Chapter 3 Differential Coding

Outline

- Introduction to DPCM
- Optimum linear prediction
- Some issues in implementation
- Delta modulation
- Interframe differential coding
- Information-preserving differential coding

Differential Coding

- ***Differential coding*** technique:
  - Instead of encoding a signal directly, encodes the difference between the signal itself and its prediction.
  - Also known as ***predictive coding***
- Example: 8 bits/sample in quantization
  - Difference signal between pixels
    - Dynamic range increase from 256 to 512
    - But variance much smaller
- Efficient and yet computationally simple

Introduction to DPCM

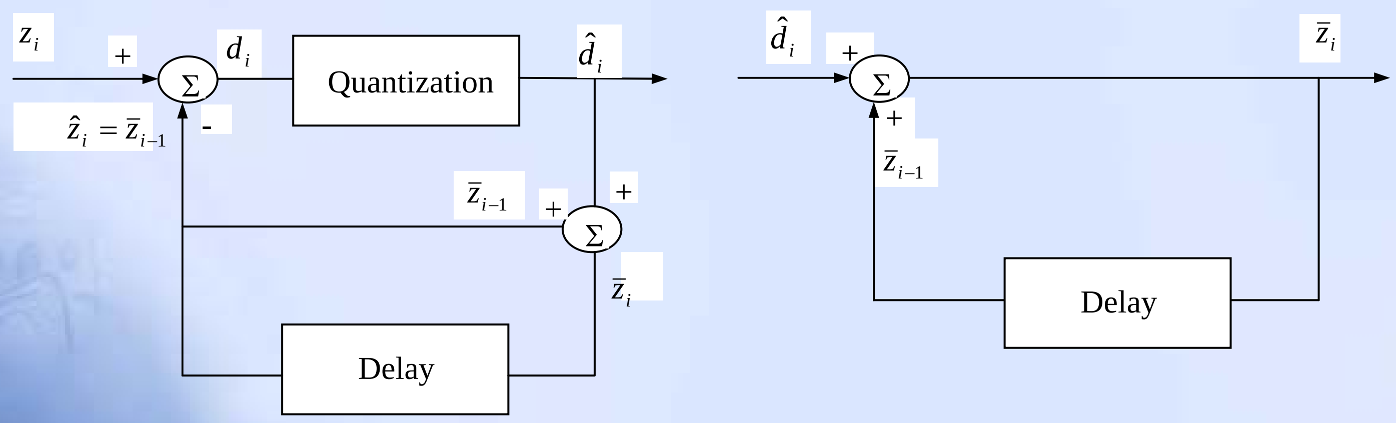
- Simple pixel to pixel DPCM
- Notations
  - $z_i, i = 1, \cdots, M$, gray level values of pixels along a row
  - $\hat{z}_i$: a prediction of gray level value of the present pixel
  - $d_i = z_i - \hat{z}_i$: the difference signal
  - $\hat{d}_i$:quantized version of the difference
    - $\hat{d}_i = Q(d_i) = d_i + e^i{}_q$ $\quad e^i{}_q$: **quantization error**
  - $\bar{z}_i = \hat{z}_i + \hat{d}_i$ Reconstructed gray value
  - Now, let $\hat{z}_i = \bar{z}_{i-1}$, i.e., previous reconstructed = prediction then

$$d_i = z_i - \bar{z}_{i-1} \quad \bar{z}_i = \bar{z}_{i-1} + \hat{d}_i = z_i + e_q{}^i$$

