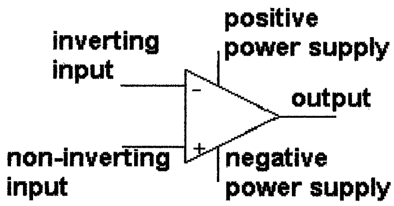
Problems

P 5.1 [a] The five terminals of the op amp are identified as follows:



[b] The input resistance of an ideal op amp is infinite, which constrains the value of the input currents to 0. Thus, $i_n = 0$ A.

[c] The open-loop voltage gain of an ideal op amp is infinite, which constrains the difference between the voltage at the two input terminals to 0. Thus, $(v_p - v_n) = 0$.

[d] Write a node voltage equation at v_n:

$$\frac{v_n - 1}{2000} + \frac{v_n - v_o}{8000} = 0$$

But $v_p = 0$ and $v_n = v_p = 0$. Thus,

$$\frac{-1}{2000} - \frac{v_o}{8000} = 0 \quad \text{so} \quad v_o = -4 \text{ V}$$

P 5.2 $\frac{v_b - v_a}{20} + \frac{v_b - v_o}{160} = 0,$ therefore $v_o = 9v_b - 8v_a$

[a] $v_a = 1.5$ V, $v_b = 0$ V, $v_o = -12$ V

[b] $v_a = 3.0$ V, $v_b = 0$ V, $v_o = -18$ V (sat)

[c] $v_a = 1.0$ V, $v_b = 2$ V, $v_o = 10$ V

[d] $v_a = 4.0$ V, $v_b = 2$ V, $v_o = -14$ V

[e] $v_a = 6.0$ V, $v_b = 8$ V, $v_o = 18$ V (sat)

[f] If $v_b = 4.5$ V, $v_o = 40.5 - 8v_a = \pm 18$

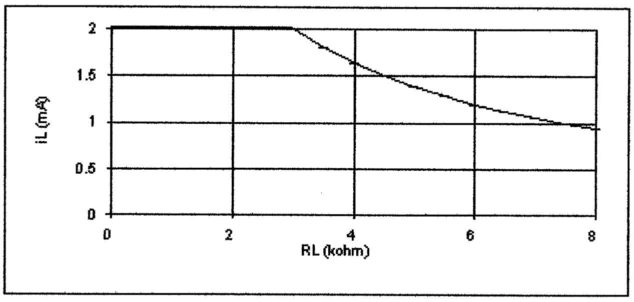
$$\therefore \quad 2.8125 \le v_a \le 7.3125 \text{ V}$$

P 5.3 $v_o = (1)(9) = 9$ V; $i_{15k\Omega} = \frac{9}{15} = 0.6\,\text{mA};$

$$i_{6k\Omega} = \frac{9}{6} = 1.5\,\text{mA}; \qquad i_{9k\Omega} = \frac{9}{9} = 1\,\text{mA}$$

$$\therefore \quad i_o = -0.6 - 1.5 - 1 = -3.1\,\text{mA}$$

[d]



P 5.36 [a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-90}{15}(-0.5) = 3 \text{ V}; \qquad v_{o2} = \frac{-120}{30}(0.4) = -1.6 \text{ V}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = -4.6\,\text{mA}$$

[b] $i_a = 0$ when $v_{o1} = v_{02}$ so from (a) $v_{o2} = 3$ V

Thus

$$\frac{-120}{30}(v_L) = 3$$

$$v_L = -\frac{90}{120} = -750 \text{ mV}$$

P 5.37 [a] $p_{16\text{k}\Omega} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\text{W}$

[b] $v_{16\text{k}\Omega} = \left(\dfrac{16}{64}\right)(320) = 80\,\text{mV}$

$$p_{16\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\text{W}$$

[c] $\dfrac{p_a}{p_b} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification.*
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.
- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 6.4 [a] $v = L\frac{di}{dt}$

$$\frac{di}{dt} = 20[e^{-5t} - 5te^{-5t}] = 20e^{-5t}(1-5t)$$

$$v = (100 \times 10^{-6})(20)e^{-5t}(1-5t)$$

$$= 2e^{-5t}(1-5t)\,\text{mV}, \quad t > 0$$

[b] $p = vi = 0.04te^{-10t}(1-5t)$

$$p(100\,\text{ms}) = 0.04(0.1)e^{-1}(1-0.5) = 735.76\,\mu\text{W}$$

[c] absorbing

[d] $i(100\,\text{ms}) = 20(0.1)e^{-0.5} = 2e^{-0.5}$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(100 \times 10^{-6})(2e^{-0.5})^2 = 73.58\,\mu\text{J}$$

