Problem 2-24

Assignment 2

(a) From exp. 1-2

$$y(t) = -\frac{1}{RC}\int_{-\infty}^{t} x(\lambda)\,d\lambda$$

If the input is a unit impulse, we obtain

$$h(t) = -\frac{1}{RC}\int_{-\infty}^{t} \delta(\lambda)\,d\lambda = -\frac{1}{RC}u(t)$$

(b) See the first equation above, or evaluate the superposition integral with the impulse response:

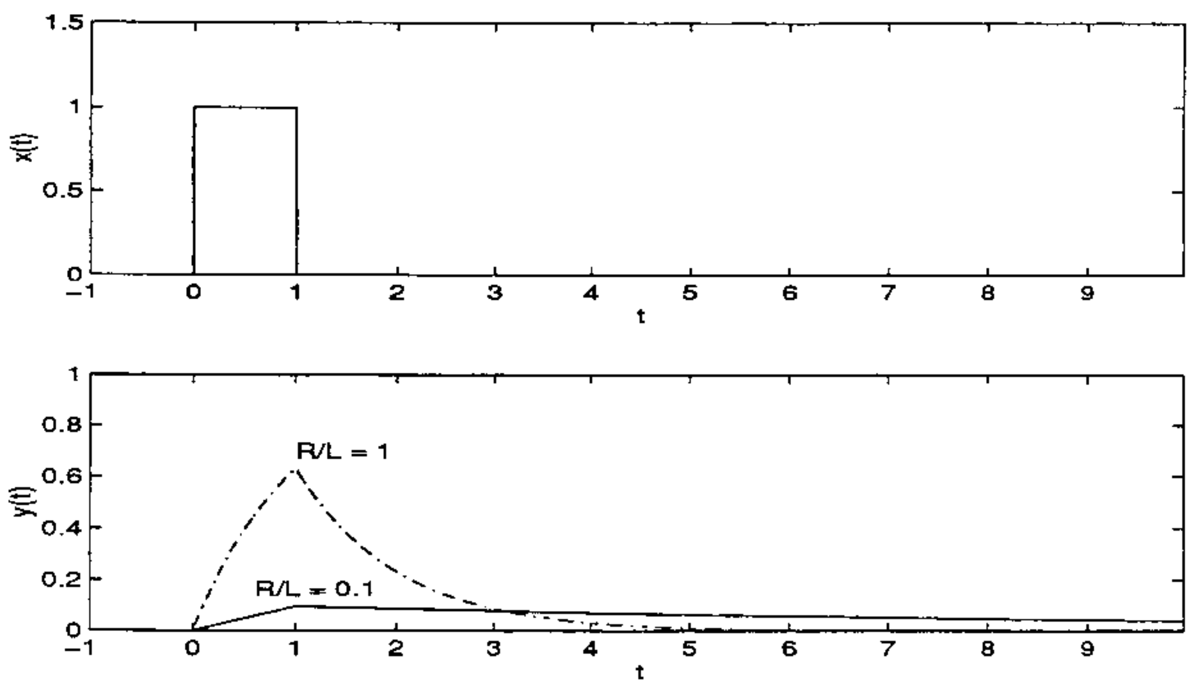
$$y(t) = \int_{-\infty}^{\infty} h(t-\lambda)x(\lambda)\,d\lambda = \int_{-\infty}^{\infty}\left[-\frac{1}{RC}u(t-\lambda)\right]x(\lambda)\,d\lambda = -\frac{1}{RC}\int_{-\infty}^{t} x(\lambda)\,d\lambda$$

Problem 2-26

We use the impulse response found in Problem 2-19 to evaluate

$$y(t) = \int_{-\infty}^{\infty} \Pi(\lambda - 0.5)\frac{R}{L}e^{-R(t-\lambda)/L}u(t-\lambda)\,d\lambda = \begin{cases} 0, & t < 0 \\ 1 - e^{-Rt/L}, & 0 < t < 1 \\ \left[1 - e^{-R/L}\right]e^{-R(t-1)/L}, & t > 1 \end{cases}$$

An alternative solution is to write the input as the difference between two steps and use superpositon after finding the step response by integration of the impulse response. A sketch is provided below.