    - **No error accumulation**

Simple Pixel-to-Pixel DPCM

Fig. 3.1: Block diagram of a practical pixel-to-pixel differential coding system

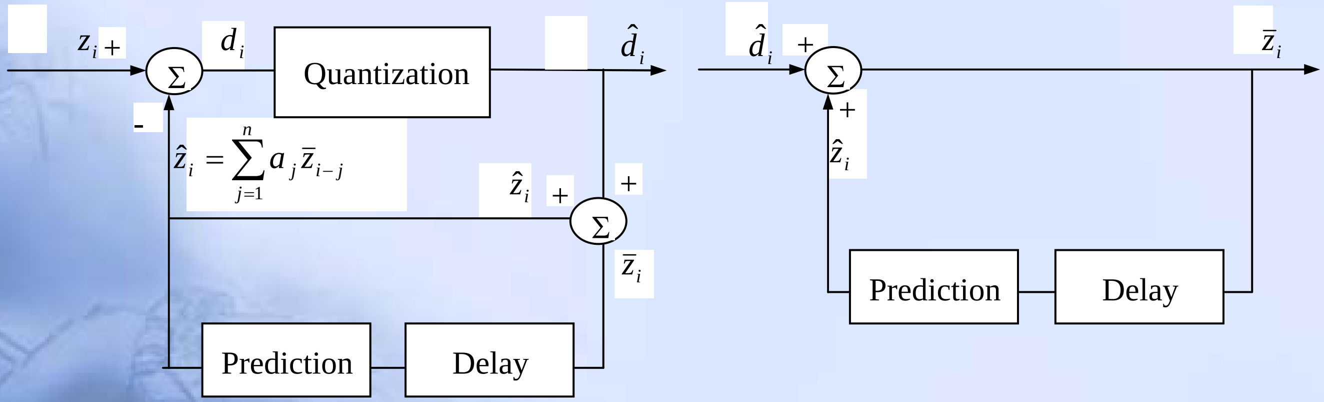


(a) Encoder (b) Decoder

General DPCM Systems

$$\hat{z}_i = f(\bar{z}_{i-1}, \bar{z}_{i-2}, \cdots, \bar{z}_{i-n})$$

- Linear prediction, i.e., *f* is linear, is of particular interest and is widely used in differential coding



General DPCM Systems

- Prediction error
  - The difference between the original input and the predicted input is called prediction error, i.e.,

$$e_p = z_i - \hat{z}_i$$

- Reconstruction (or coding) error
  - Difference between the original signal z_i, and the reconstructed signal $\bar{z}_i$

$$e_r = z_i - \bar{z}_i$$

General DPCM Systems

- **Quantization error** is equal to the reconstruction error or coding error when the transmission is error free [Gish' 67]

$$e_q = d_i - \hat{d}_i = (z_i - \hat{z}_i) - (\bar{z}_i - \hat{z}_i) = e_r$$

- Meaning: quantization error is the only source of information loss with an error free transmission channel

General DPCM Systems

- Applications: the differential coding technique has played an important role in image/video coding
  - In JPEG, international coding standard for still images
    - The differential coding is used in lossless mode
    - In DCT based mode for coding DC coefficients
  - In all international video coding standards, such as H.261, H.263, H.264, MPEG 1/2/4
  - Motion compensated (MC) coding is essentially predictive coding.

Optimal Linear Prediction

- Problem formulation
  - Consider a discrete-time random process *z*. At a typical moment *i*, it is a random variable. We have *n* previous observations available, and would like to form a prediction of z_i ,denoted by $\hat{z}_i$. The output of the predictor, $\hat{z}_i$, is a linear function of the *n* previous observations.

$$\hat{z}_i = \sum_{j=1}^{n} a_j \bar{z}_{i-j}$$

with $a_j, j = 1, 2, \cdots, n$ being a set of real coefficients.

Optimal Linear Prediction

- The mean square prediction error, MSE_p, is

$$MSE_p = E[(e_p)^2] = E[(z_i - \hat{z}_i)^2]$$

- The optimum prediction refers to the determination of a set of coefficients, $a_j, j = 1,2,\cdots,n$ such that the mean square prediction error, MSE_p, is minimized.

Optimal Linear Prediction

- This optimization problem turns out to be computationally intractable for most practical cases due to the feedback around the quantizer shown in Fig. 3.2, and the nonlinear nature of the quantizer.
- Therefore, the optimization is solved in two separate stages,
  - The best linear predictor is first designed ignoring the quantizer
  - Then, the quantizer is optimized for the distribution of the difference signal [Habibi'71]

Optimal Linear Prediction

- Orthogonality condition and minimum mean square error
  - By taking the differentiation of MSE_p, with respect to coefficient a_j, one can derive the following **necessary** conditions, usually referred to as the ***orthogonality condition***.

$$E[e_p z_{i-j}] = 0 \quad for \qquad j = 1,2,\cdots,n$$

  - The interpretation of this equation is that the prediction error e_p, must be orthogonal to all the observations $z_{i-j},\ j = 1,2,\cdots,n$

Optimal Linear Prediction

- These are equivalent to

$$R_z(m) = \sum_{j=1}^{n} a_j R_z(m-j) \qquad for \qquad m = 1,2,\cdots,n$$

where R_z represents the autocorrelation function of *z*

- In Matrix format (**Yule-Walker equations**)

$$\begin{bmatrix} R_z(1) \\ R_z(2) \\ \vdots \\ \vdots \\ R_z(n) \end{bmatrix} = \begin{bmatrix} R_z(0) & R_z(1) & \cdots & \cdots & R_z(n-1) \\ R_z(1) & R_z(0) & \cdots & \cdots & R_z(n-2) \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \vdots \\ R_z(n-1) & R_z(n-2) & \cdots & \cdots & R_z(0) \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_n \end{bmatrix}$$

Optimal Linear Prediction

- The minimum mean square prediction error is then found to be

$$MSE_p = R_z(0) - \sum_{j=1}^{n} a_j R_z(j)$$

- Solution to Yule-Walker Equations
  - Solved by matrix inversion.
  - A recursive procedure was developed by Levinson to solve the equations [Leon-Garcia'94].
    - More attractive when the matrix dimension is high
  - Autocorrelation function can be derived from measurements.

