the solar wind
- Can we use it in a cube?
  - Sail to the…?



Atmospheric Drag

Drag Coefficient

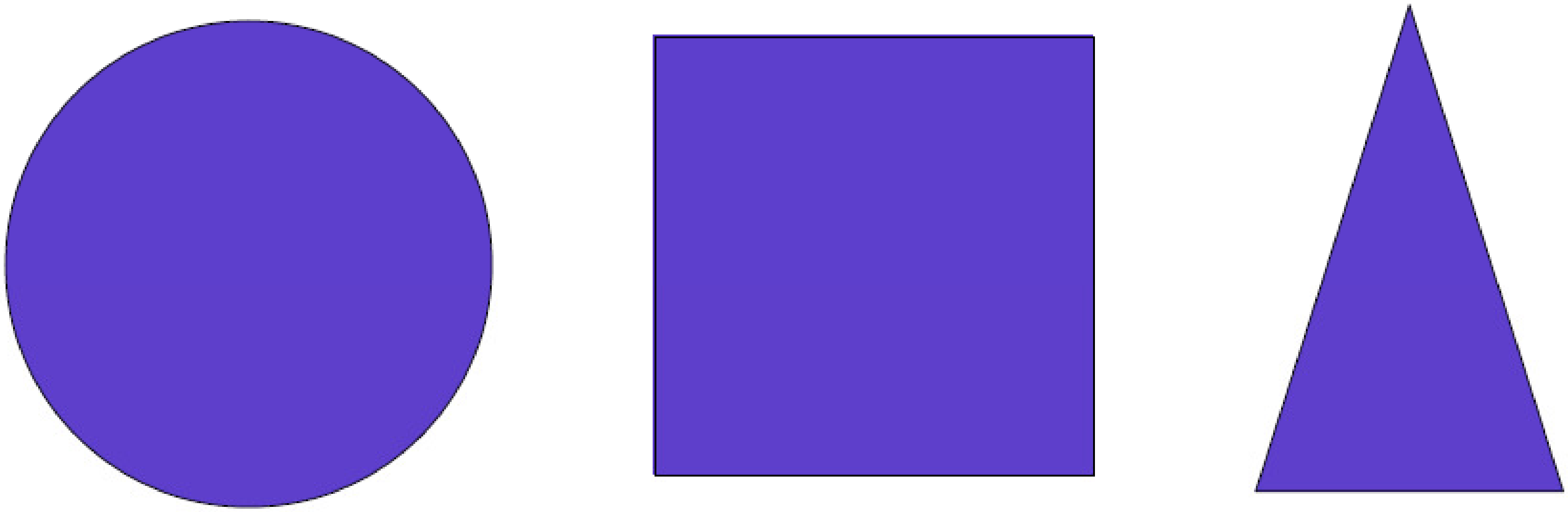
$$a_D = -\frac{1}{2}\rho(C_D A/m)V^2 \tag{6.21}$$



Ballistic Coefficient

Ballistic Coefficient

Bc = K (Mass/Cross Sectional Area)



How do they go through the atmosphere?

Which stays in orbit longer – a bowling ball or a balloon of the same size?

The Earth is not a Sphere!