Problem 2-31

(a) The impulse response is

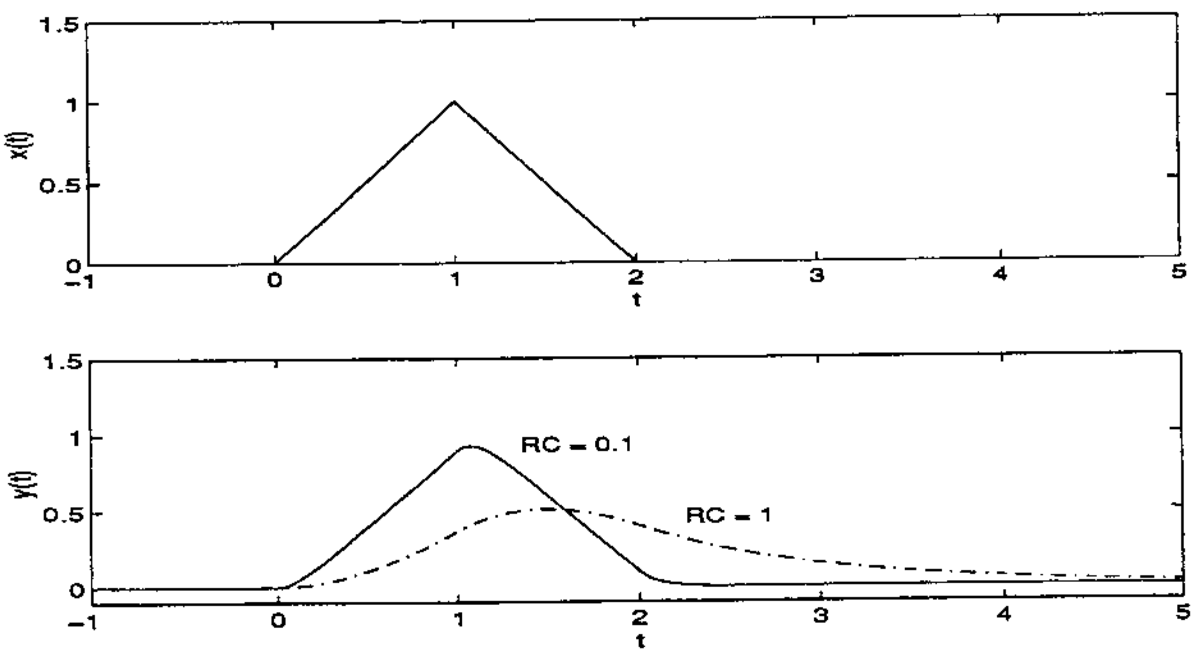
$$h(t) = \frac{1}{RC}\exp(-t/RC)u(t)$$

and, by integration, the step and ramp responses are

$$a_s(t) = [1 - \exp(-t/RC)]u(t) \text{ and } a_r(t) = r(t) - RC[1 - \exp(-t/RC)]u(t)$$

respectively. Using superposition, the output is

$$\begin{aligned} y(t) &= a_r(t) - 2a_r(t-1) + a_r(t-2) \\ &= r(t) - RC[1 - \exp(-t/RC)]u(t) \\ &\quad - 2\{r(t-1) - RC[1 - \exp(-(t-1)/RC)]u(t-1)\} \\ &\quad + r(t-2) - RC[1 - \exp(-(t-2)/RC)]u(t-2) \end{aligned}$$



(b) The response to $dx(t)/dt$ is the derivative of the response given in (a), which is

$$\begin{aligned} y(t) &= a_s(t) - 2a_s(t-1) + a_s(t-2) = [1 - \exp(-t/RC)]u(t) \\ &\quad - 2\{[1 - \exp(-(t-1)/RC)]u(t-1)\} + [1 - \exp(-(t-2)/RC)]u(t-2) \end{aligned}$$

(a) The frequency response function is given in terms of the impulse response by

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

From the impulse response given in Problem 2-29, this gives

$$H(j\omega) = \int_{-\infty}^{\infty} \left[\delta(t) - \frac{R}{L} \exp(-Rt/L) u(t) \right] \exp(-j\omega t) dt = \frac{j\omega}{R/L + j\omega}$$

(b) In terms of $f = \omega/2\pi$, the frequency response function is

$$H(f) = \frac{jf/f_3}{1 + jf/f_3} \text{ where } f_3 = \frac{R}{2\pi L}$$

Taking the magnitude, the amplitude response function is

$$A(f) = \frac{|f|/f_3}{\sqrt{1 + (f/f_3)^2}}$$

(c) The phase response function is the argument of the frequency response function. It is given by

$$\theta(f) = \frac{\pi}{2} - \tan^{-1}(f/f_3)$$

Plots of the amplitude and phase response functions are given below: