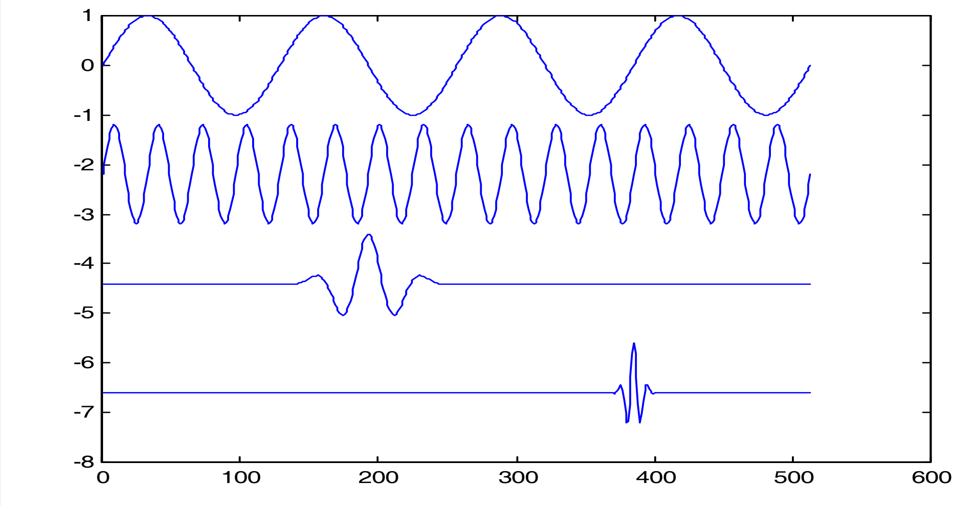
Chapter 8 Wavelet Transform (WT) for Image Coding

A Review of WT

- <u>Wave vs. Wavelet</u>
  - Figure 8.1



  - Wave: Does not have compact support (extends to infinity)
  - Transient signal (Anomaly, burst) : have compact support (non-zero only in a short interval)
  - Many image features (e.g., edges) highly localized in spatial position.

A Review of WT

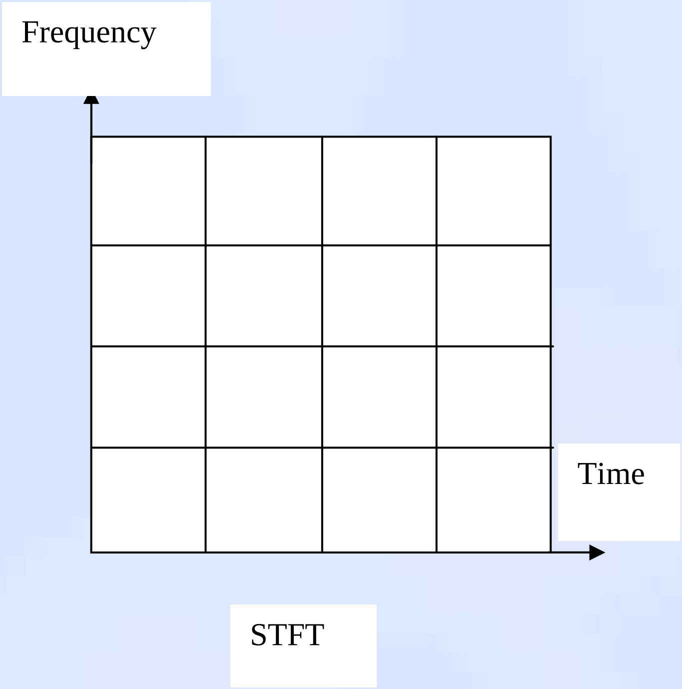
- **<u>Non-Stationary Signal Analysis</u>**
  - Stationary signal:
    - Properties not evolve in time
    - Fourier transform (FT) is suitable
  - Non-Stationary signal:
    - Properties evolve in time
    - Time-Frequency Analysis
      - 2-D time-frequency space (can be derived from Figure 8.1)
      - Started with Gabor's **windowed FT**
        - short-time Fourier transform (STFT)
      - Another approach: WT

A Review of WT

- **<u>STFT vs. WT</u>**
  - STFT:
    - Resolution in time and frequency cannot be arbitrarily small, due to Heisenberg inequality:

      $$\Delta t \cdot \Delta f \geq 1/(4\pi)$$

    - Once window is chosen: Δf and Δt are fixed
    - Meaning anomaly (burst) and trend cannot be analyzed with good resolution *simultaneously*

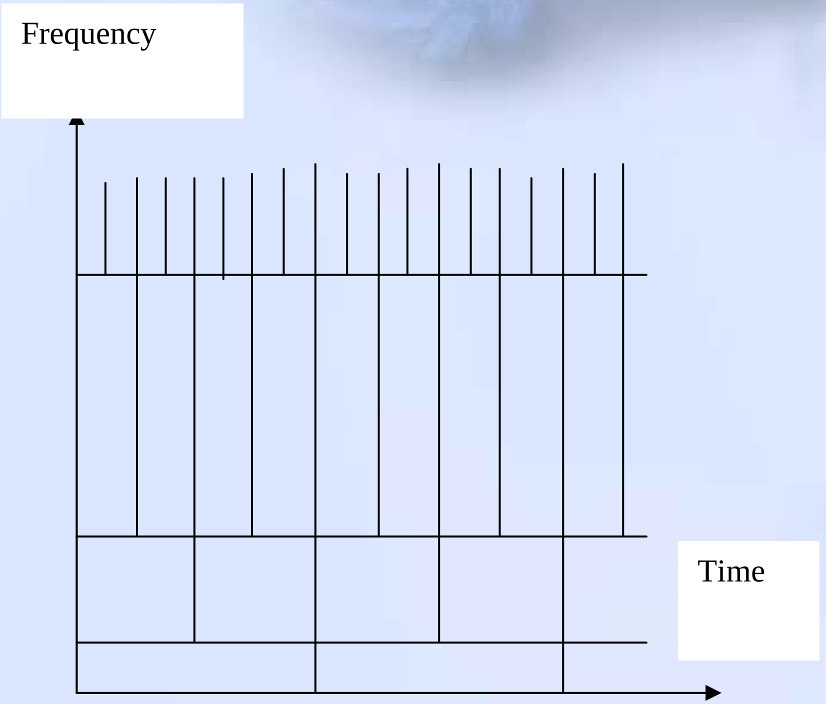


A Review of WT

- WT:
  - Constant relative bandwidth (const. Q):

