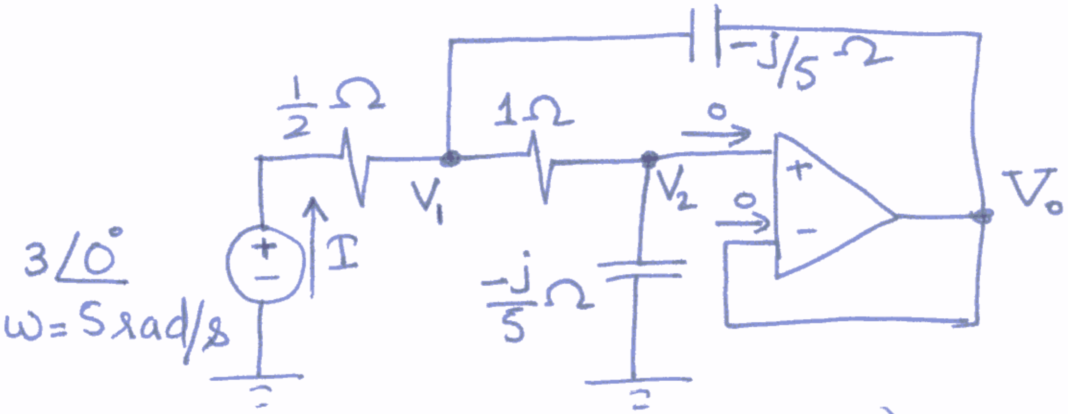
Homework #2 Solution

Q1 ⓐ



$V_2 = V_o$ (because of op amp characteristics).

Nodal at V_1 $\quad 2(V_1 - 3) + \frac{V_1 - V_2}{1} + (V_1 - V_o)(5j) = 0$

$\Rightarrow 2V_1 - 6 + V_1 - V_o + 5jV_1 - 5jV_o = 0$

$\Rightarrow (3+5j)V_1 - (1+5j)V_o = 6$ ——— ①.

Nodal at V_2 $\quad \frac{V_2 - V_1}{1} + (V_2)(5j) = 0$

$\Rightarrow V_o - V_1 + 5jV_o = 0 \Rightarrow V_1 = (1+5j)V_o$

Substitute in eq ①

① $\Rightarrow (3+5j)(1+5j)V_o - (1+5j)V_o = 6$

$\Rightarrow (3+20j-25-1-5j)V_o = 6 \Rightarrow (-23+15j)V_o = 6$

$\Rightarrow V_o = \frac{6}{-23+15j} = \frac{6}{27.4\angle 146.9} \Rightarrow V_o = 0.22\angle -146.9$

$= -0.18 - j0.12$

ⓑ Impedance seen by voltage source $= \frac{3\angle 0^\circ}{I}$.

~~Apply KCL at V_1~~ $\cancel{\Rightarrow I = \frac{V_1 - V_2}{1} + (V_1 - V_o)(5j)}$

$\cancel{\Rightarrow I = V_1 - V_o + 5jV_1 - 5jV_o}$

$\cancel{= (1+5j)V_o - V_o + 5j(1+5j)V_o - 5jV_o}$

$=$

$I = (3\angle 0^\circ - V_1)2 = (3 - (1+5j)V_o)2$

$= [3 - (1+5j)(-0.18 - j0.12)]2 = [3 - (-0.18 - j1.08 + 0.6)]2$

$= (2.58 + j1.08)2 = 5.16 + j2.16 = 5.59\angle 22.71$

$\therefore Z = \frac{3\angle 0^\circ}{5.59\angle 22.71} = 0.537\angle -22.71^\circ$

Q2. $T = 3$ sec.