[e] The energy is a maximum where the current is a maximum:

$$\frac{di_L}{dt} = 0 \quad \text{when} \quad 1 - 5t = 0 \quad \text{or} \quad t = 0.2\,\text{s}$$

$$i_{\text{max}} = 20(0.2)e^{-1} = 4e^{-1}\,\text{A}$$

$$w_{\text{max}} = \frac{1}{2}(100 \times 10^{-6})(4e^{-1})^2 = 108.27\,\mu\text{J}$$

P 6.5 [a] $0 \le t \le 2\,\text{s}:$

$$v = -25t$$

$$i = \frac{1}{2.5}\int_0^t -25x\,dx + 0 = -10\frac{x^2}{2}\Big|_0^t$$

$$i = -5t^2\,\text{A}$$

$2\,\text{s} \le t \le 6\,\text{s}:$

$$v = -100 + 25t$$

$$i(2) = -20\,\text{A}$$

$$\begin{aligned} \therefore\ i &= \frac{1}{2.5}\int_2^t (25x - 100)\,dx - 20 \\ &= 10\int_2^t x\,dx - 40\int_2^t dx - 20 \\ &= 5(t^2 - 4) - 40(t-2) - 20 \\ &= 5t^2 - 40t + 40\,\text{A} \end{aligned}$$

$6\,\text{s} \le t \le 10\,\text{s}:$

$v = 200 - 25t$

$i(6) = 5(36) - 240 + 40 = -20\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_6^t (200 - 25x)\,dx - 20 \\ &= 80\int_6^t dx - 10\int_6^t x\,dx - 20 \\ &= 80(t-6) - 10(t^2 - 36)/2 - 20 = 80t - 5t^2 - 320\,\text{A} \end{aligned}$$

$10\,\text{s} \le t \le 12\,\text{s}:$

$v = 25t - 300$

$i(10) = 800 - 500 - 320 = -20\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_{10}^t (25x - 300)\,dx - 20 \\ &= 10\int_{10}^t x\,dx - 120\int_{10}^t dx - 20 \\ &= 5(t^2 - 100) - 120(t - 10) - 20 \\ &= 5t^2 - 120t + 680\,\text{A} \end{aligned}$$

$t \ge 12\,\text{s}:$

$v = 0$

$i(12) = 5(12)^2 - 120(12) + 680 = -40\,\text{A}$

$$\begin{aligned} i &= \frac{1}{2.5}\int_{12}^t 0\,dx - 40 \\ &= -40\,\text{A} \end{aligned}$$

[b] For $0 \le t \le 2\,\text{s}$, $v = -25t\,\text{V}$; $i = -5t^2\,\text{A}$

$v = 0$ when $t = 0$ so $i = 0\,\text{A}$

For $2 \le t \le 6\,\text{s}$, $v = -100 + 25t\,\text{V}$; $i = 5t^2 - 40t + 40\,\text{A}$

$v = 0$ when $t = 4\,\text{s}$ so $i = 5(4)^2 - 40(4) + 40 = -40\,\text{A}$

For $6 \le t \le 10\,\text{s}$, $v = 200 - 25t\,\text{V}$; $i = -5t^2 + 80t - 320\,\text{A}$

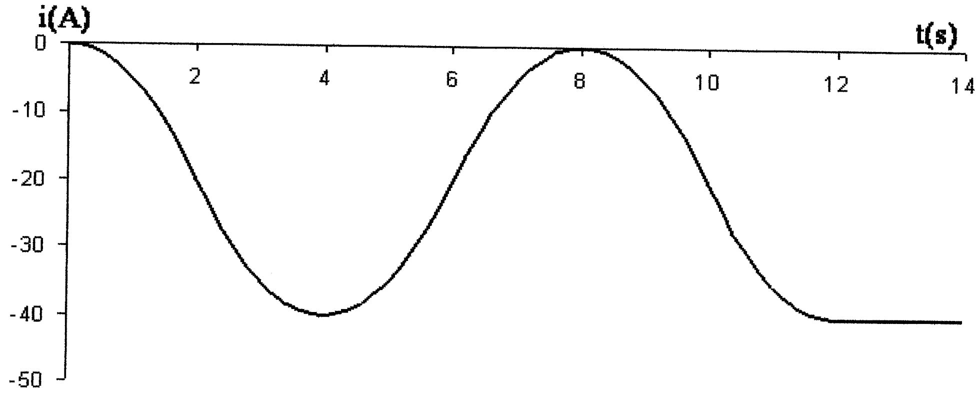
$v = 0$ when $t = 8\,\text{s}$ so $i = -5(8)^2 + 80(8) - 320 = 0\,\text{A}$

For $10 \le t \le 12\,\text{s}$, $v = 25t - 300\,\text{V}$; $i = 5t^2 - 120t + 680\,\text{A}$