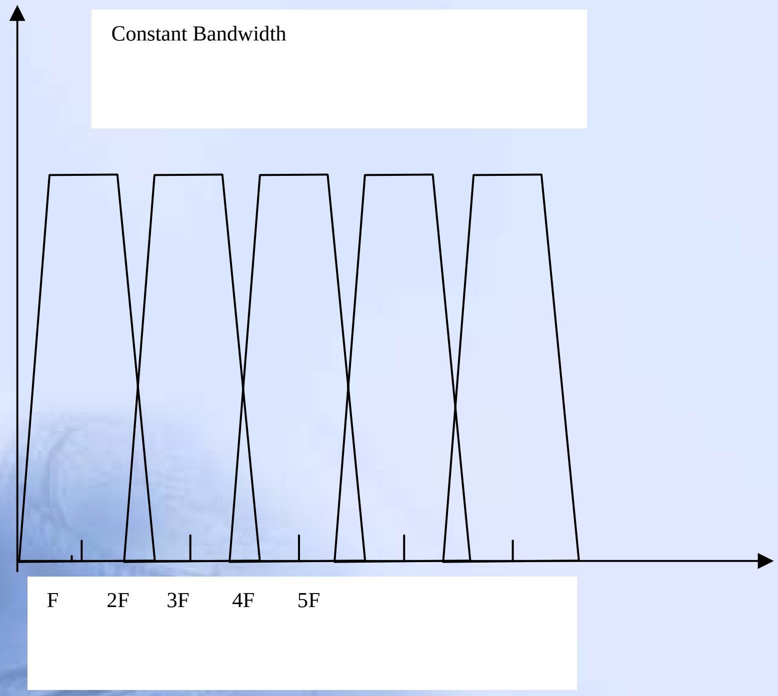
    $$\Delta f / f = constant$$

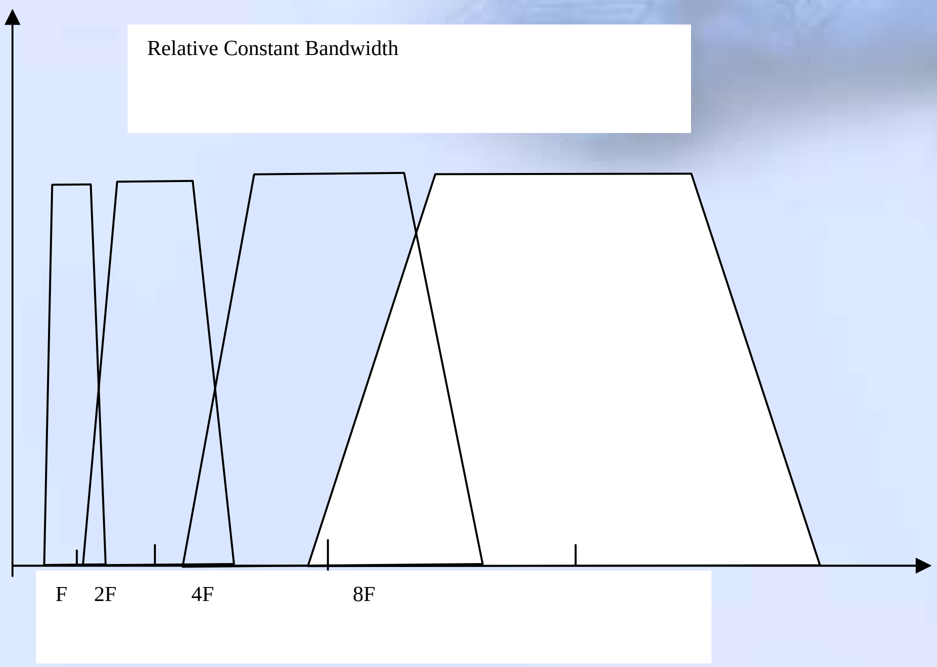
  - Meaning:
    - $\Delta t \downarrow$ as $\Delta f \uparrow (f \uparrow)$, and $\Delta f \downarrow$ as $f \downarrow$
    - as $f \uparrow$, high time resolution obtained
    - as $f \downarrow$, high freq. resolution obtained