Some Issues in the Implementation of DPCM

- 1-D, 2-D and 3D DPCM
- Order of predictor
  - The # of coefficients in the linear prediction, *n*, is referred to as the order of the predictor.
- The relation between the MSE_p and the order *n*:
  - MSE_p decreases as *n* increases
  - The performance improvement becomes negligible as *n > 3* [Habibi'71]

Prediction MSE vs. Prediction Order

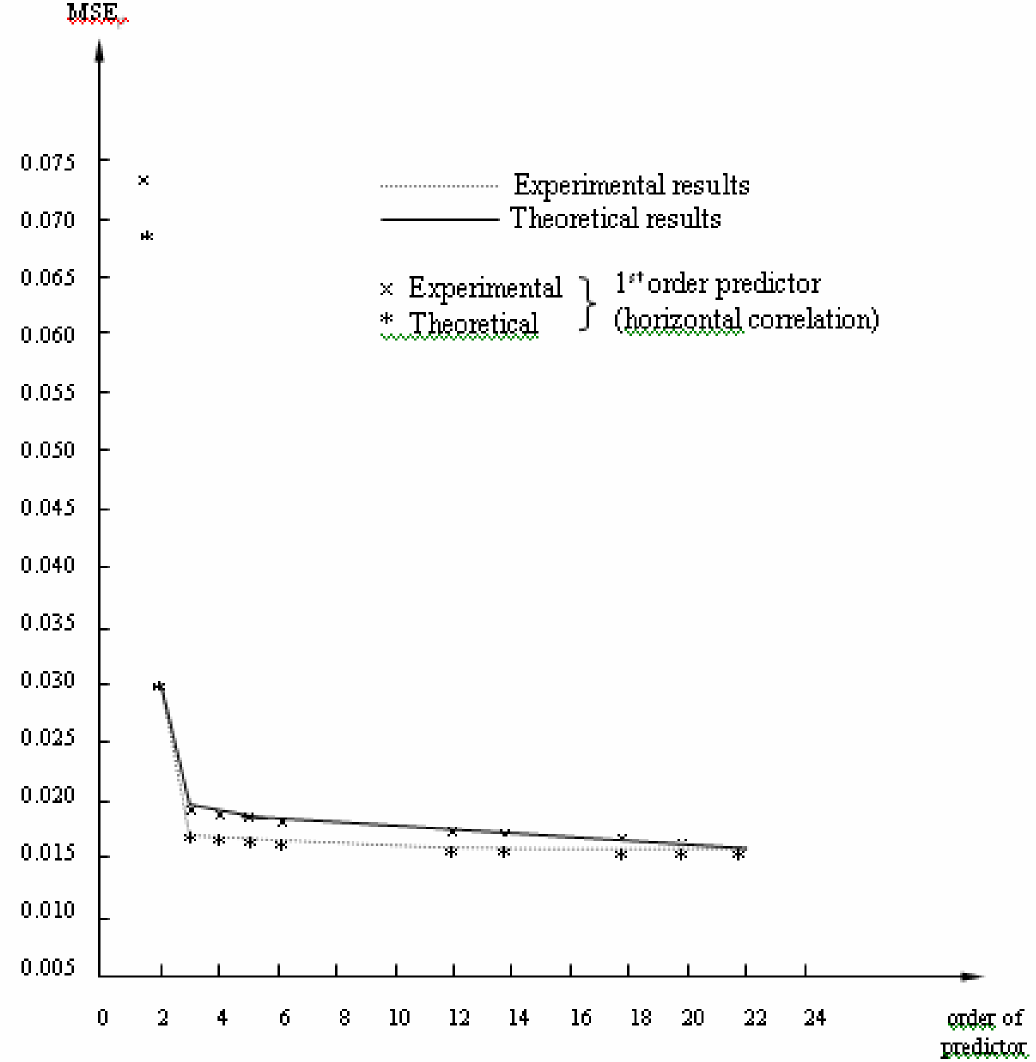


Figure 3.5 Mean square prediction error versus order of predictor after [habibi 1971]

Adaptive Prediction

- Adaptive prediction can be done in two different ways:
  - Forward adaptive prediction
    - Based on the input of a DPCM system
    - More sensitive to variation of local statistics
    - Side information: prediction coefficients
  - Backward adaptive prediction
    - Based on the output of a DPCM system
    - Less sensitive to variation of local statistics
    - No side information
- In either case, the data has to be buffered.

