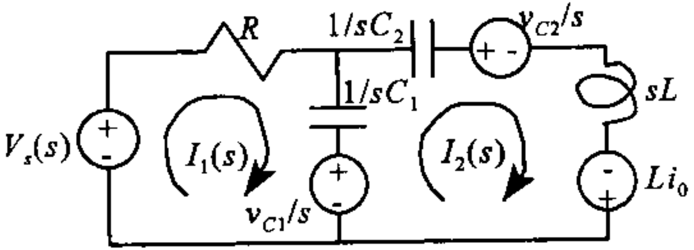
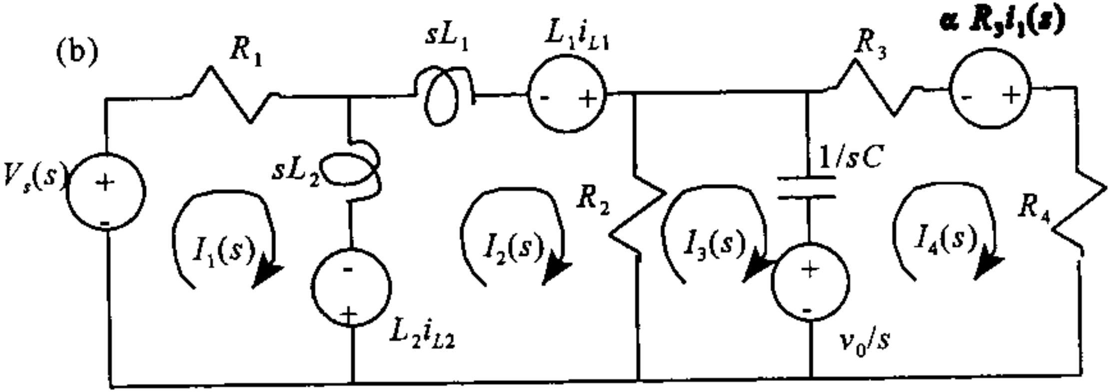
CHAPTER 6

Problem 6-1

(a)

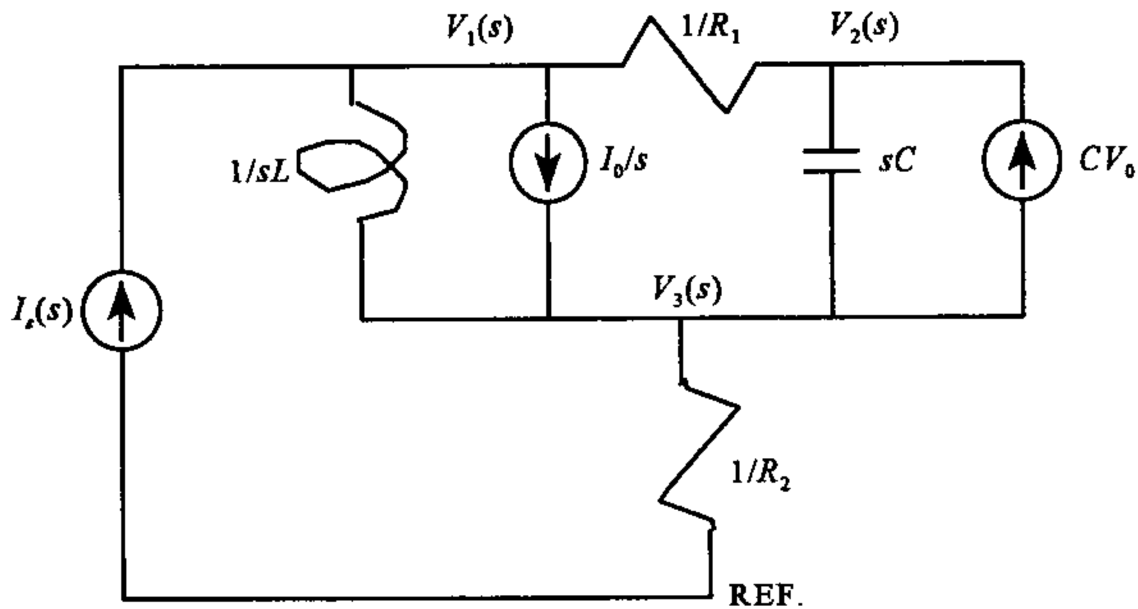


(b)



Problem 6-7

The Laplace transform equivalent circuit with admittances and current sources for initial conditions is shown below. This form is appropriate for writing KCL equations.