Wavelet Transform

A Review of WT

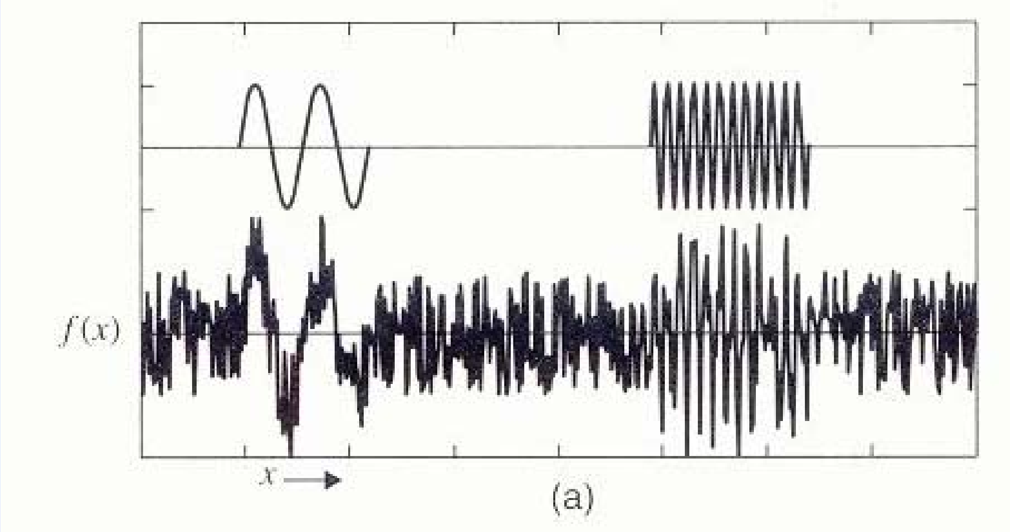


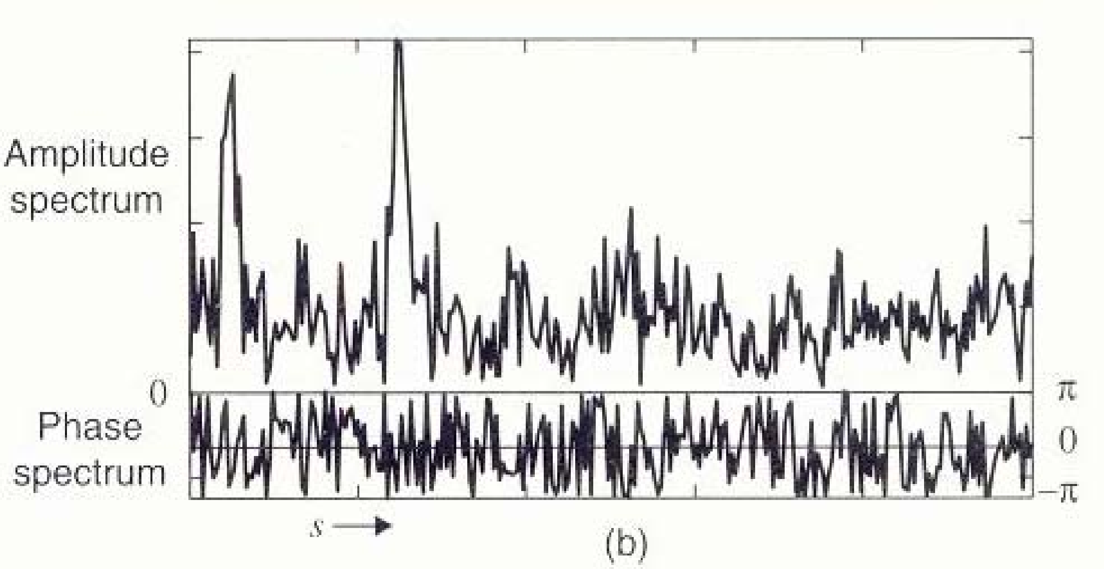


- Figure 8.3 Constant bandwidth analysis (for FT) and relative constant bandwidth analysis (for WT)

A Review of WT

- **Example**
  - A two tone bursts corrupted by random noise
  - FT: not easy to be interpreted , in particular, phase spectrum

