Key Fourier Transform Properties and their Application.

Lecture Outline

- **Time Scaling**
- **Duality**
- **Frequency Shifting (Modulation)**
- **Multiplication in Time**
- **Convolution in Time**

1. The **key** Fourier Transform properties that will be used throughout the course are *time scaling*, *duality*, *frequency shifting*, *multiplication*, and *convolution*.

2. **Time-Scaling:** $x(at) \Leftrightarrow \frac{1}{|a|}X(f/a)$
   - Indicates that streching of time axis leads to contraction of frequency axis and vice versa.

3. **Duality:** If $x(t) \Leftrightarrow X(f)$, then $X(t) \Leftrightarrow x(-f)$.
   - Symmetry between time and frequency domains: intuition gained from one transform pair can be applied to its dual.
   - Eliminates half the transform calculations.

4. **Frequency Shifting (Modulation):** $e^{j2\pi f_c t}x(t) \Leftrightarrow X(f - f_c)$.
   - Modulates a signal to be centered at a different frequency.
   - Radio signal example: modulation and demodulation.

5. **Multiplication in Time:**
$$x_1(t)x_2(t) \Leftrightarrow \int_{-\infty}^{\infty} X_1(\lambda)X_2(f-\lambda)d\lambda.$$
   - Simple multiplication in time becomes a complicated convolution in frequency.
   - Most modulation and demodulation techniques involve multiplication.
   - Can use multiplication property to show that a time limited signal cannot be bandlimited (and vice versa by duality).

6. **Convolution in Time:**
$$\int_{-\infty}^{\infty} x_1(t)x_2(t-\tau)d\tau \Leftrightarrow X_1(f)X_2(f).$$
   - Convolution in time much easier to analyze in the frequency domain.
   - Convolution in time defines the output of a time-invariant filter to a given input.
   - Analysis and design of filters typically done in the frequency domain.

7. **Convolution Review**

    - Convolution integral involves product of two signals that are functions of integration variable.
    - To get product, flip one signal and drag it across the other.
    - Area under product at drag offset is convolution integral.
    - Convolution best understood through pictures and practice.

Main Points:

- Time scaling contracts a signal along the time axis, which stretches it along the frequency axis.
- The time and frequency domains are duals of each other in Fourier analysis.
- Frequency shifting is obtained by modulating a signal in time.
- Multiplication in time becomes convolution in frequency: time windowing becomes frequency convolution with sinc function.
- Convolution is a drag (and a flip).
- Filter outputs obtained by convolution: analysis much simpler in the frequency domain.