$v = 0$ when $t = 12\,\text{s}$ so $i = 5(12)^2 - 120(12) + 680 = -40\,\text{A}$

For $t \ge 12\,\text{s}$, $v = 0$; $i = -40\,\text{A}$

[c]



P 6.6 [a] $v_L = L\dfrac{di}{dt} = [56\cos 140t + 92\sin 140t]e^{-20t}\,\text{mV}$

$$\therefore \quad \frac{dv_L}{dt} = [11{,}760\cos 140t - 9680\sin 140t]e^{-20t}\,\text{mV/s}$$

$$\frac{dv_L}{dt} = 0 \quad \text{when} \quad \tan 140t = \frac{11{,}760}{9680} = 1.21$$

$$\therefore \quad t = 6.30\,\text{ms}$$

Also $\quad 140t = 0.8821 + \pi \qquad$ etc.

Because of the decaying exponential v_L will be maximum the first time the derivative is zero.

[b] $v_L(\text{max}) = [56\cos 0.8821 + 92\sin 0.8821]e^{-0.12602} = 93.997\,\text{mV}$

$v_L\,\text{max} \approx 94\,\text{mV}$

Note: When $\quad t = \dfrac{0.8821 + \pi}{140}; \qquad v_L = -60\,\text{mV}$

P 6.7 [a]

$$\begin{aligned} i &= \frac{1000}{50}\int_0^t 250\sin 1000x\,dx - 5 \\ &= 5000\int_0^t \sin 1000x\,dx - 5 \\ &= 5000\left[\frac{-\cos 1000x}{1000}\right|_0^t - 5 \\ &= 5(1 - \cos 1000t) - 5 \\ i &= -5\cos 1000t\,\text{A} \end{aligned}$$

P 6.18 [a]
$$v = 5\times 10^6\int_0^{250\times10^{-6}} 100\times 10^{-3}e^{-1000t}\,dt - 60.6$$
$$= 500\times 10^3\frac{e^{-1000t}}{-1000}\Big|_0^{250\times10^{-6}} - 60.6$$
$$= 500(1-e^{-0.25}) - 60.6 = 50\,\text{V}$$
$$w = \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.2)(10^{-6})(50)^2 = 250\,\mu\text{J}$$

[b] $v = 500 - 60.6 = 439.40\,\text{V}$
$$w = \frac{1}{2}(0.2)\times 10^{-6}(439.40)^2 = 19.31\,\text{mJ} = 19{,}307.24\,\mu\text{J}$$

P 6.19 [a] $w(0) = \frac{1}{2}C[v(0)]^2 = \frac{1}{2}(0.40)\times 10^{-6}(25)^2 = 125\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-1500t}$

$v(0) = A_2 = 25\,\text{V}$
$$\frac{dv}{dt} = -1500e^{-1500t}(A_1t + A_2) + e^{-1500t}(A_1)$$
$$= (-1500A_1t - 1500A_2 + A_1)e^{-1500t}$$
$$\frac{dv}{dt}(0) = A_1 - 1500A_2$$
$$i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$
$$\therefore\ \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{90\times 10^{-3}}{0.40\times 10^{-6}} = 225\times 10^3$$
$$\therefore\ 225\times 10^3 = A_1 - 1500(25)$$

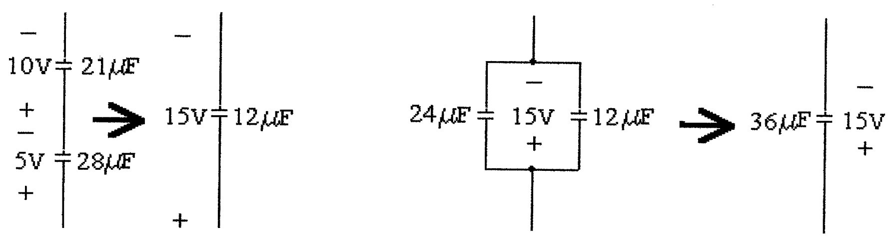
Thus, $A_1 = 2.25\times 10^5 + 3.75\times 10^4 = 262{,}500\,\frac{\text{V}}{\text{s}}$

[c] $v = (262{,}500t + 25)e^{-1500t}$
$$i = C\frac{dv}{dt} = 0.40\times 10^{-6}\frac{d}{dt}(262{,}500t + 25)e^{-1500t}$$
$$i = \frac{d}{dt}[(0.105t + 10\times 10^{-6})e^{-1500t}]$$
$$= (0.105t + 10\times 10^{-6})(-1500)e^{-1500t} + e^{-1500t}(0.105)$$
$$= (-157.5t - 15\times 10^{-3} + 0.105)e^{-1500t}$$
$$= (0.09 - 157.5t)e^{-1500t}\,\text{A}, \qquad t\ge 0$$
$$= (90 - 157{,}500t)e^{-1500t}\,\text{mA}, \qquad t\ge 0$$