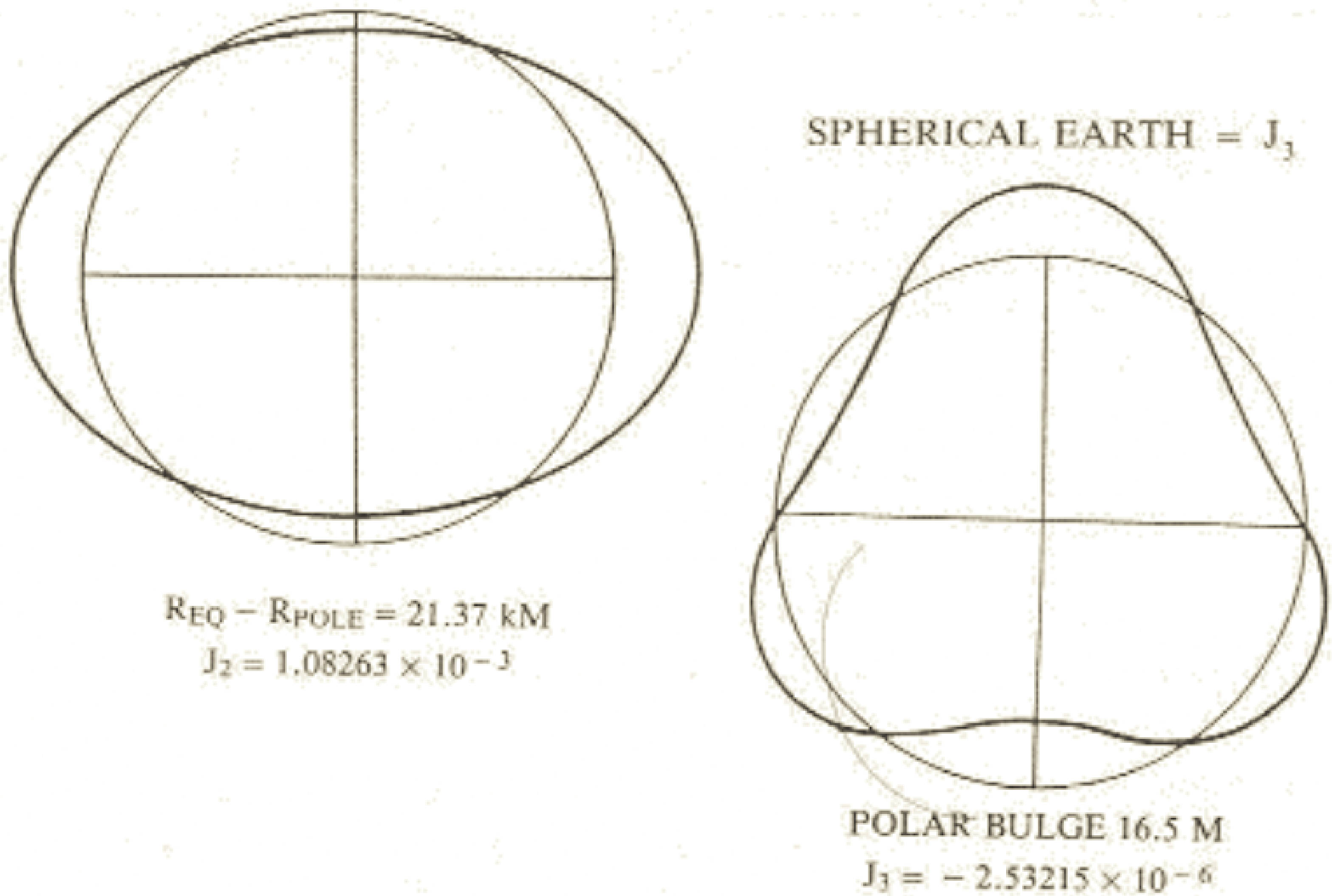
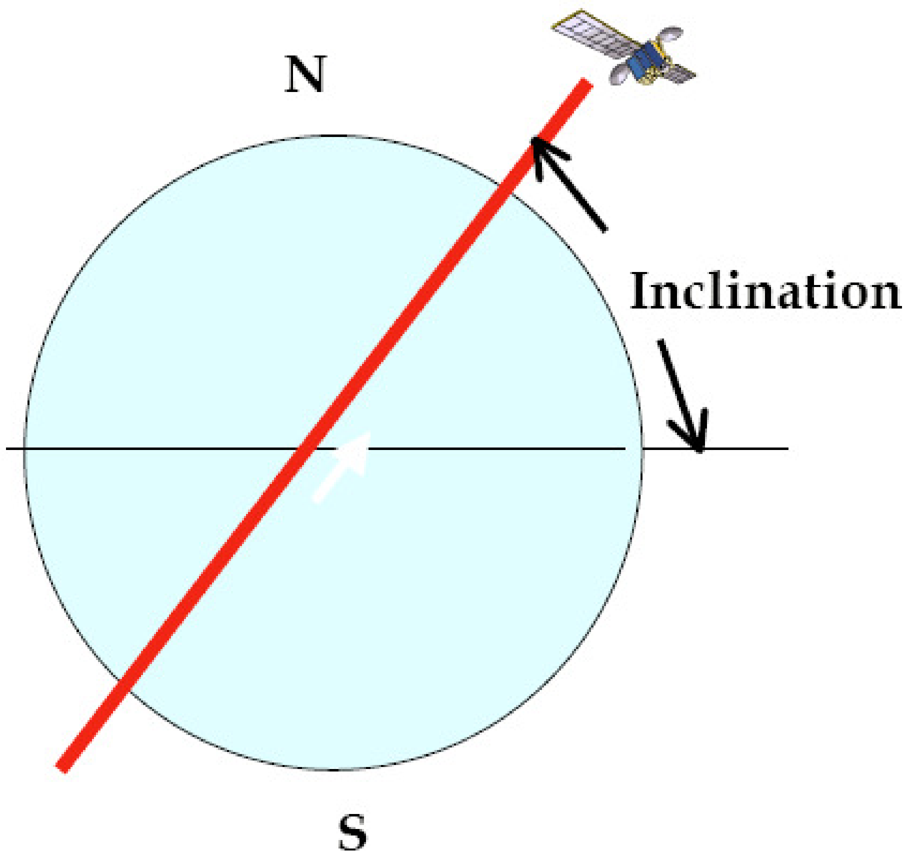