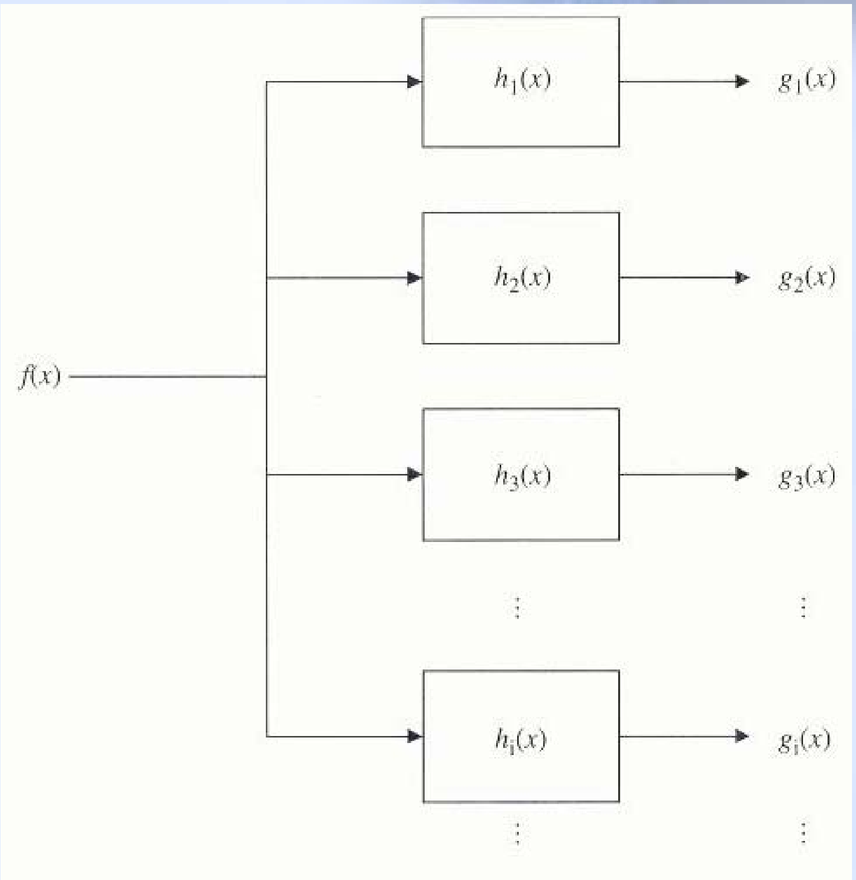




A Review of WT

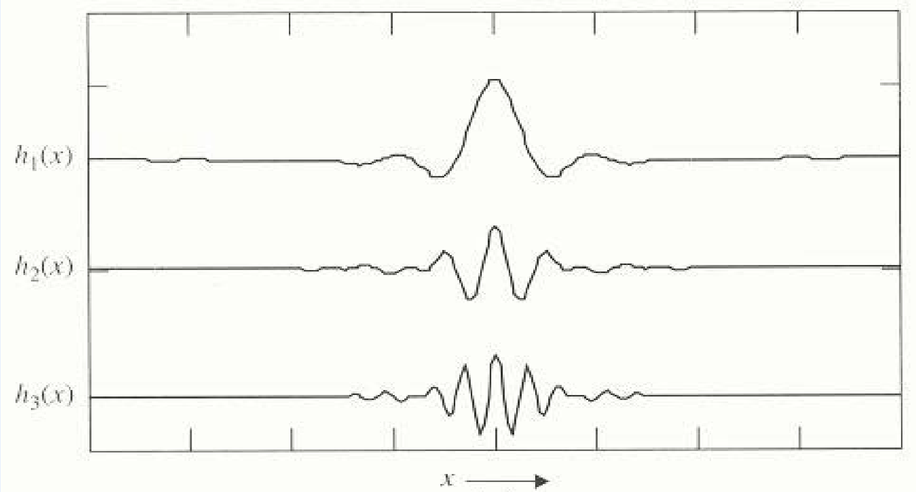
- Filter Bank Theory (WT)
  - Implementation of a bandpass filter bank

$$\sum_{\forall i} H_i(S) = 1 \Rightarrow \sum_{\forall i} g_i(x) = f(x)$$



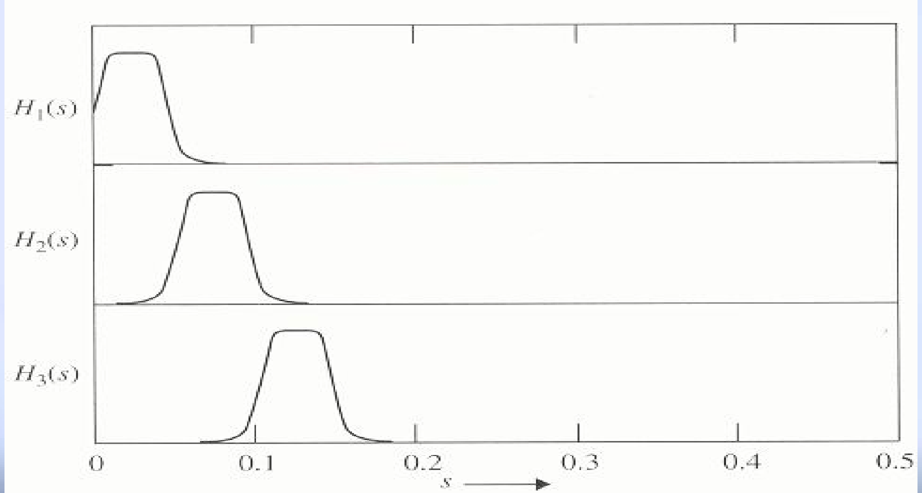
A Review of WT

- Smooth bandpass filters
  - impulse responses



(a)

