Lecture 5 - EE 104: Introduction to Communications - Winter 2003

Fourier Transforms.

Lecture Outline

- **Introduction to Fourier Transforms**
- **Fourier Transforms from Fourier Series**
- **Fourier Tranform Pair**
- **Signal Spectrum**
- **Fourier Transform for Rectangular Pulse**
- **Time/Bandwidth Tradeoffs**

1. Introduction to Fourier Transforms
   - The Fourier transform of a signal represents its spectral components.
   - The Fourier transform and inverse provide a 1-1 mapping between time and frequency domains.

2. Fourier Transforms from Fourier Series
   - To get Fourier transform from Fourier series, repeat a nonperiodic signal $x(t)$ at periodic intervals T_0.
   - This periodic repetition reflects the spectral content of the repeated signal at integer multiples of the fundamental frequency $nf_0 = n/T_0$.
   - As $T_0 \to \infty$ the periodic repetition converges to the original signal, and the samples of the spectrum at n/T_0 become a continuous function of f.

3. Fourier Tranform Pair
   - The Fourier transform pair is given by
   $$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt,$$
   and
   $$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} df.$$
   - The transform pair only exists when the Dirichlet conditions are satisfied ($x(t)$ absolutely integrable with a finite number of maxima/minima in any interval and a finite number of finite discontinuities).
   - Notation for transform pairs $x(t)$ and $X(f)$: $x(t) \Leftrightarrow X(f)$, $x(t) = \mathcal{F}^{-1}(X(f))$, $X(f) = \mathcal{F}(x(t))$.
   - A signal has a Fourier transform if it satisfies the Dirichlet conditions (integral is well behaved).

4. Signal Spectrum
   - The Fourier transform $X(f)$ is generally a complex function.
   - $X(f)$ typically represented in terms of its amplitude $|X(f)|$ and phase $\angle X(f)$.
   - For real signals, $|X(f)| = |X(-f)|$ and $\angle X(f) = -\angle X(-f)$.

5. Rectangular Pulse Example
   - The rectangular pulse is a very important signal in signal processing and communication systems.
   - The rectancular pulse in time acts as a time window.
   - The Fourier transform of a rectangular pulse is a sinc function with infinite frequency content.
   - Stretching the rectangular pulse along the time axis causes its Fourier transform to shrink along the frequency axis.

6. Time/Bandwidth Tradeoffs
   - In general, time limited signals have infinite bandwidth.
   - Shrinking a signal in the time domain causes it to stretch in the frequency domain.

Main Points:

- The Fourier tranform represents the spectral components of a signal.
- The Fourier transform pair allows a signal to be uniquely represented in either the time or frequency domains.
- The Fourier transform usually represented in terms of amplitude and phase, which have special symmetries for real functions.
- The Fourier transform pair of a rectangular pulse in time with sinc functions in frequency is critical in the study of communication systems.
- Shrinking a signal along the time axis causes it to stretch along the frequency axis.
- A signal cannot be both time-limited and band-limited.