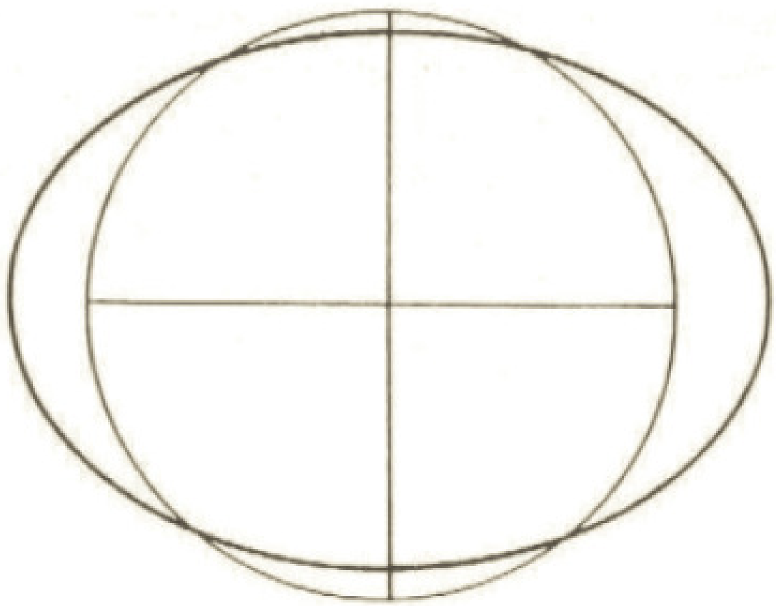