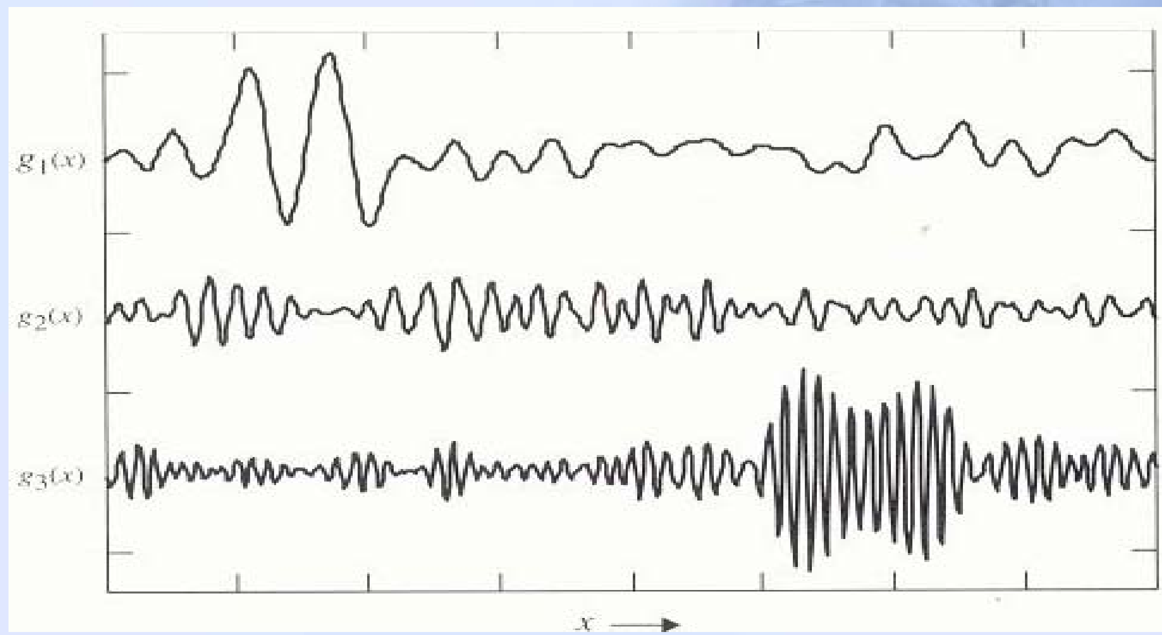
  - transfer functions



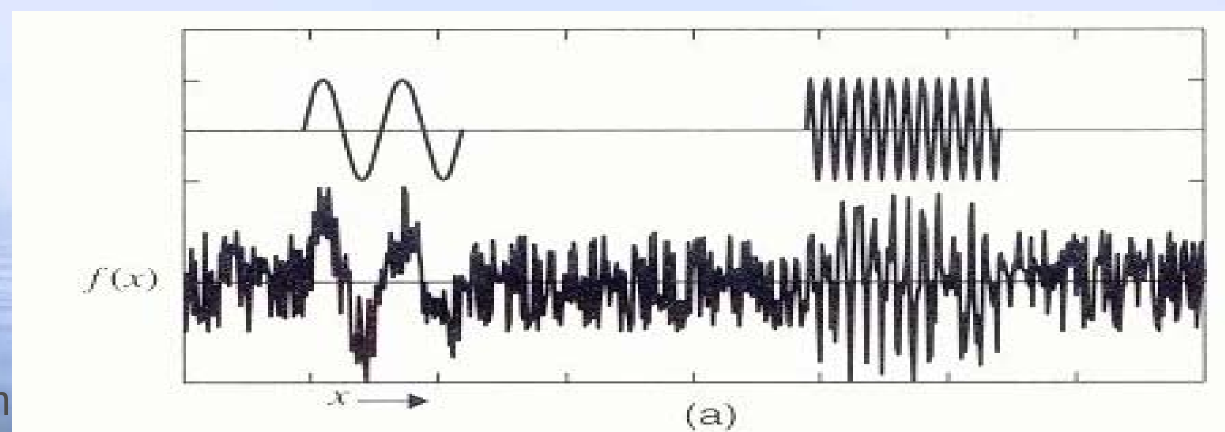
(b)

A Review of WT

- **Example**
  - Smooth bandpass filter bank output



  - Original signal and corrupted signal



A Review of WT

- **WT: Unification of Several Techniques**
  - Filter Bank Analysis
  - Pyramid Coding
  - Subband Coding
- **Three Types of WT**
  - CWT (Continuous WT)
  - Wavelet series expansion
  - DWT (Discrete WT)

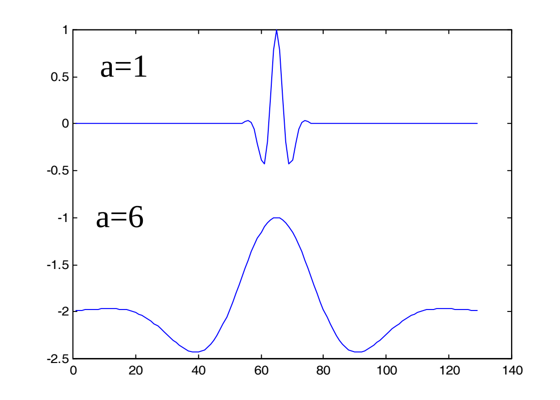
Discrete WT

- **DWT**
  - Most closely resembles unitary transforms
  - Most useful in image compression
  - Given a set of orthonormal basis functions, DWT acts just like unitary transform
  - Orthonormal wavelets with compact support (by Daubechies):

$$\{_r\psi(x)\} = \{2^{j/2}\,_r\psi(2^j x - k)\}$$

    - $\psi(x)$:mother wavelet
    - *j*,*k*: integers
    - compact support: [*0,2r-1*]
    - shift: *k*
    - dilation (scaling): 2^j
  - *N*-point signal $\Rightarrow$ N coefficients

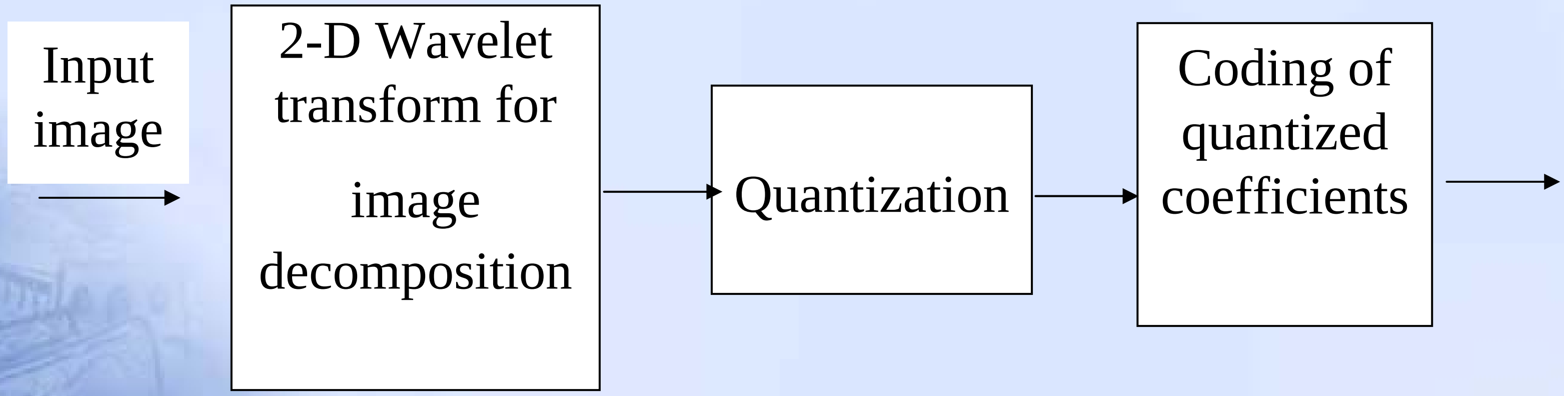
    $N \times N$ image $\Rightarrow$ N^2 coefficients
- HW #5: Ex. 8-1