By inspection, the KCL equations written at each node are given by the matrix equation

$$\begin{bmatrix} \frac{1}{sC}+\frac{1}{R_1} & -\frac{1}{R_1} & -\frac{1}{sL} \\ -\frac{1}{R_1} & \frac{1}{R_1}+sC & -sC \\ -\frac{1}{sL} & -sC & \frac{1}{sL}+\frac{1}{R_2}+sC \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} = \begin{bmatrix} I_s(s)-\frac{I_0}{s} \\ CV_0 \\ \frac{I_0}{s}-CV_0 \end{bmatrix}$$

(b) Replace the initial condition generators with series voltage sources. Let the voltage across the current source be $V_s(s)$. Let the mesh currents be $I_1(s)$ and $I_2(s)$. Note that $I_1(s) = I_s(s)$, so $I_1(s)$ is known. Thus the first KVL equation determines $V_s(s)$. The KVL equations are

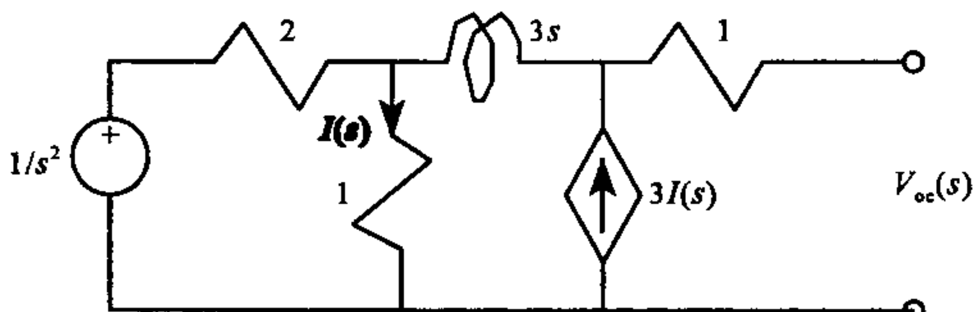
$$\begin{bmatrix} sL+R_2 & -sL \\ -sL & sL+R_1+\frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} V_s(s)+LI_0 \\ -LI_0-\frac{V_0}{s} \end{bmatrix}$$

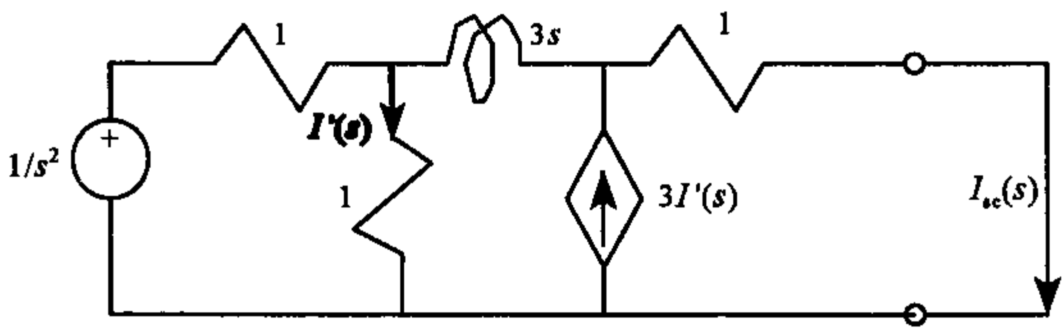
Problem 6-10

(a) Use the fact that

$$Z_{eq}(s) = V_{oc}(s)/I_{sc}(s)$$

The first circuit below may be used to find $V_{oc}(s)$ and the second circuit to find $I_{sc}(s)$.





The KCL equations for the first circuit are

$$\frac{1}{2}\left(V_a - \frac{1}{s^2}\right) + I + \frac{V_a - V_b}{3s} = 0,\ V_a = I$$

$$\frac{V_b - V_a}{3s} = 3I = 3V_a$$

Solve the second equation for V_b in terms of V_a:

$$V_b = (9s + 1)V_a$$

Substitute into the first equation to get

$$V_b = V_{oc} = (9s + 1)V_a = \frac{3(9s + 1)}{s^2}$$

Now work with the second circuit to get the short circuit current. Note that all voltages and currents are different than for the first circuit. The node voltage equations are

$$\frac{1}{2}\left(V_a - \frac{1}{s^2}\right) + I' + \frac{V_a - V_b}{3s} = 0,\ V_a = I'$$