Fig. 11.21 Shape of the Earth.

Non-Spherical Effect





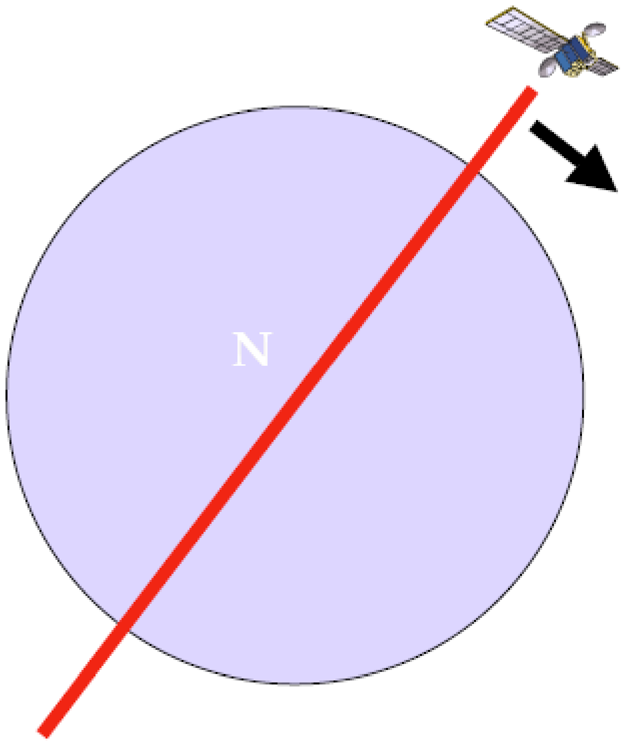
$R_{EQ} - R_{POLE} = 21.37$ kM

$J_2 = 1.08263 \times 10^{-3}$

What is the effect of this?