Scaling

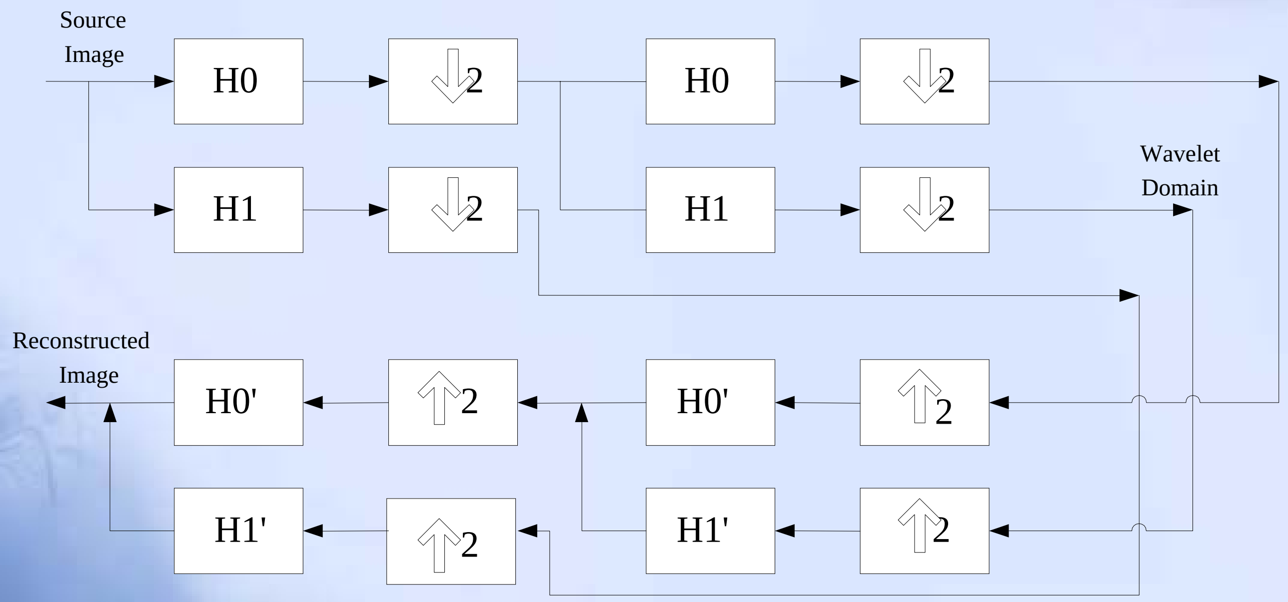
DWT for Image Compression

- **Block Diagram**



DWT for Image Compression

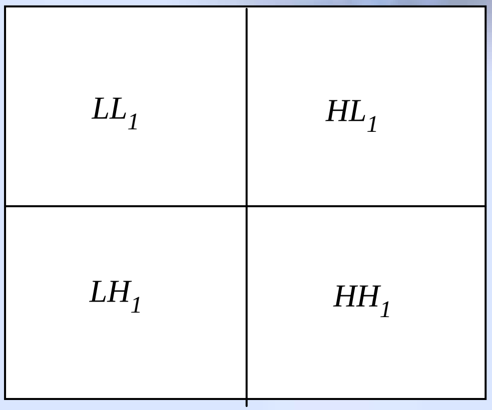
- Two level (1-D) wavelet decomposition and reconstruction



- 2-D decomposition is realized by filtering along horizontal direction, then along vertical direction.

DWT for Image Compression

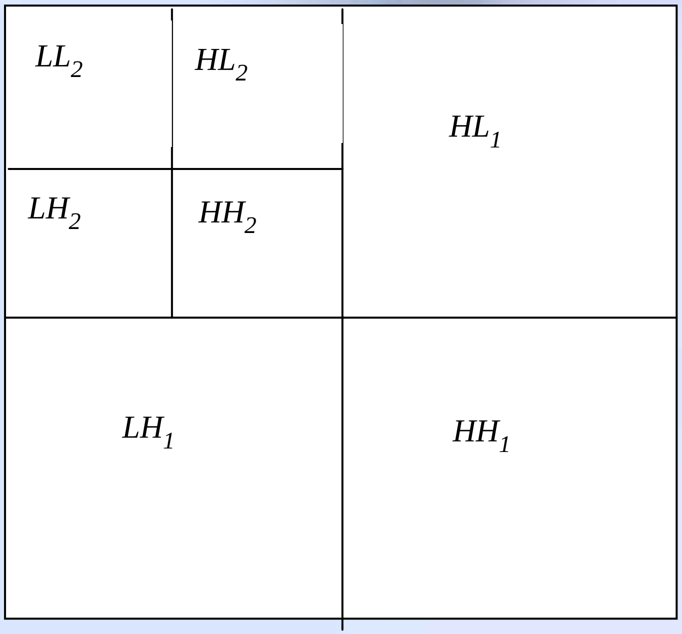
- **Image Decomposition**
  - Scale 1



    - 4 subbands: LL_1, HL_1, LH_1, HH_1
    - Each coeff. ↔ a 2*2 area (*not exactly*) in the original image
    - Low frequencies: $0 < |\omega| < \pi/2$
    - High frequencies: $\pi/2 < \omega < \pi$

DWT for Image Compression

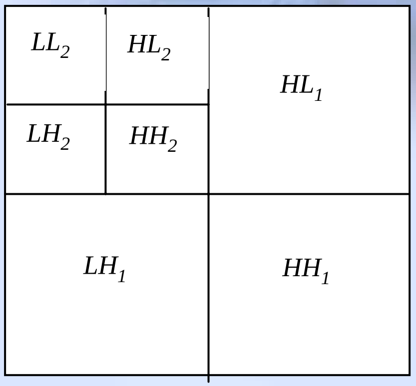
- **Image Decomposition**
  - Scale 2
    - 4 subbands: LL_2, HL_2, LH_2, HH_2
    - Each coeff. $\leftrightarrow$ a 2*2 area in Scale 1 image
    - Low Frequency: $0 < |\omega| < \pi/4$
    - High frequencies: $\pi/4 < \omega < \pi/2$