$$\frac{V_b - V_a}{3s} - 3I' + V_b = 0$$

Solve the second for V_a in terms of V_b:

$$V_a = \frac{1 + 3s}{1 + 9s}V_b$$

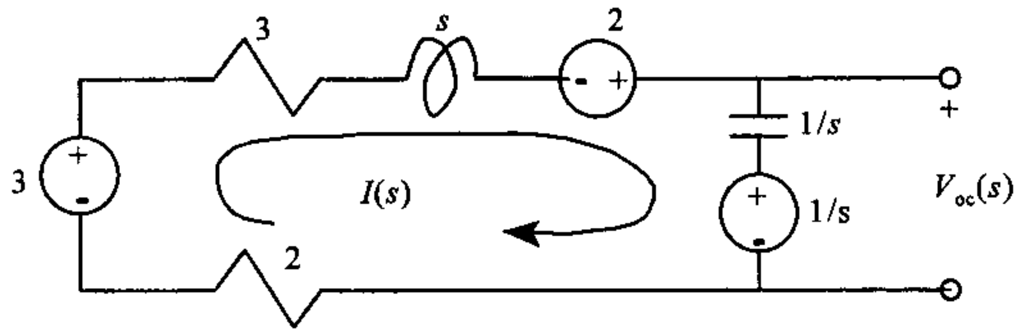
Substitute into the first equation and solve for V_b. Note that $I_{sc} = V_b/1$:

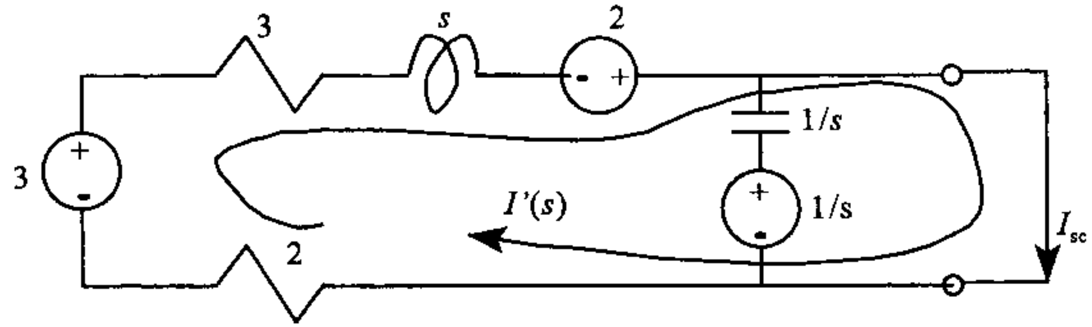
$$I_{sc} = \frac{V_b}{1} = \frac{9s + 1}{9s - 1}\frac{1}{2s^2}$$

Divide V_{oc} by I_{sc} to get Z_T. These results may then be used in the circuits of Fig. 6-9.

$$Z_T = 6(9s - 1)$$

(b) First do a Thevenin-to-Norton equivalent of the left-hand current source and parallel resistor. The resulting open-circuit and short-circuit Laplace-transform euqivalent circuits are shown below:





Use the top circuit to get open circuit voltage:

$$\left(2 + 3 + 5 + \frac{1}{s}\right) I(s) = 3 + 2 - \frac{1}{s} \text{ or } I(s) = \frac{5s - 1}{s^2 + 5s + 1}$$

Use Ohm's law and KVL to get the open circuit voltage:

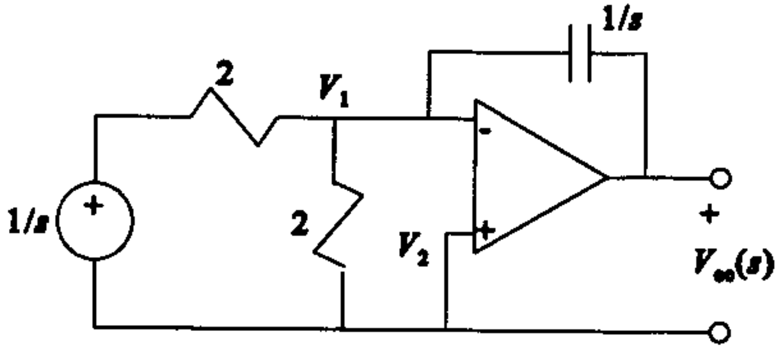
$$V_{oc}(s) = I(s)/s + 1/s = (s + 10)/(s^2 + 5s + 1)$$