Effect of Transmission Errors

- Transmission error caused by channel noise reverses the binary bit information from 0 to 1 or 1 to 0 with certain bit error rate
- The effect of transmission error on reconstructed image varies depending on different coding techniques
- PCM-coding: each pixel is coded independently.
  - Bit reversal in transmission only affects the gray level of corresponding pixel, not other pixels.

Effect of Transmission Errors

- In DPCM, the effect caused by transmission errors becomes more severe.
  - The transmission error propagates
  - It is reported that error propagation is more severe in 1-D differential coding than in 2-D differential coding
    - Dependency on the other dimension mitigates error propagation effect.
- Bit error rate required by DPCM coding is lower than that required by PCM coding.
  - E.g., for broadcast TV quality
    - PCM: bit error rate <= $5 \cdot 10^{-6}$
    - 2-D DPCM: <= 10^{-7}
    - 1-D DPCM: <= 10^{-9}

Effect of Transmission Errors

- Channel encoding with an error correction capability was applied to lower the bit error rate.
  - E.g., to lower the bit error rate from the order of 10^{-6} to the order of 10^{-9} for DPCM coding with 1-D prediction, an error correction by adding 3% redundancy in channel coding has been used [Bruders'78]

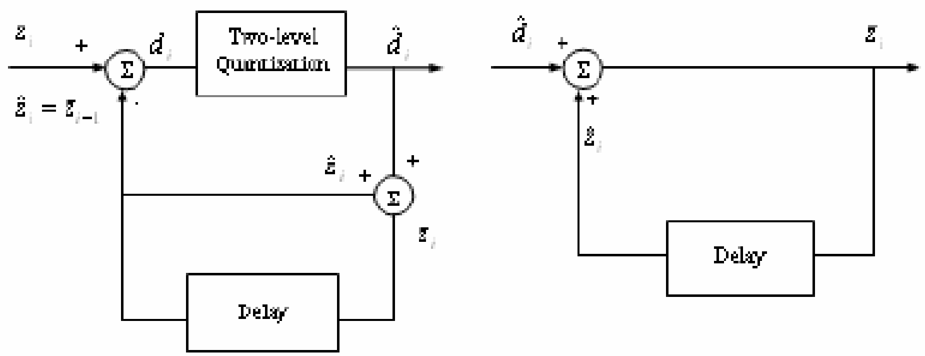
Delta Modulation (DM)

- DM is essentially a special type of DPCM, with the following two features:
  - The linear predictor is of first order, with the coefficient equal to 1.
  - The quantizer is a one-bit quantizer, i.e., depending on whether the difference signal is positive or negative, the output is either $+\Delta/2$ or $-\Delta/2$.
- DM is simple, and has been widely applied.

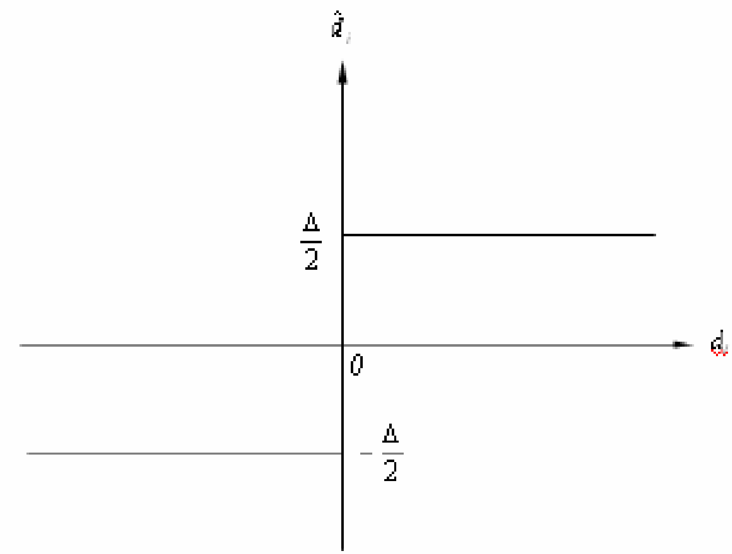
Delta Modulation (DM)

$$\hat{z}_i = \bar{z}_{i-1}. \qquad (3.13)$$

$$\hat{d}_i = \begin{cases} +\Delta/2 & \text{if } z_i > \bar{z}_{i-1} \\ -\Delta/2 & \text{if } z_i < \bar{z}_{i-1} \end{cases}. \qquad (3.14)$$



