4.4-1 In Fig. 4.14, when the local carrier is $\cos[(\omega_c+\Delta\omega)t+\delta]$ or $\sin[(\omega_c+\Delta\omega)t+\delta]$, we have

$$x_1(t) = 2[m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t]\cos[(\omega_c+\Delta\omega)t+\delta]$$

$$= 2m_1(t)\cos\omega_c t\cos[(\omega_c+\Delta\omega)t+\delta] + 2m_2(t)\sin\omega_c t\cos[(\omega_c+\Delta\omega)t+\delta]$$

$$= m_1(t)\{\cos(\Delta\omega t+\delta) + \cos[(2\omega_c+\Delta\omega)t+\delta]\} + m_2(t)\{\sin[(2\omega_c+\Delta\omega)t+\delta] - \sin(\Delta\omega t+\delta)\}$$

Similarly

$$x_2(t) = m_1(t)\{\sin[(2\omega_c+\Delta\omega)t+\delta] + \sin(\Delta\omega t+\delta)\} + m_2(t)\{\cos(\Delta\omega t+\delta) - \cos[(2\omega_c+\Delta\omega)t+\delta]\}$$

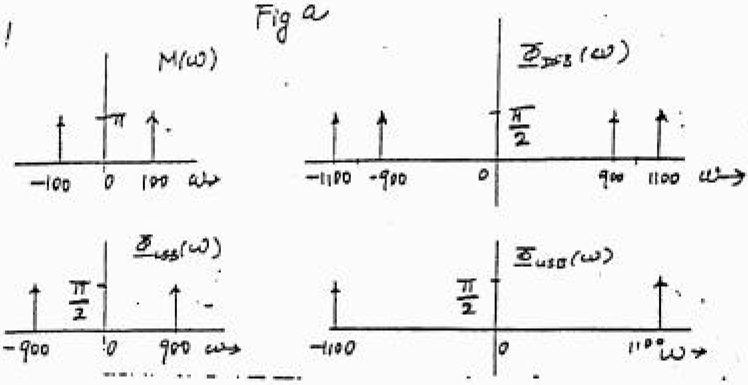
After $x_1(t)$ & $x_2(t)$ are passed through lowpass filters, the outputs are

$$m_1'(t) = m_1(t)\cos(\Delta\omega t+\delta) - m_2(t)\sin(\Delta\omega t+\delta)$$

$$m_2'(t) = m_1(t)\sin(\Delta\omega t+\delta) + m_2(t)\cos(\Delta\omega t+\delta)$$

4.5-1

Fig a



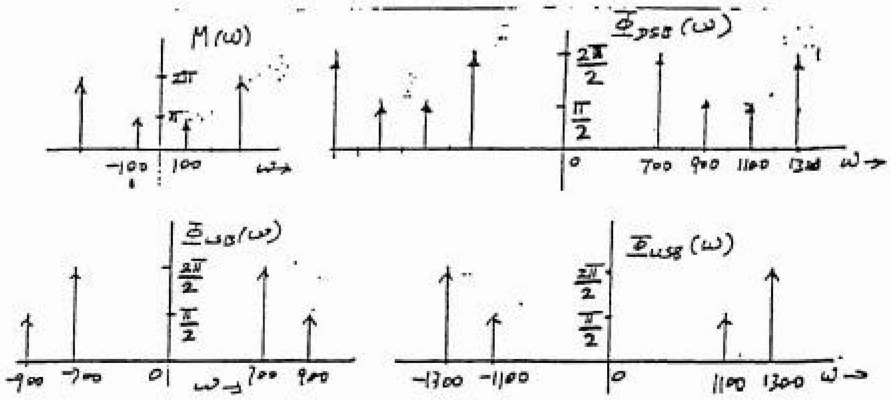
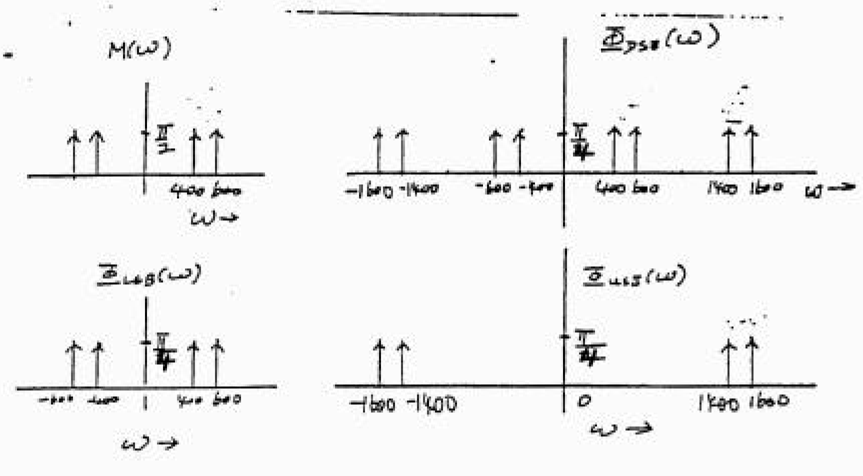


Fig b.