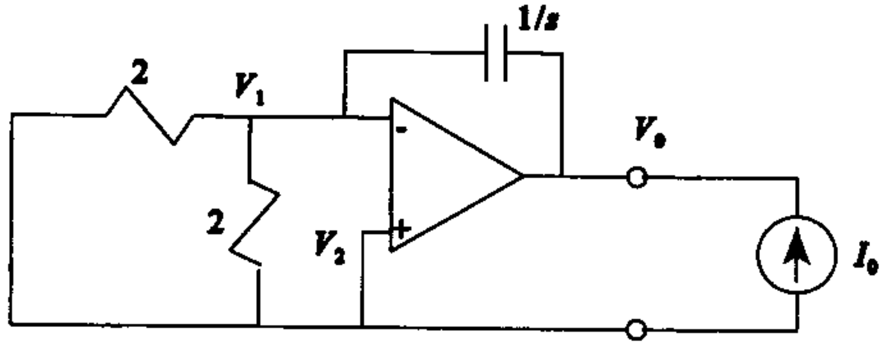
Use the lower circuit to get the short circuit current:

$$(2 + 3 + s)I'(s) = 3 + 2 \text{ or } I'(s) = I_{sc}(s) = 5/(s + 5)$$

The Thevenin equivalent impedance is $Z_T(s) = V_{oc}/I_{sc} = (s + 5)(s + 10)/[5(s^2 + 5s + 1)$. These results may then be used in the circuits of Fig. 6-9.

(c) Find the open circuit voltage for this circuit and then use the test source at the output method. The circuits shown below are gemane.





For the top circuit, KCL at node 1 gives

$$\frac{V_1 - 1/s}{2} + \frac{V_1}{2} + \frac{V_1 - V_{oc}}{1/s} = 0$$

By the high gain/infinite input impedance properties of the operational amplifier $V_1 = V_2 = 0$, so

$$V_{oc}(s) = -\frac{1}{2s^2}$$

To find the equivalent impedance, use the lower circuit, remembering that the voltages are different than for the first circuit. The KCL equation at the input is

$$\frac{V_1}{2} + \frac{V_2}{2} + \frac{V_1 - V_2}{1/s} = 0$$

Again use the fact that the voltages at the two op-amp inputs are approximately equal and 0. From this and from the above equation, we deduce that $V_0 = 0$ for arbitrary I_0. Therefore, $Z_T = 0$ and the Thevenin equivalent circuit consists solely of a voltage source of value V_{oc}. There is no Norton equivalent.

Problem 6-16

(a) With the open circuit, $I_2 = 0$ and

$$Y_{21}V_1 = -Y_{22}V_2$$

from the equation defining the two-port parameters (see the statement for Problem 6-15). Thus,

$$H(s) = \frac{V_2}{V_1} = -\frac{Y_{21}}{Y_{22}}$$

But

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \quad \text{and} \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

The former equation means that we short circuit the input and apply V_2 to the output; the latter equation means that we short circuit the output and apply V_1 to the input. Using the former equation (i.e., short circuiting the input and driving the output by V_2), the nodal equations become

$$sCV_2 + sC(V_3 - V_2) + 2V_3/R = 0 \ (1)$$
$$V_4/R + (V_4 - V_2)/R + 2CsV_4 = 0 \ (2)$$
$$sC(V_2 - V_1) + (V_2 - V_4)/R = I_2 \ (3)$$

where the definitions for V_3 and V_4 given in the problem statement are used. Solve (1) and (2) for V_3 and V_4 and substitute into (3). The result is

$$V_2\left[\frac{1}{R} + sC - \frac{(sC)^2}{2/R + 2Cs} - \frac{(1/R)^2}{2/R + 2Cs}\right] = I_2$$

Solve for the ratio I_2/V_2 (recall that this is for $V_1 = 0$):

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1/R^2 + 4Cs/R + (sC)^2}{2/R + 2Cs} \quad \text{(A)}$$

Now assume the output is short circuited with the input being driven by V_1. The nodal equations are

$$sC(V_3 - V_1) + sCV_3 + 2V_3/R = 0 \ (4)$$
$$(V_4 - V_1)/R + 2CsV_4 + V_4/R = 0 \ (5)$$
$$sCV_3 + V_4/R = I_2 \ (6)$$

Solve (4) and (5) for V_3 and V_4 in terms of V_1 and substitute into (6). The result is

$$I_2 = \left[\frac{(sC)^2}{2Cs + 2/R} + \frac{(1/R)^2}{2Cs + 2/R}\right]V_1$$

(Recall that this is for $V_2 = 0$.) Now solve for the ratio I_2/V_1:

