Problem 8-1

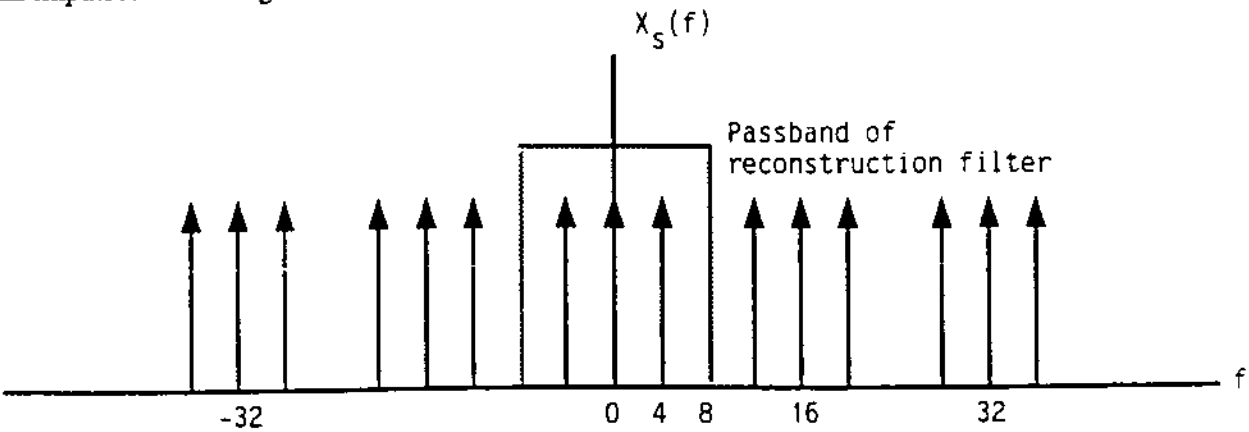
With

$$x(t) = 4 + 8\cos 8\pi t$$

and

$$X(f) = 4\delta(f) + 4\delta(f-4) + 4\delta(f+4)$$

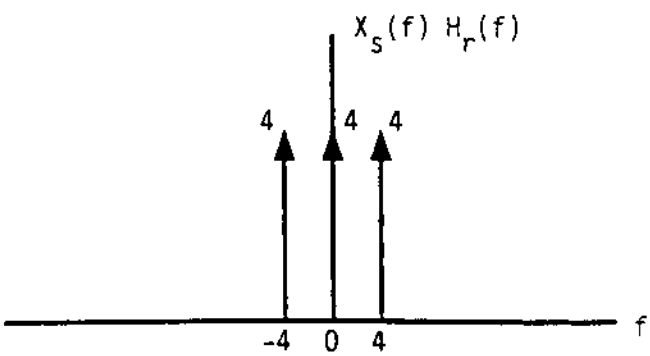
the specturm of the sampled signal appears as shown below. For a sampling frequency of 16 Hz all impulses have wieght 64.



The reconstruction filter has the transfer function

$$H_r(f) = \begin{cases} \dfrac{1}{16}, & |f| \le 8 \\ 0, & \text{otherwise} \end{cases}$$

The spectrum at the output of the reconstruction filter is



Clearly

$$X_s(f)\, H_r(f) = X(f)$$

which corresponds exactly to the time domain signal $x(t)$.