(a) Encoder (b) Decoder

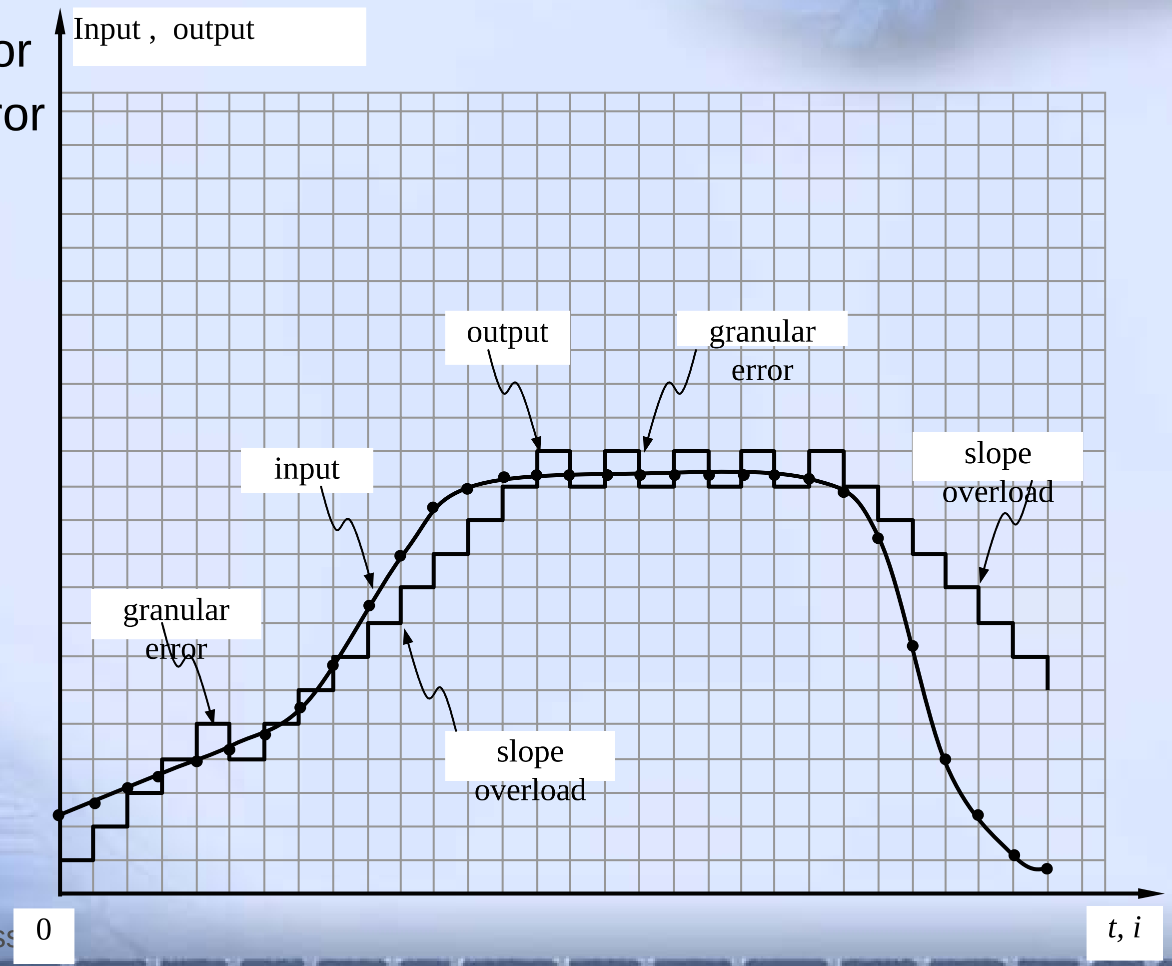


Two-level quantizer

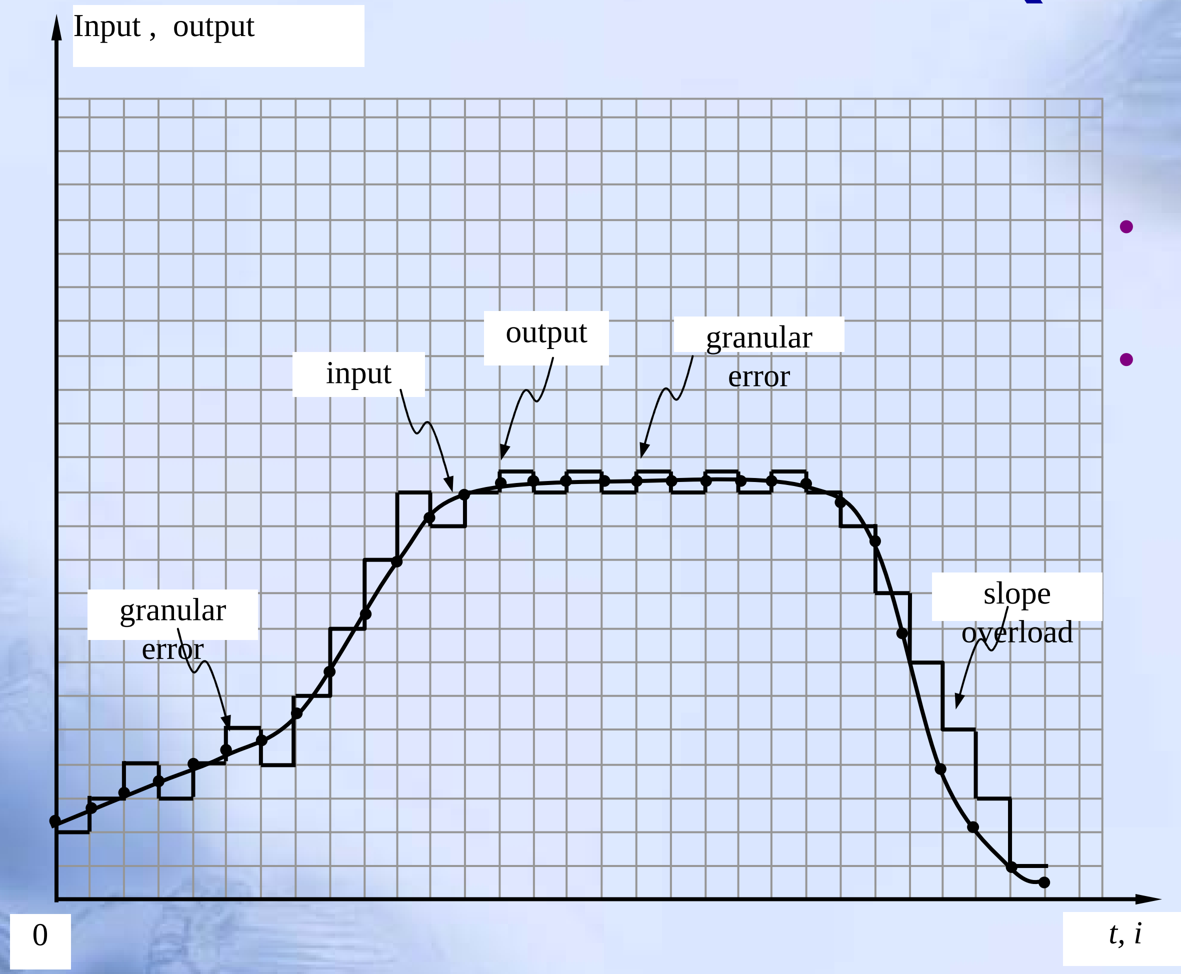
Delta Modulation (DM)

Fixed step size Δ

- Large Δ -> large granular error
- Small Δ -> slope overload error



Delta Modulation (DM)