P 6.25 $\frac{1}{21} + \frac{1}{28} = \frac{7}{84} \quad \therefore \quad C_{eq} = 12\,\mu\text{F}$

$-10\,\text{V} - 5\,\text{V} = -15\,\text{V}$

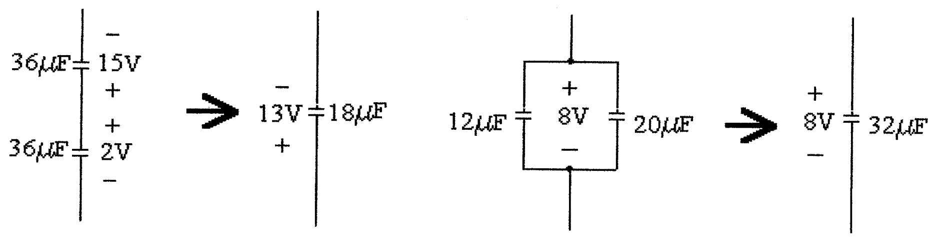
$24 + 12 = 36\,\mu\text{F}$



$\frac{1}{36} + \frac{1}{36} = \frac{2}{36} \quad \therefore \quad C_{eq} = 18\,\mu\text{F}$

$-15\,\text{V} + 2\,\text{V} = -13\,\text{V}$

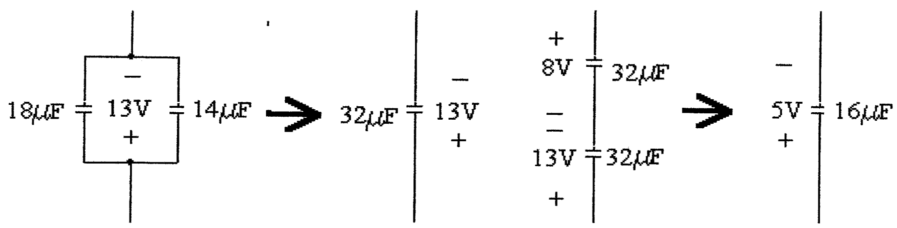
$12 + 20 = 32\,\mu\text{F}$



$18 + 14 = 32\,\mu\text{F}$

$\frac{1}{32} + \frac{1}{32} = \frac{2}{32} \quad \therefore \quad C_{eq} = 16\,\mu\text{F}$

$8\,\text{V} - 13\,\text{V} = -5\,\text{V}$



P 6.32

$$\begin{aligned} v_2(t) &= 20\times 10^{-3}\frac{di_o}{dt} \\ &= (20\times 10^{-3})(50\times 10^{-3})\{e^{-8000t}[-6000\sin 6000t + 12{,}000\cos 6000t] \\ &\quad +(-8000e^{-8000t})[\cos 6000t + 2\sin 6000t]\} \\ &= e^{-8000t}\{4\cos 6000t - 22\sin 6000t\}\ \text{V} \end{aligned}$$

$\therefore\ v_2(0) = 4\,\text{V}$

$i_0(0) = 50\,\text{mA}$

$v_R(0) = 320(50\times 10^{-3}) = 16\,\text{V}$

$v_1(0) = 16 + 4 = 20\,\text{V}$

P 6.33

$$\begin{aligned} v_c &= \frac{-10^6}{20}\int_0^t e^{-80x}\sin 60x\,dx - 300 \\ &= 5e^{-80t}[80\sin 60t + 60\cos 60t] + 300 - 300 \\ &= 400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t\ \ \text{V} \\ v_L &= 5\frac{di_o}{dt} \\ &= 5[-80e^{-80t}\sin 60t + 60e^{-80t}\cos 60t] \\ &= -400e^{-80t}\sin 60t + 300e^{-80t}\cos 60t\ \ \text{V} \\ v_o &= v_c - v_L \\ &= (300e^{-80t}\cos 60t - 300e^{-80t}\cos 60t + 400e^{-80t}\sin 60t + \\ &\quad 400e^{-80t}\sin 60t) \\ &= 800e^{-80t}\sin 60t\ \ \text{V} \end{aligned}$$

P 6.34 [a] $5\frac{di_g}{dt} + 40\frac{di_2}{dt} + 90i_2 = 0$

$$40\frac{di_2}{dt} + 90i_2 = -5\frac{di_g}{dt}$$

[b] $i_2 = e^{-t} - 5e^{-2.25t}\,\text{A}$

$$\frac{di_2}{dt} = -e^{-t} + 11.25e^{-2.25t}\,\text{A/s}$$

$$i_g = 10e^{-t} - 10\,\text{A}$$