Problem 8-3

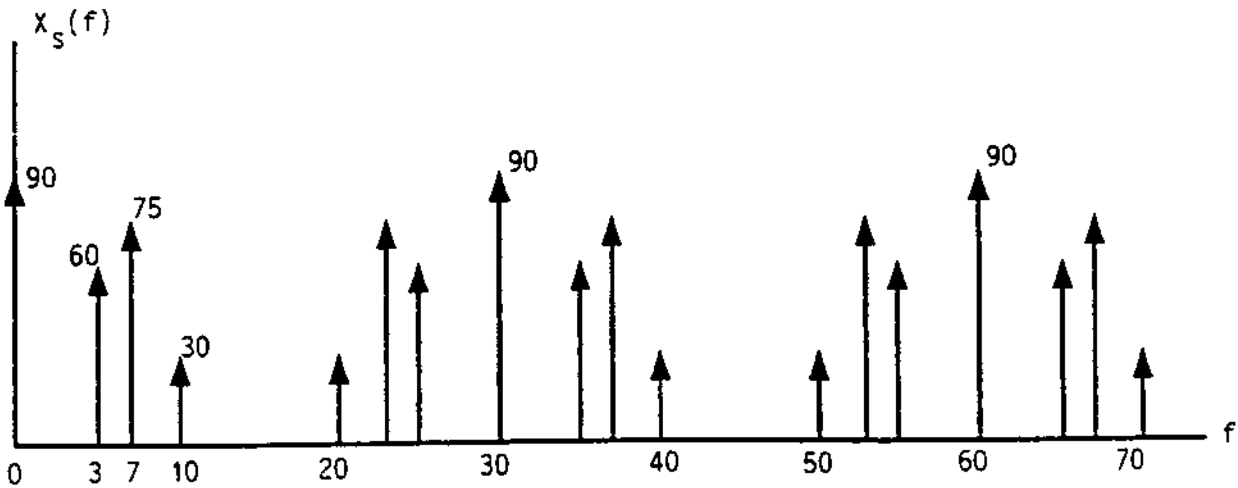
For

$$x(t) = 3 + 4\cos 10\pi t + 5\cos 14\pi t + 2\cos 20\pi t$$

we have

$$X(f) = 3\delta(f) + 2\delta(f+5) + 2\delta(f-5) + \frac{5}{2}\delta(f+7) + \frac{5}{2}\delta(f-7) + \delta(f+10) + \delta(f-10)$$

The dc component and the positive-frequency portion of the spectrum of the sampled signal is shown below for $f \leq 70$.



The original signal, $x(t)$, can be reconstructed from the sampled signal, $x_s(t)$, using a lowpass filter having the transfer function

$$H_r(f = \begin{cases} \frac{1}{30}, & |f| < 15 \\ 0, & \text{otherwise} \end{cases}$$

Problem 8-19

(a) The z-transform of $x(nT) = \left(\frac{1}{5}\right)^n u(n)$ is

$$X(z) = \sum_{n=0}^{\infty}\left(\frac{1}{5}\right)^n z^{-n} = \sum_{n=0}^{\infty}\left(\frac{1}{5}z^{-1}\right)^n$$

which is

$$X(z) = \frac{1}{1-\frac{1}{5}z^{-1}}, \quad |z| > \frac{1}{5}$$

(b) The z-transform of $x(nT) = \left(-\frac{1}{5}\right)^n u(n)$ is

$$X(z) = \sum_{n=0}^{\infty}\left(-\frac{1}{5}\right)^n z^{-n} = \sum_{n=0}^{\infty}\left(-\frac{1}{5}z^{-1}\right)^n$$

which is

$$X(z) = \frac{1}{1+\frac{1}{5}z^{-1}}, \quad |z| > \frac{1}{5}$$

(c) The z-transform of $x(nT) = u(n) + \left(\frac{3}{4}\right)^n u(n-4)$ is

$$X(z) = \sum_{n=0}^{\infty} z^{-n} + \sum_{n=4}^{\infty}\left(\frac{3}{4}\right)^n z^{-n}$$

which can be written

$$X(z) = \sum_{n=0}^{\infty}\left(z^{-1}\right)^n + \sum_{n=4}^{\infty}\left(\frac{3}{4}z^{-1}\right)^n$$

With the change of index $k = n-4$ in the second sum, this becomes

$$X(z) = \frac{1}{1-z^{-1}} + \sum_{k=0}^{\infty}\left(\frac{3}{4}z^{-1}\right)^{k+4}$$

This gives

$$X(z) = \frac{1}{1-z^{-1}} + \frac{\left(\frac{3}{4}z^{-1}\right)^4}{1-\frac{3}{4}z^{-1}}, \quad |z| > 1$$

The region of convergence results by recognizing that the first term exists for $|z| > 1$ and the second term exists for $|z| > 3/4$. The z-transform exists only when both terms are defined. This requires that $|z| > 1$.