Fig S4.5-1



a) $\varphi_{USB}(t) = \frac{1}{2}\cos(1100t)$, $\varphi_{LSB}(t) = \frac{1}{2}\cos(900t)$

b) $\varphi_{USB}(t) = \frac{1}{2}\cos(1100t) + \cos(1300t)$

$\varphi_{LSB}(t) = \cos(700t) + \frac{1}{2}\cos(900t)$

c) $\varphi_{USB}(t) = \frac{1}{4}[\cos(1400t) + \cos(1600t)]$

$\varphi_{LSB}(t) = \frac{1}{4}[\cos(400t) + \cos(600t)]$

4.5-2 $\phi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$

$\phi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t$

(a) $m(t) = \cos 100t$ and $m_h(t) = \sin 100t$. Hence

$$\phi_{LSB}(t) = \cos 100t \cos 1000t + \sin 100t \sin 1000t = \cos(1000-100)t = \cos 900t$$

$$\phi_{USB}(t) = \cos 100t \cos 1000t - \sin 100t \sin 1000t = \cos(1000+100)t = \cos 1100t$$

(b) $m(t) = \cos 100t + 2\cos 300t$ and $m_h(t) = \sin 100t + 2\sin 300t$

$$\phi_{LSB}(t) = (\cos 100t + 2\cos 300t)\cos 1000t + (\sin 100t + 2\sin 100t)\sin 1000$$
$$= \cos 900t + 2\cos 1300t$$

$$\phi_{USB}(t) = (\cos 100t + 2\cos 300t)\cos 1000t - (\sin 100t + 2\sin 100t)\sin 1000t$$
$$= \cos 1100t + 2\cos 1300t$$

(c) $m(t) = \cos 100t \cos 500t = \frac{1}{2}\cos 400t + \frac{1}{2}\cos 600t$

$m_h(t) = \frac{1}{2}\sin 400t + \frac{1}{2}\sin 600t$

$$\phi_{LSB}(t) = \left(\tfrac{1}{2}\cos 400t + \tfrac{1}{2}\cos 600t\right)\cos 1000t + \left(\tfrac{1}{2}\sin 400t + \tfrac{1}{2}\sin 600t\right)\sin 1000t$$
$$= \tfrac{1}{2}\cos 400t + \tfrac{1}{2}\cos 600t$$

$$\phi_{USB}(t) = \left(\tfrac{1}{2}\cos 400t + \tfrac{1}{2}\cos 600t\right)\cos 1000t - \left(\tfrac{1}{2}\sin 400t + \tfrac{1}{2}\sin 600t\right)\sin 1000t$$
$$= \tfrac{1}{2}\cos 1400t + \tfrac{1}{2}\cos 1600t$$

4.5-5

The incoming SSB signal at the receiver is given by† [Eq. 4.17b ~~(4.19)~~]

$$\varphi_{SSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t$$

Let the local carrier be $\cos[(\omega_c + \Delta\omega)t + \delta]$. The product of the incoming signal and the local carrier is $e_d(t)$, given by

$$e_d(t) = \varphi_{SSB}(t)\cos[(\omega_c + \Delta\omega)t + \delta]$$

$$= 2[m(t)\cos\omega_c t + m_h(t)\sin\omega_c t]\cos[(\omega_c + \Delta\omega)t + \delta]$$

The lowpass filter suppresses the sum frequency components centered at $(2\omega_c + \Delta\omega)$ leaving the difference frequency components centered $\Delta\omega$. Hence the output $e_o(t)$ is

$$e_o(t) = \{m(t)\cos[(\Delta\omega)t + \delta] - m_h(t)\sin[(\Delta\omega)t + \delta]\} \qquad (1)$$

Observe that if $\Delta\omega$ and δ are both zero, the output is

$$e_o(t) = m(t)$$

as expected. It is interesting to compare the effects of phase and frequency errors for DSB and SSB systems.

(a) It can be seen by setting $\delta = 0$ in Eq. (1) that the effect of a frequency error in SSB transmission is equivalent to generating another SSB signal with a carrier frequency $\Delta\omega$. This means each component of $m(t)$ is shifted by $\Delta\omega$ (Fig. S4.5-5b) Again, this may make the voice sound only slightly different, as long as $\Delta\omega$ is within limits. For voice signals, a frequency shift of ±20 Hz is tolerable. Most U.S. systems, however, restrict the shift to ±2 Hz.

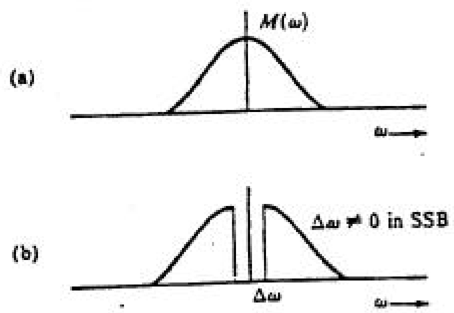


Fig. S4.5-5

(b) If $\Delta\omega = 0$, we observed that for DSB, the signal remains undistorted, although it is attenuated by a factor $\cos\delta$. If δ is close to $\pm\pi/2$, however, the signal attenuation can be very high. For SSB signals, on the other hand, when $\Delta\omega = 0$, the output is given by

$$e_o(t) = [m(t)\cos\delta - m_h(t)\sin\delta] \qquad (2)$$

We shall now show that the distortion in Eq. (2) is a phase distortion where each frequency component of $M(\omega)$ acquires a phase shift δ. From Eq. (2), we have

$$E_o(\omega) = [M(\omega)\cos\delta - M_h(\omega)\sin\delta]$$