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2 = 0} = \frac{(sC)^2 + 1/R^2}{2Cs + 2/R} \quad \text{(B)}$$

(b) Use (A) and (B) in $H(s) = -Y_{21}/Y_{22}$. This gives the transfer function

$$H(s) = \frac{s^2 + 1/(RC)^2}{s^2 + \frac{4}{RC}s + \frac{1}{(RC)^2}}$$

The zeros are at

$$z_{1,2} = \pm j(1/RC)$$

and the poles are given by

$$p_{1,2} = -\frac{2}{RC} \pm \sqrt{\frac{3}{(RC)^2}} = -\frac{3.73}{RC}, -\frac{0.27}{RC}$$

(c) The amplitude response is given by

$$|H(j\omega)| = \frac{\left|\omega^2 - \frac{1}{(RC)^2}\right|}{\sqrt{\left(\frac{1}{(RC)^2} - \omega^2\right)^2 + \frac{16\omega^2}{(RC)^2}}}$$

Note that $H(0) = 1$. We want the amplitude response at frequency ω_c to be 0.707, or the amplitude response squared to be 0.5. That is,

$$\frac{1}{2} = \frac{\left|\omega_c^2 - \frac{1}{(RC)^2}\right|}{\sqrt{\left(\frac{1}{(RC)^2} - \omega_c^2\right)^2 + \frac{16\omega_c^2}{(RC)^2}}}$$

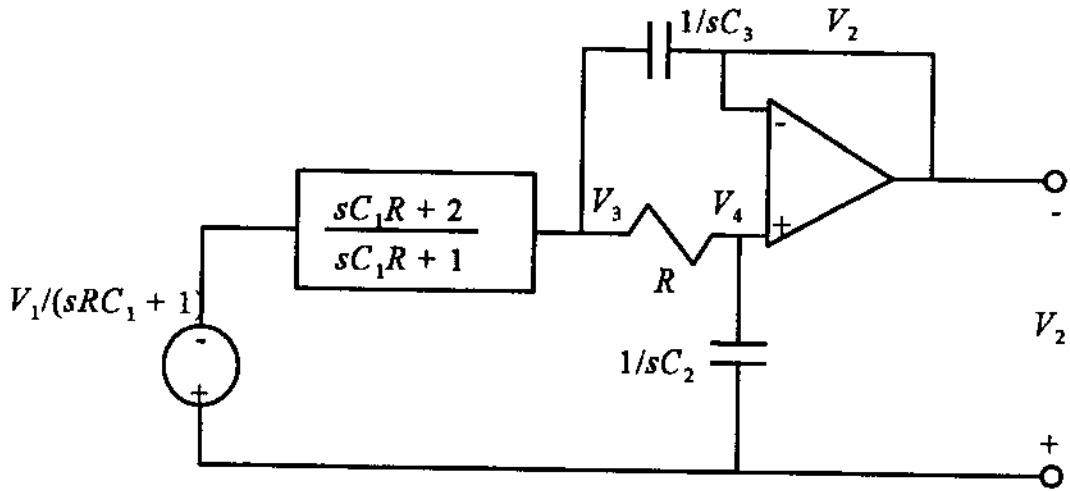
The solutions of this equation are

$$\omega_c = \frac{2}{RC} \pm \frac{\sqrt{5}}{RC} \quad \text{or} \quad (\omega_c RC)^2 = (\sqrt{5} \pm 2)^2$$

(d) We want the zero at $\omega_z = (2\pi)(60) = 1/RC$. For $C = 10^{-6}$ F, we get $R = 10^6/120\pi = 2.65\ \text{k}\Omega$.

Problem 6-17

(a) Do a Thevenin-to-Norton equivalent of the input, and a parallel combination of R and $1/sC_1$; then combine $R \| 1/sC_1$ and the second R in series to finally end up with the Laplace-equivalent circuit shown below:



Write a KCL equation at node 3:

$$\frac{V_3 - V_1/(sC_1R + 1)}{(sC_1R + 2)/(sC_1R + 1/R)} + \frac{V_3 - V_2}{R} + sC_3(V_3 - V_2) = 0 \tag{1}$$