Lecture 6 - EE 104: Introduction to Communications - Winter 2003

Fourier Transform Properties.

Lecture Outline

- **Useful Properties of Fourier Transforms**
  - **Linearity**
  - **Time shifting**
  - **Time differentiation and integration**
  - **DC calculation**
  - **Conjugation**
  - **Parseval's relation**
- **Key Fourier Transform Properties**
  - **Time Scaling**
  - **Duality**
  - **Frequency Shifting (Modulation)**

1. The properties of *linearity*, *time shifting*, *time differentiation and integration*, *DC calculation*, *conjugation*, and *Parseval's relation* are useful in Fourier analysis.

2. **Linearity:** $ax_1(t) + bx_2(t) \Leftrightarrow aX_1(f) + bX_2(f)$
   - Important property that is often used with other properties to get new transforms.

3. **Time Shift:** $x(t-\tau) \Leftrightarrow e^{-j2\pi f\tau}X(f)$.
   - Can use time shift to get transform of alternating or staircase pulse

4. **Differentiation in Time:**
$$\frac{d}{dt}x(t) \Leftrightarrow j2\pi f X(f).$$
   - Simplifies the solution to linear systems equations (control systems).
   - Extends to nth order derivatives.

5. **Integration in Time:** For signals with $X(0) = 0$ (zero initial conditions),
$$\int_{-\infty}^{t} x(t)dt \Leftrightarrow \frac{1}{j2\pi f}X(f).$$
   - Integration in time becomes simple multiplication in frequency.
   - Nonzero initial conditions will be studied shortly using unit step function.
   - Use integration property to find Fourier transform of triangle pulse.

6. **DC Calculation:** $X(0) = \int x(t)dt$ similarly $x(0) = \int X(f)df$.
   - Fourier transform at frequency $f = 0$ is integral of signal over time.

- Can use this property to solve hard integrals of signals using their Fourier transforms.

7. **Conjugation:** $x^*(t) \Longleftrightarrow X^*(-f)$.

   - Indicates symmetries of time and frequency domains.
   - Shows that for real signals, $X(f) = X^*(-f)$.

8. **Parseval's Relation:**

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 dt.$$

   - Can compute signal power in either time or frequency domain.

9. The properties of *time scaling*, *duality*, *frequency shifting*, *multiplication*, and *convolution* are key for understanding the relationship between time and frequency domains and for the study of communication systems.

10. **Time-Scaling:** $x(at) \Leftrightarrow \frac{1}{|a|}X(f/a)$

   - Indicates that streching of time axis leads to contraction of frequency axis and vice versa.

11. **Duality:** If $x(t) \Leftrightarrow X(f)$, then $X(t) \Leftrightarrow x(-f)$.

   - Symmetry between time and frequency domains: intuition gained from one transform pair can be applied to its dual.
   - Eliminates half the transform calculations.

12. **Frequency Shifting (Modulation):** $e^{j2\pi f_c t}x(t) \Leftrightarrow X(f - f_c)$.

   - Modulates a signal to be centered at a different frequency.
   - Radio signal example: modulation and demodulation.

Main Points:

- Time delays lead to linear phase shifts.
- Signal power can be computed in time or frequency domains.
- Time scaling contracts a signal along the time axis, which stretches it along the frequency axis.
- The time and frequency domains are duals of each other in Fourier analysis.
- Frequency shifting is obtained by modulating a signal in time.