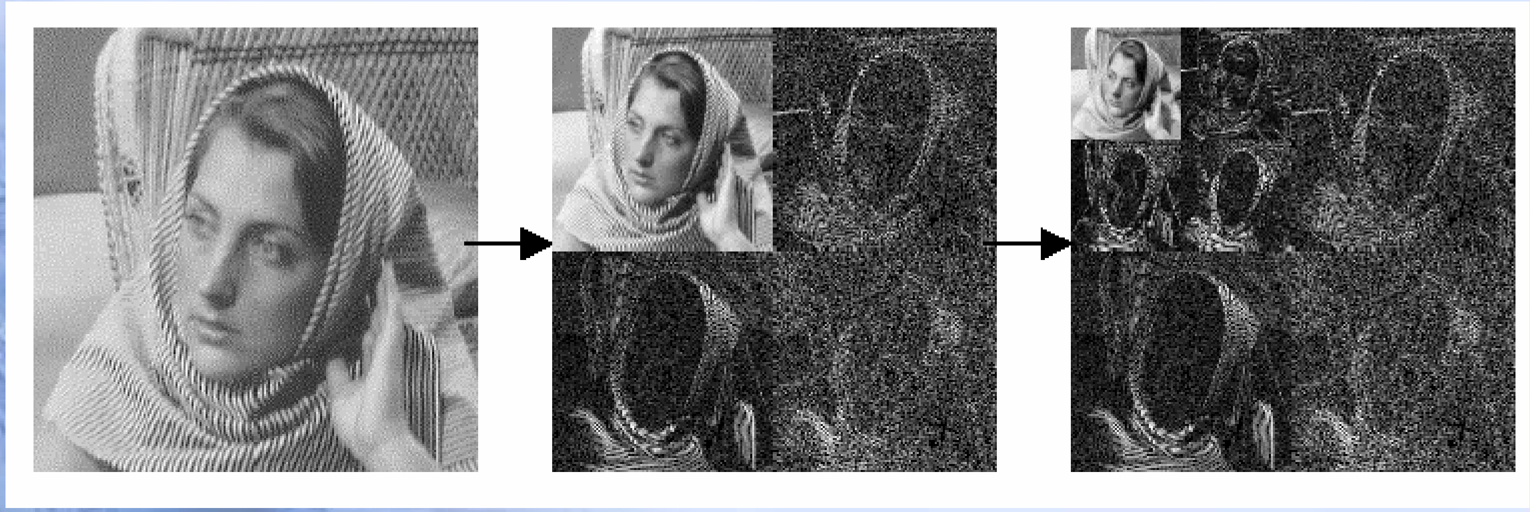


At a coarser scale, coefficients represent a larger spatial area of the image but a narrow band of frequencies.

DWT for Image Compression

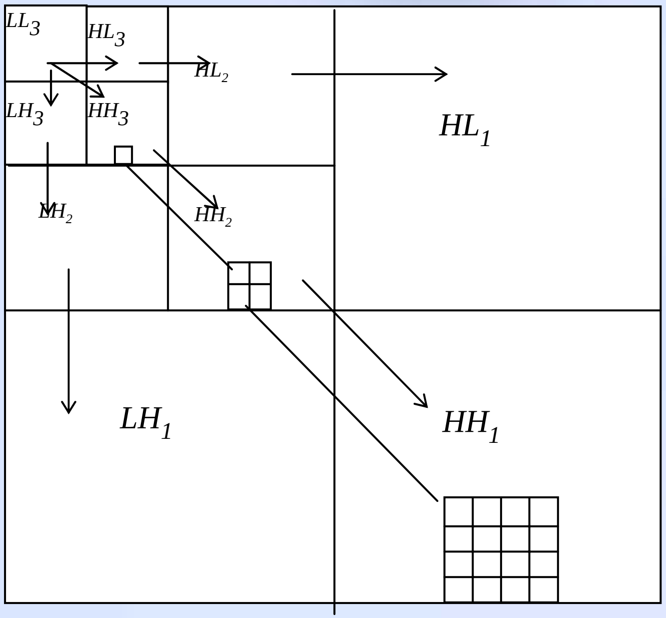
- **Image Decomposition**





DWT for Image Compression

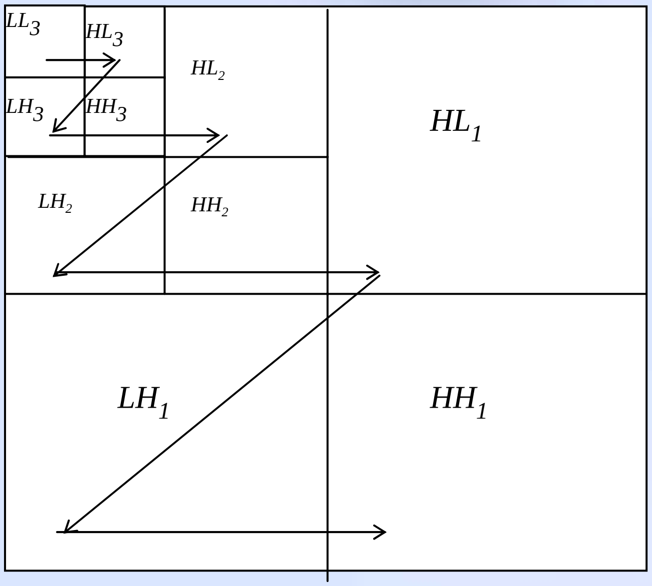
- **Image Decomposition**
  - Parent vs. Children
  - *Descendants*: corresponding coeff. at finer scales
  - *Ancestors*: corresponding coeff. at coarser scales



Parent-children dependencies of subbands: arrow points from the subband of parents to the subband of children.

DWT for Image Compression

- **Image Decomposition**
  - Feature 1:
    - Energy distribution similar to other TC: concentrated in low frequencies
  - Feature 2:
    - Spatial self-similarity across subbands



The scanning order of the subbands for encoding the significance map.

DWT for Image Compression

- Differences from DCT Technique
  - In conventional TC
    - Anomaly (edge) $\leftrightarrow$ many nonzero coeff.
      insignificant energy
    - TC allocates too many bits to “trend”, few bits left to “anomalies”
    - Problem at Very Low Bit-rate Coding : block artifacts
  - DWT
    - Trends & anomalies information available
    - Major difficulty: fine detail coefficients associated with anomalies $\Leftrightarrow$ the largest no. of coeff.
    - Problem: how to efficiently represent *position* information?

