Convolution. Signal Bandwidth. Dirac Delta Functions. Filter Impulse and Frequency Response.

Lecture Outline

- **Convolution Review**
- **Signal Bandwidth**
- **Dirac Delta Function and its Properties**
- **Filter Impulse and Frequency Response**

1. Convolution Review
   - Convolution integral involves product of two signals that are functions of integration variable.
   - To get product, flip one signal and drag it across the other.
   - Area under product at drag offset is convolution integral.
   - Convolution best understood through pictures and practice.

2. Signal Bandwidth
   - For bandlimited signals, bandwidth B defined as range of *positive* frequencies for which $|X(f)| > 0$.
   - In practice all signals are time-limited and therefore are not band-limited.
   - Need alternate definitions of bandwidth that indicate how much spectrum a signal occupies.
   - Common definitions include null-to-null and 3dB bandwidth definitions.
   - When a real baseband signal is upconverted to a carrier frequency, its bandwidth (under any definition) typically doubles.

3. Dirac Delta Function ($\delta(t)$)
   - A mathematical construct useful in analyzing signals and filters
   - Defined as a signal that is zero everywhere except at zero, and integrates to one.
   - Alternate definition is that it is the limit of contracting rectangle functions of unit area.
   - Also called an impulse function.

4. Properties of Delta Functions
   - Any signal convolved with the delta function yields the original signal.
   - The Fourier transform of $\delta(t)$ is one.
   - Constant signals in time become delta functions in frequency.

5. Filter Impulse and Frequency Response

   - We define a filter's impulse response $h(t)$ to be the filter output in response to a delta function as input.
   - We define the frequency response of a filter $H(f)$ to be the Fourier transform of its impulse response.
   - This implies that the filter frequency response is the output $Y(f)$ of the filter with input 1 in the frequency domain.
   - It is easier in practice to measure a filter's frequency response than it is to measure its impulse response.
   - Recall that the output of a filter with exponential input is the same exponential weighted by the filter frequency response at the exponential frequency. To see this, we need the Fourier Transform of exponential functions.

Main Points:

- Convolution is a drag (and a flip).
- Signal bandwidth definition depends on its use.
- Dirac delta function is idealized signal useful in signal analysis. item Filter impulse response defined as the filter output to a delta function input.
- Filter frequency response is Fourier transform of its impulse response.