(d) The z-transform of $x(nT) = 2u(n) - 2u(n-8)$ is

$$X(z) = \sum_{n=0}^{\infty} 2z^{-n} - \sum_{n=8}^{\infty} 2z^{-n}$$

Letting $k = n - 8$ in the second sum is

$$X(z) = \sum_{n=0}^{\infty} 2\left(z^{-1}\right)^{n} - \sum_{k=0}^{\infty} 2\left(z^{-1}\right)^{k+8}$$

This gives

$$X(z) = \frac{2}{1 - z^{-1}} - \frac{2z^{-8}}{1 - z^{-1}}$$

or

$$X(z) = \frac{2\left(1 - z^{-8}\right)}{1 - z^{-1}}, \quad |z| \neq 0$$

The above expression gives $X(z)$ in closed form. The region of convergence is justified by recognizing that $X(z)$ can also be written in terms of the finite sum.

$$X(z) = 2 + 2z^{-1} + 2z^{-2} + 2z^{-3} + 2z^{-4} + 2z^{-5} + 2z^{-6} + 2z^{-7}$$

The terms in the above series of the form z^{-k} for $k > 0$ are clearly defined for all z except $z = 0$.

(d) This part of the problem is solved using the same approach as was used in part (c). The result is the following MATLAB script.

```
syms n z                    % Make n and z symbolic
xn = 2;                     % Define x(n) for first term
x1z = ztrans(xn,n,z);       % z-transform
x2z = x1z*z^(-8);           % Define second term
xz = x1z+x2z;               % Combine terms
xz                          % Display result

xz =

2*z/(z-1)+2/z^7/(z-1)
```

We see that the result is in agreement with Problem 8-19(d).

Problem 8-21

(a) The z-transform of $x(nT) = \left(\frac{2}{3}\right)^n u(n-4)$ is

$$X(z) = \sum_{n=4}^{\infty} \left(\frac{2}{3}\right)^n z^{-n} = \sum_{n=4}^{\infty} \left(\frac{2}{3} z^{-1}\right)^n$$

With the change of index $k = n-4$ we have

$$X(z) = \sum_{k=0}^{\infty} \left(\frac{2}{3} z^{-1}\right)^{k+4}$$

Thus

$$X(z) = \frac{\left(\frac{2}{3}\right)^4 z^{-4}}{1 - \frac{2}{3} z^{-1}}, \quad |z| > \frac{2}{3}$$

(b) The z-transform of $x(nT) = \left(\frac{2}{3}\right)^{n-4} u(n-4)$ is

$$X(z) = \sum_{n=4}^{\infty} \left(\frac{2}{3}\right)^{n-4} z^{-n}$$

We once again use the change of index $k = n-4$. This gives

$$X(z) = \sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k z^{-(k+4)} = z^{-4} \sum_{k=0}^{\infty} \left(\frac{2}{3} z^{-1}\right)^k$$

Thus $X(z) = \frac{z^{-4}}{1 - \frac{2}{3} z^{-1}}, \quad |z| > 2/3$

Problem 3-29

The long division is shown below:

$$
\begin{array}{r}
1+0.30z^{-1}-0.21z^{-2}+0.347z^{-3}+0.3171z^{-4} \\
1-0.30z^{-1}+0.30z^{-2}-0.50z^{-3}\overline{)\,1\qquad\qquad\qquad\qquad\qquad\qquad\qquad} \\
\underline{1-0.30z^{-1}+0.30z^{-2}-0.50z^{-3}\qquad\qquad} \\
0.30z^{-1}-0.30z^{-2}+0.50z^{-3}\qquad\qquad \\
\underline{0.30z^{-1}-0.09z^{-2}+0.09z^{-3}-0.15z^{-4}} \\
-0.21z^{-2}+0.41z^{-3}+0.15z^{-4} \\
\underline{-0.21z^{-2}+0.063z^{-3}-0.063z^{-4}} \\
0.347z^{-3}+0.213z^{-4} \\
\underline{0.347z^{-3}-0.1041z^{-4}} \\
0.3171z^{-4}
\end{array}
$$