- Reduce granular error
- Avoid overload error

Adaptive step size Δ

Interframe Differential Coding

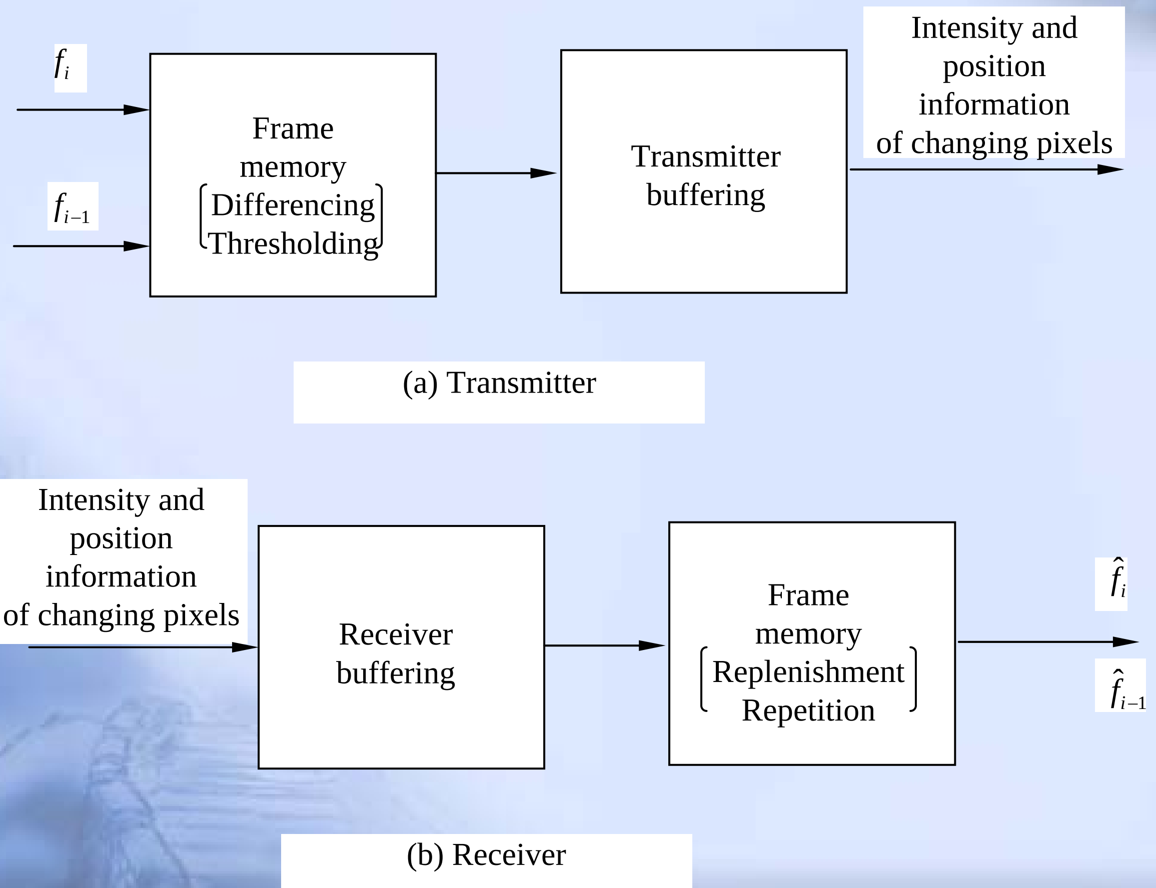
- 3-D differential coding involves an image sequence
- Consider applications such as video telephony and videoconferencing
  - The sensor is fixed in position for a while and it takes pictures. As time goes by, the captured images form a temporal image sequence.
  - The coding of such an image sequence is referred to as **interframe coding**.
- The subject of image sequence and video coding is addressed in Part IV. In this section, we briefly discuss how differential coding is applied to interframe coding

Interframe Differential Coding

- Conditional replenishment
  - Frame replenishment (FR) [Mounts'69]: first real demonstration of interframe coding exploiting interframe redundancy [Netravali'79]
  - Previous frame is used as reference for present frame.
  - Consider a pair of pixels, locating in the same spatial location but in two consecutive frames
    - If the gray level difference between this pair exceeds certain criterion, the pixel is considered ***changing***
      - The present pixel value and its position info are transmitted to receiver where the pixel value is replenished
    - Otherwise, the previous pixel value is repeated for the present pixel value

