Problem 3-4

This an even square wave with zero average value. Because of the zero average value, $a_0 = 0$. Because of the eveness, $b_n = 0$ for all n. To find a_n, evaluate (3-15):

$$a_n = \frac{2}{T_0}\left[\int_{-T_0/2}^{-T_0/4} -A\cos(n\omega_0 t)\,dt + \int_{-T_0/4}^{T_0/4} A\cos(n\omega_0 t)\,dt + \int_{T_0/4}^{T_0/2} -A\cos(n\omega_0 t)\,dt\right]$$

$$= \frac{2A}{T_0}\left[-\frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_{-T_0/2}^{-T_0/4} + \frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_{-T_0/4}^{T_0/4} - \frac{\sin(n\omega_0 t)}{n\omega_0}\Bigg|_{T_0/4}^{T_0/2}\right] = \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right), \text{ using } \omega_0 = \frac{2\pi}{T_0}$$

$$= \begin{cases} 0, & n \text{ even} \\ (-1)^{(n-1)/2}\dfrac{4A}{n\pi}, & n \text{ odd} \end{cases}$$

Problem 3-8

(a) By trigonometric identities (see Appendix F), we may write $x(t)$ as

$$x(t) = [1 - \cos(5000\pi t)]\cos(20000\pi t) = \cos(20000\pi t) - 1/2\cos(15000\pi t) - 1/2\cos(25000\pi t)$$

Using the fact that the power in a sinusoid is ½ the square of its amplitude, and the fact that we can add powers of the separate harmonics of a harmonic sum of sinusoids, we find that (b

$$P_{\text{ave},\,x(t)} = \frac{1}{2}(1)^2 + \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{3}{4}\ \text{W}$$

(b) From (a) it is seen that the signal $x(t)$ consists of components with frequencies 7500, 10000, and 12500 Hz. Only the first two are passed by the telephone system, so the output power is

$$P_{\text{ave},\,y(t)} = \frac{1}{2}(1)^2 + \frac{1}{2}\left(\frac{1}{2}\right)^2 = \frac{5}{8}\ \text{W};\ \text{Ratio} = \frac{5}{6}$$

Problem 3-13

(a) Using Euler's theorem, the exponential Fourier series is

$$x_a(t) = \left[\frac{e^{j20\pi t} + e^{-j20\pi t}}{2}\right]^2 \frac{e^{j10\pi t} - e^{-j10\pi t}}{2j}$$

$$= \frac{1}{8j}\left[e^{j40\pi t} + 2 + e^{-j40\pi t}\right]\left[e^{j10\pi t} - e^{-j10\pi t}\right]$$

$$= -\frac{1}{8j}e^{-j50\pi t} + \frac{1}{8j}e^{-j30\pi t} - \frac{1}{4j}e^{-j10\pi t} + \frac{1}{4j}e^{j10\pi t} - \frac{1}{8j}e^{j30\pi t} + \frac{1}{8j}e^{j50\pi t}$$

(b) A series of steps similar to those used in part (a) results in

$$x_b(t) = \left[\frac{e^{j30\pi t} - e^{-j30\pi t}}{2j}\right]^3 + 2\frac{e^{j25\pi t} + e^{-j25\pi t}}{2}$$

$$= \frac{j}{8}\left[-e^{-j90\pi t} + 3e^{-j30\pi t} - 3e^{j30\pi t} + e^{j90\pi t}\right] + e^{j25\pi t} + e^{-j25\pi t}$$

$$= -\frac{j}{8}e^{-j90\pi t} + j\frac{3}{8}e^{-j30\pi t} + e^{-j25\pi t} + e^{j25\pi t} - j\frac{3}{8}e^{j30\pi t} + \frac{j}{8}e^{j90\pi t}$$

(c) A series of steps used to those above gives

$$x_c(t) = \left[\frac{e^{j40\pi t} - e^{-j40\pi t}}{2j}\right]^2\left[\frac{e^{j20\pi t} + e^{-j20\pi t}}{2}\right]^2 + \frac{e^{j10\pi t} - e^{-j10\pi t}}{2j}\,\frac{e^{j5\pi t} - e^{-j5\pi t}}{2j}$$

$$= -\frac{1}{16}e^{-j120\pi t} - \frac{1}{8}e^{-j80\pi t} + \frac{1}{16}e^{-j40\pi t} - \frac{1}{4j}e^{-j15\pi t} - \frac{1}{4j}e^{-j5\pi t}$$

$$+ \frac{1}{4} + \frac{1}{4j}e^{j5\pi t} + \frac{1}{4j}e^{j15\pi t} + \frac{1}{16}e^{j40\pi t} - \frac{1}{8}e^{j80\pi t} - \frac{1}{16}e^{j120\pi t}$$

Problem 3-17

Property	a	b	c	d	e	f
Real coefficients		X			X	
Imaginary coefficients	X			X		
Complex coefficients			X			X
Even-indexed coefficient = 0	X	X	X	X	X	X
$X_0 = 0$	X	X	X	X	X	X

Problem 3-24

Apply (3-73). The transfer function of the system is

$$H(j\omega) = \frac{1}{1 + j\omega L/R} = \frac{1}{\sqrt{1 + (\omega L/R)^2}} e^{-j\tan^{-1}(\omega L/R)}$$

From Table 3-1, the exponential Fourier series coefficients for a triangular signal are

$$X_n = \frac{4A}{(n\pi)^2}, \; n \text{ odd}$$

Thus, (3-73) becomes

$$y(t) = \sum_{n=-\infty,\, n \text{ odd}}^{\infty} \left(\frac{8A}{(n\pi)^2 \sqrt{1 + n^2}} \right) \cos[n\omega_0 t - \tan^{-1}(n)]$$