EZW Image Coding

- **<u>Embedded Coding</u>**
  - Having all lower bit rate codes of the same image embedded at the beginning of the bit stream
  - Bits are generated in order of importance
    - Bit plane coding, coarser scale to finer scale
  - Encoder can terminate encoding at any point, allowing a target rate to be met exactly
  - Suitable for applications with scalability

EZW Image Coding

- **Zerotree of DWT Coefficients**
  - Significance map: binary decision as to a pixel = 0 or not, w.r.t. a threshold *T* (*T* is decreased by half in each scan)
  - Total encoding cost = cost of encoding significance map + cost of encoding nonzero values
  - **An element of zerotree**:
    - **A coeff.:** itself and all of its descendants are insignificant w.r.t. threshold *T*
    - **Zerotree root**: An element of zerotree, & not a descendant of a zero element at a coarser scale
    - **Isolated zero**: Insignificant, but has some significant descendant
  - Significance map can be efficiently represented as a string of four symbols:

* Zerotree root	* Isolated zero
* Positive significant coeff.	* Negative significant coeff.

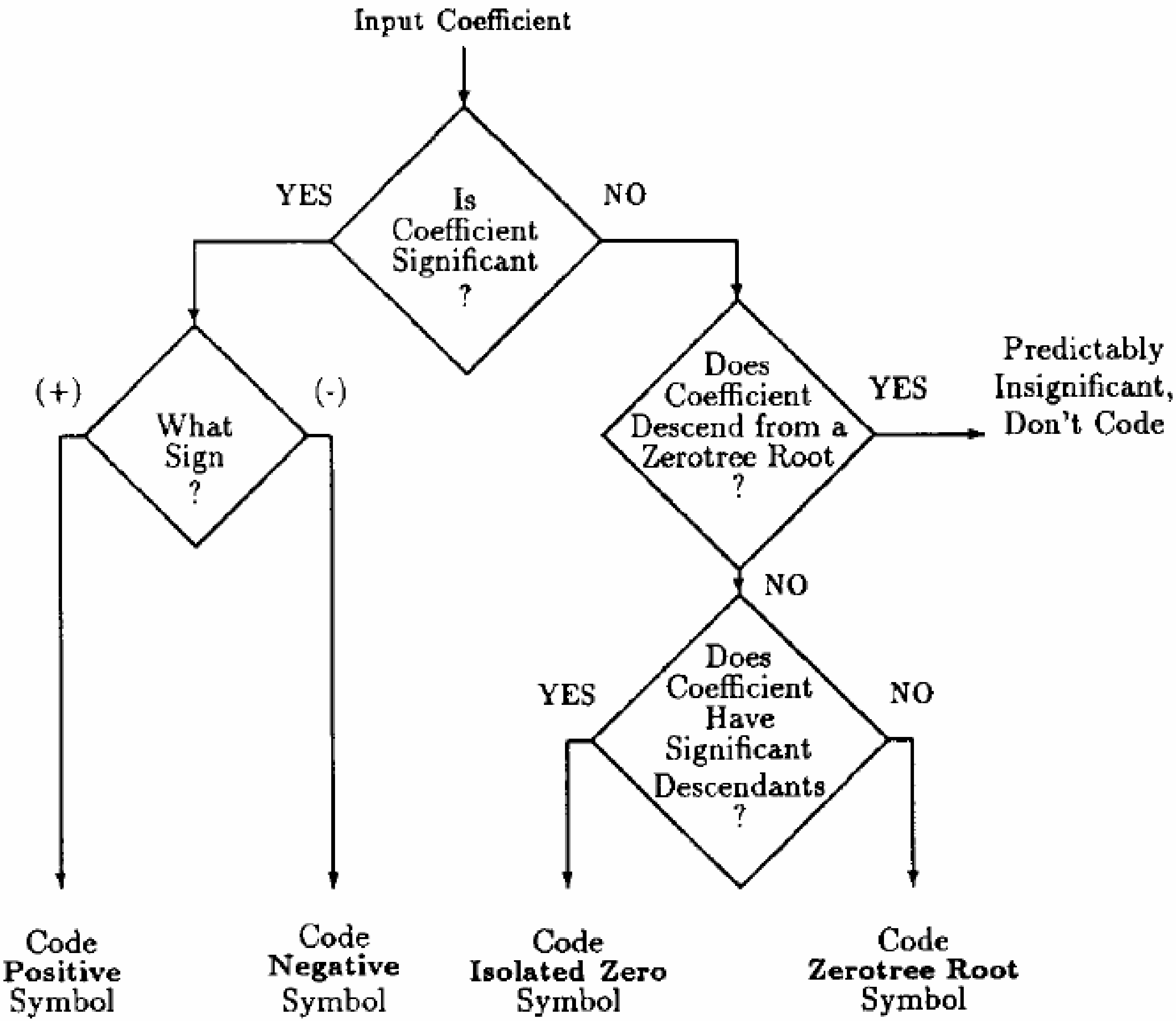


Fig. 6. Flow chart for encoding a coefficient of the significance map.

EZW Image Coding

- **<u>Comparison</u>**
  - DCT coding
    - Run-length (RLC) [within the same scale]
    - End of block (EOB) [within the same scale]
  - EZW coding
    - More efficient due to using self-similarity across different scales
    - Bit plane coder for scalability and efficient exploitation of self-similarity
    - Higher quality of reconstructed image:
      - Due to more efficient in position encoding
      - *No* possibility that a significant coeff. be obscured by a statistical energy measure
    - Experimental results reported: “Barbara” at very low bit rate
      - 2.4 dB better for same bit rate and 0.12 bpp savings for the same PSNR