Interframe Differential Coding

- Conditional replenishment



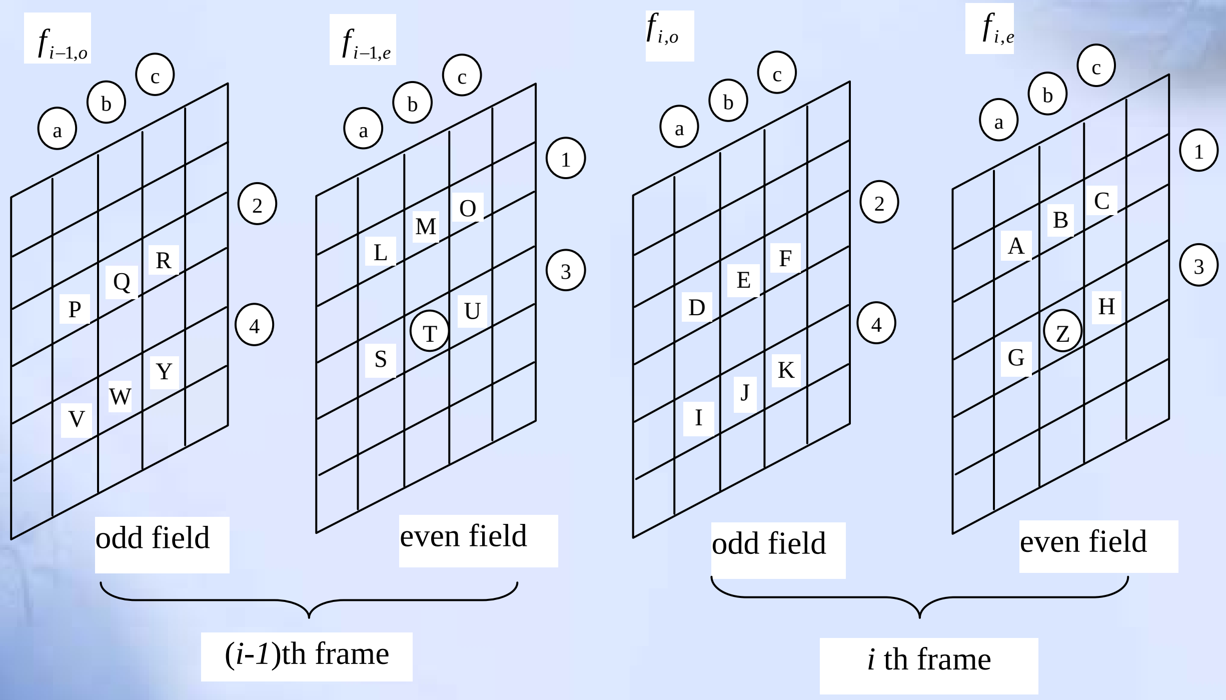
Interframe Differential Coding

- A buffer is used in the transmitter to smooth the transmission data rate.
  - Necessary because data rate varies from region to region within an image frame, and from frame to frame
- Experiments using “head and shoulder” view of a person in animated conversation as the video source demonstrated an average bit rate of **1 bit/pixel** with quality comparable with standard **8 bit/pixel** PCM transmission [Mounts’69]
- Compared to pixel to pixel 1-D DPCM, the most popularly used coding technique at the time, conditional replenishment is more efficient due to exploitation of high inter-frame redundancy.
  - Temporal redundancy is higher than spatial redundancy for TV signals.

3-D DPCM

- It is soon realized it is more efficient to transmit the gray level difference than the gray level itself -> **interframe differential coding**
- Furthermore, instead of treating each pixel independent of its spatial neighboring pixels, it is more efficient to utilize spatial redundancy as well, → **3D DPCM**

3-D DPCM



Pixel arrangement in two TV frames after [Haskell'79]

3-D DPCM

	Original signal (Z)	Prediction signal ($\hat{Z}$)	Differential signal (d_z)
Element difference	Z	G	Z-G
Field difference	Z	$\frac{E+J}{2}$	$Z-\frac{E+J}{2}$
Frame difference	Z	T	Z-T
Element difference of frame difference	Z	T+G-S	(Z-G)-(T-S)
Line difference of frame difference	Z	T+B-M	(Z-B)-(T-M)
Element difference of field difference	Z	$T+\frac{E+J}{2}-\frac{Q+W}{2}$	$(Z-\frac{E+J}{2})-(T-\frac{Q+W}{2})$

Different 3-D DPCM approaches

3-D DPCM

- It was found that element difference of field difference generally corresponds to lowest entropy -> most efficient
  - Frame difference and element difference correspond to higher entropy.
- To prevent transmission error propagation, linear predictor should use only pixels from the same line or the same line in the previous frame
- Combining the above two factors, element difference of *frame difference* prediction is preferred.

Motion Compensated Predictive Coding

- When frames taken densely enough, changes in successive frames can be attributed to the object motion.
- Under this assumption, if we can analyze object motion from successive frames, then we should be able to predict objects in next frame based on their positions in the previous frame, using the estimated motion.
- The difference between original frame and the predicted frame, and the motion vectors are then quantized and coded - **Motion Compensated Predictive Coding**.
  - If motion estimation is accurate enough, the motion compensated prediction error can be smaller than that in 3-D DPCM.
  - A major development in image sequence coding since 1980s.
  - Adopted by all international video coding standards.

Information Preserving Differential Coding

- DPCM involves quantization, hence is *lossy* coding
- Information preserving - Lossless coding
- In applications such as those involving scientific measurements, information preserving is required.
  - E.g., medical image coding
- Lossless differential coding

Information Preserving Differential Coding

- No quantizer
- Differential technique applied
- An efficient *lossless* coder is used
  - Huffman coding
  - Arithmetic coding