Therefore

$$x(0)=1$$
$$x(T)=0.3$$
$$x(2T)=-0.21$$
$$x(3T)=0.347$$
$$x(4T)=0.3171$$

(b) Since

$$X(z)=\frac{1-0.72^{-1}}{\left(1+0.5z^{-1}\right)^2}=\frac{1-0.72^{-1}}{1+z^{-1}+0.25z^{-2}}$$

we write the long division as

$$
\begin{array}{r}
1-1.70z^{-1}+1.45z^{-2}-1.025z^{-3}+0.6625z^{-4} \\
1+z^{-1}+0.25z^{-2}\overline{)\,1-0.70z^{-1}\qquad\qquad\qquad\qquad\qquad\qquad} \\
\underline{1+z^{-1}+0.25z^{-2}\qquad\qquad\qquad\qquad\qquad} \\
-1.70z^{-1}-0.25z^{-2}\qquad\qquad\qquad\qquad \\
\underline{-1.70z^{-1}-1.70z^{-2}-0.425z^{-3}\qquad\qquad} \\
1.45z^{-2}+0.425z^{-3}\qquad\qquad \\
\underline{1.45z^{-2}+1.45z^{-3}+0.3625z^{-4}} \\
-1.025z^{-3}-0.3625z^{-4} \\
\underline{-1.025z^{-3}-1.025z^{-4}} \\
0.6625z^{-4}
\end{array}
$$

Problem 8-33

(a) Since

$$X(z)=\frac{1}{\left(1-z^{-1}\right)\left(1+0.5z^{-1}\right)\left(1-0.2z^{-1}\right)}$$

we can write

$$\frac{X(z)}{z}=\frac{z^2}{(z-1)(z+0.5)(z-0.2)}=\frac{A}{z-1}+\frac{B}{z+0.5}+\frac{C}{z-0.2}$$

The value of A is

$$A=\frac{1}{1.5(0.8)}=\frac{2}{3}\frac{5}{4}=\frac{5}{6}$$

The value of B is

$$B=\frac{(-0.5)^2}{(-1.5)(-0.7)}=\frac{1}{3}\frac{5}{7}=\frac{5}{21}$$

The value of C is

$$C=\frac{(0.2)^2}{(-0.8)(0.7)}=-\frac{1}{4}\frac{2}{7}=-\frac{1}{14}$$

Thus

$$X(z)=\frac{5}{6}\frac{1}{1-z^{-1}}+\frac{5}{21}\frac{1}{1+0.5z^{-1}}-\frac{1}{14}\frac{1}{1-0.2z^{-1}}$$

so that

$$x(nT)=\left[\frac{5}{6}+\frac{5}{21}(-0.5)^n-\frac{1}{14}(0.2)^n\right]u(n)$$

(b) For

$$X(z)=\frac{1-0.5z^{-1}}{(1-z^{-1})(1+0.5z^{-1})(1-0.2z^{-1})}$$

the partial fraction expansion is

$$\frac{X(z)}{z}=\frac{z(z-0.5)}{(z-1)(z+0.5)(z-0.2)}=\frac{A}{z-1}+\frac{B}{z+0.5}+\frac{C}{z-0.2}$$

The value of A is

$$A=\frac{0.5}{(1.5)(0.8)}=\frac{1}{3}\frac{5}{4}=\frac{5}{12}$$

The value of B is

$$B=\frac{-0.5(-1)}{(-1.5)(-0.7)}=\frac{1}{3}\frac{10}{7}=\frac{10}{21}$$

The value of C is

$$C = \frac{0.2(-0.3)}{(-0.8)(0.7)} = \frac{1}{4}\frac{3}{7} = \frac{3}{28}$$

Thus

$$X(z) = \frac{5}{12}\frac{1}{1-z^{-1}} + \frac{10}{21}\frac{1}{1+0.5z^{-1}} + \frac{3}{28}\frac{1}{1-0.2z^{-1}}$$

which yields

$$x(nT) = \left[\frac{5}{12} + \frac{10}{21}(-0.5)^n + \frac{3}{28}(0.2)^n\right]u(n)$$

Problem 8-50

As with the previous problem, the MATLAB command **conv** is easily applied to this problem. For the sample values given the result is

```
h =

     0     1     2     2     1     0    -1    -1    -1     0     0

x =

     1     1     2     2     1     0    -1    -1     0     0     0

y =

  Columns 1 through 12

     0     1     3     6     9    10     7     1    -6    -9    -8    -4

  Columns 13 through 21

     0     2     2     1     0     0     0     0     0
```

Once again, see Problem 8-51 for the figure.