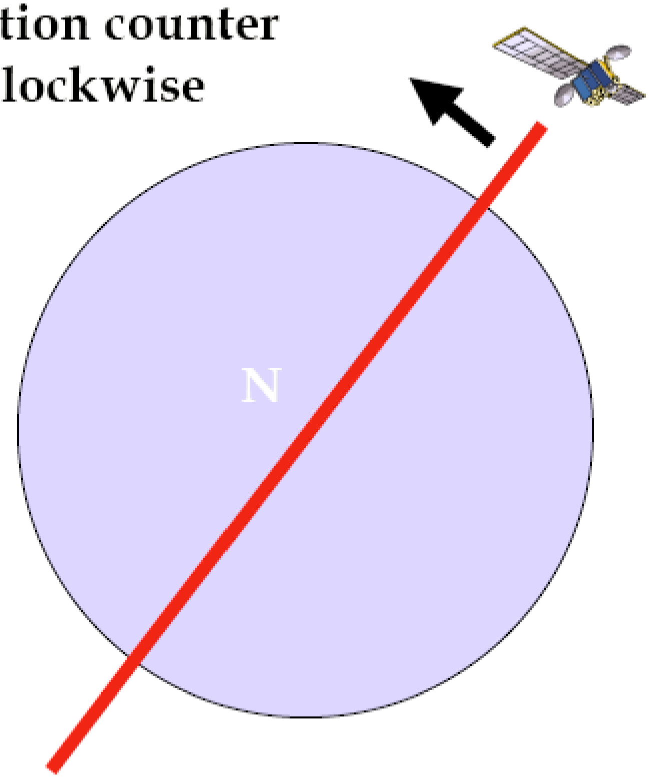
Inclination and J_2

Oblatness causes rotation clockwise



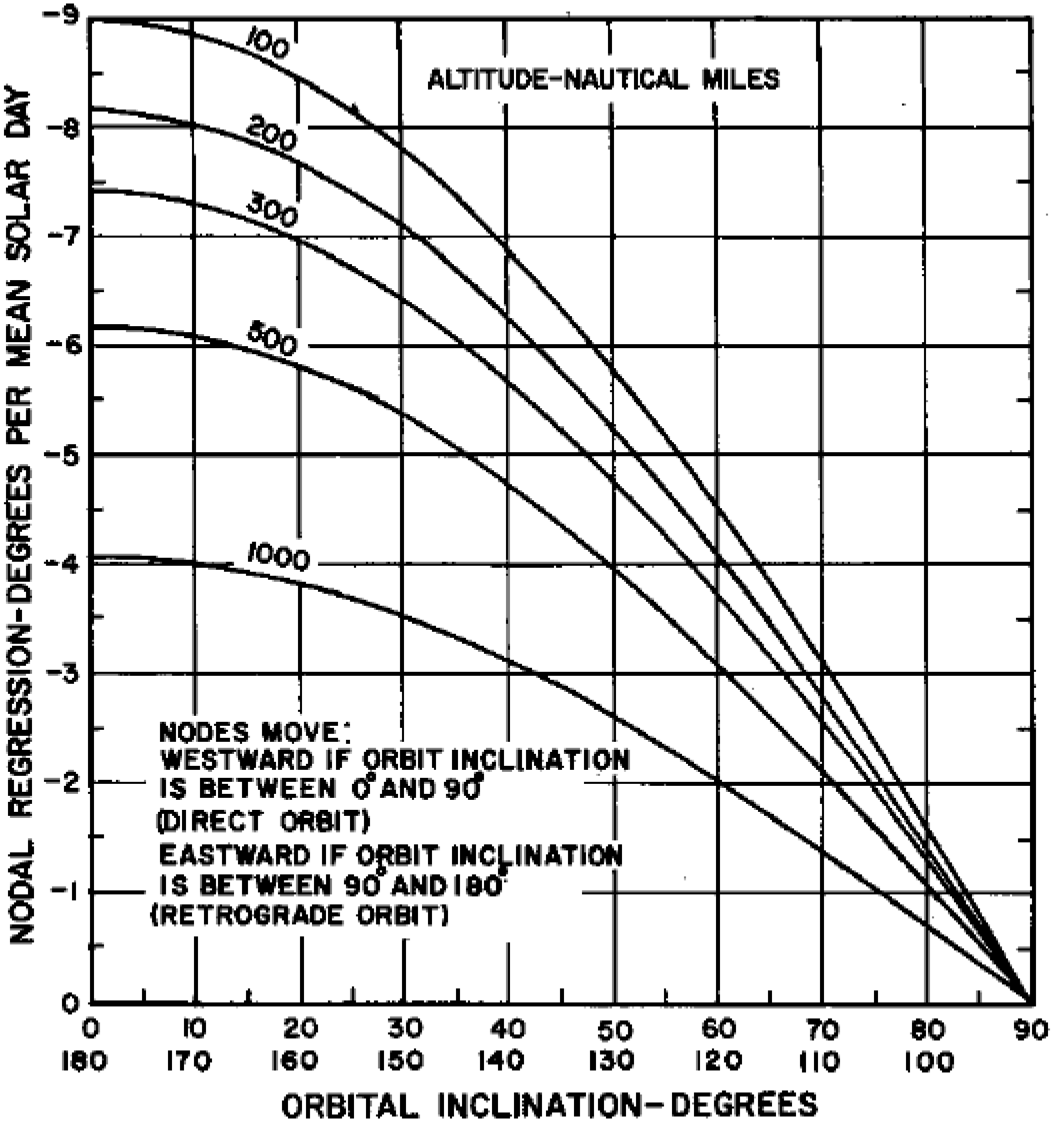
Prograde Orbit
$I < 90_o$

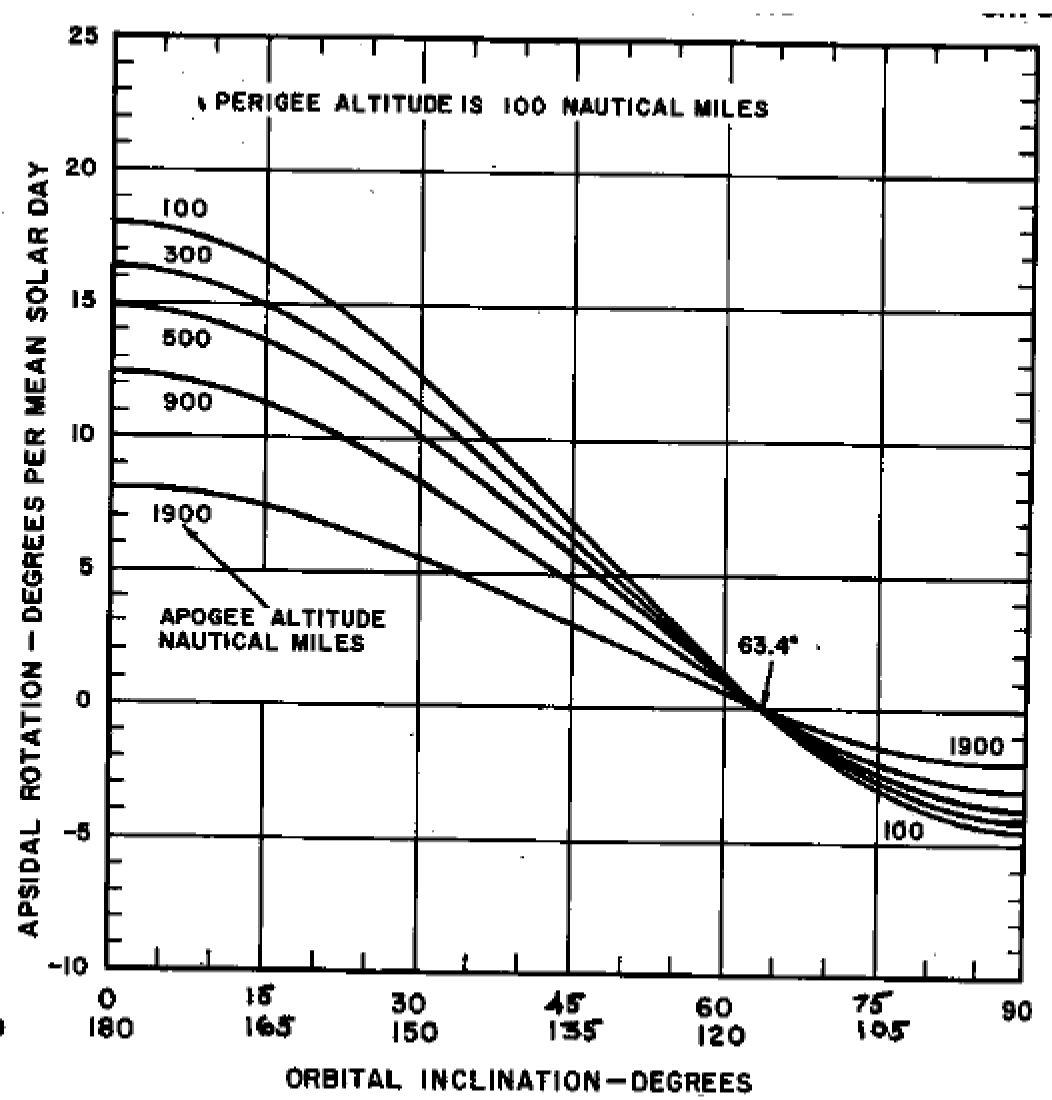
Oblatness causes rotation counter clockwise



Retrograde Orbit
$I > 90_o$

J_2 Effects


ALTITUDE-NAUTICAL MILES
100
200
300
500
1000
NODES MOVE:
WESTWARD IF ORBIT INCLINATION IS BETWEEN 0° AND 90° (DIRECT ORBIT)
EASTWARD IF ORBIT INCLINATION IS BETWEEN 90° AND 180° (RETROGRADE ORBIT)
NODAL REGRESSION-DEGREES PER MEAN SOLAR DAY
ORBITAL INCLINATION-DEGREES


PERIGEE ALTITUDE IS 100 NAUTICAL MILES
100
300
500
900
1900
APOGEE ALTITUDE NAUTICAL MILES
63.4°
APSIDAL ROTATION – DEGREES PER MEAN SOLAR DAY
ORBITAL INCLINATION – DEGREES

Orbital Debris

Orbital Debris

Importance of Orbits to a Mission

- How does the orbit affect mission design?
  - Thermal issues
  - Ground tracking, satellite visibility
    - Link budget
  - Power generation
  - Navigation and tracking
    - Sun sensor visibility, eclipse times
  - Propulsion, orbit maintenance
  - Payload concerns
  - Radiation amounts
  - Launch site and providers
    - Cost of the launch
    - Are you the primary?
    - Range safety
- When should you start analyzing orbits to satisfy mission requirements?

Software and Links

- Keplerian elements online
  - http://www.amsat.org/amsat/keps/menu.html
  - http://celestrak.com/
- Tracking Software
  - STK
  - Linux
    - Predict
    - Sattrack
  - Windows
    - Nova
- Websites
  - Heavens-above.com

AA236 Guidelines

- Design for worse case orbits
  - We generally don’t know what orbit we’ll get
  - Consider worse case for power, thermal, and communication
  - Rule out orbits?
- Possible delta-V capability
  - Cold gas thrusters
  - Electric propulsion
  - Solar Sail
  - What can we do with it?