$\underline{0 < t < 1}$ $\quad f(t) = t$

$\underline{1 < t < 2}$ $\quad f(t) = +2$

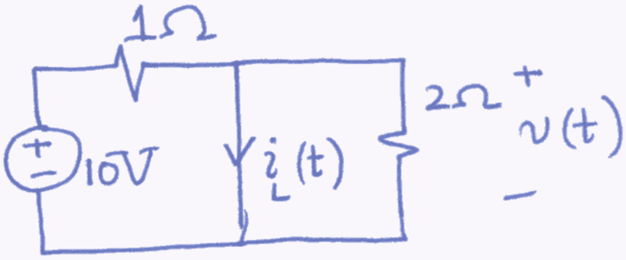
$\underline{2 < t < 3}$ $\quad f(t) = -1$

$$\therefore \quad f_{rms} = \sqrt{\frac{1}{3}\left[\int_0^1 t^2\,dt + \int_1^2 2^2\,dt + \int_2^3 (-1)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{3}\left[\frac{1}{3}t^3\Big|_0^1 + 4t\Big|_1^2 + t\Big|_2^3\right]}$$

$$= \sqrt{\frac{1}{3}\left[\frac{1}{3} + 4 + 1\right]} = \sqrt{\frac{16}{9}} = \frac{4}{3} = 1.77$$

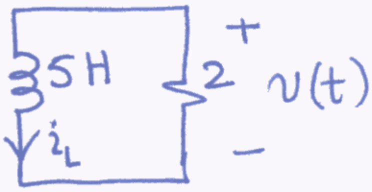
Q3. For $t < 0$



$i_L(t) = \frac{10}{1} = 10\,A$; for $t \le 0$

$v(t) = 0$ (because short ckt)

For $0 < t < 2$.



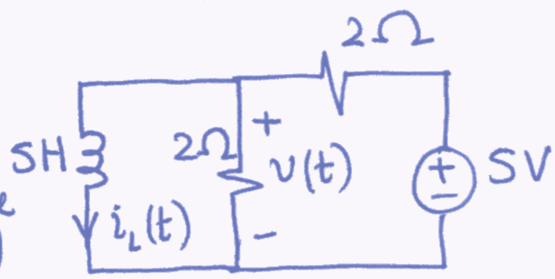
$\tau = L/R = \frac{5}{2}$; $i_L(0) = 10A$

$i_L(t) = i_L(0)\,e^{-t/\tau} = 10e^{-2t/5}$

$v_L(t) = v(t) = L\frac{di_L}{dt} = -20e^{-2t/5}$

At $t = 2$sec $\quad i_L(t) = 4.49\,A$

For $t > 2$



$i_L(t) = F + (I - F)\,e^{-(t-2)/\tau}$

i_L final value $= \frac{5}{2}$ A. (inductor behave as short ckt)

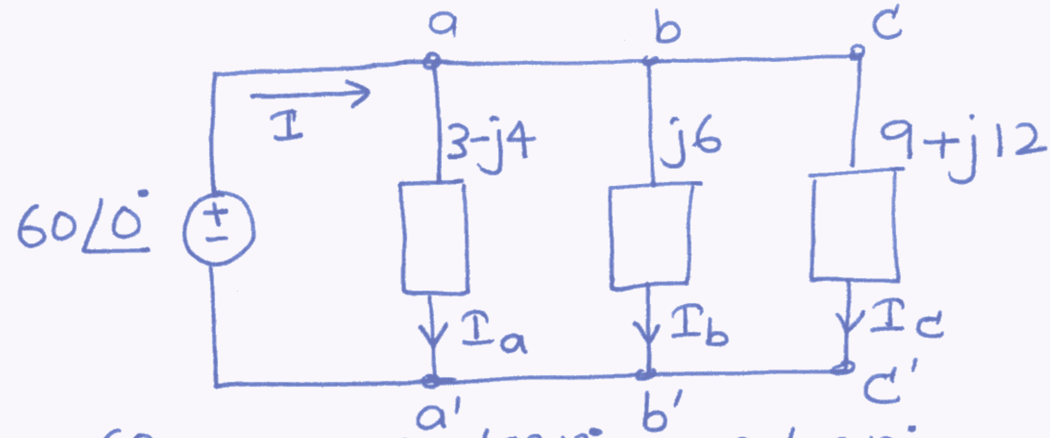
i_L initial value $= i_L(2) = 4.49\,A$.

$\tau = L/R_{eq} = 5/1 = 5$ sec.

$i_L(t) = \frac{5}{2} + \left(4.49 - \frac{5}{2}\right)e^{-(t-2)/5} = \frac{5}{2} + 2e^{-\frac{1}{5}(t-2)}$; For $t \geq 2$

$v_L(t) = v(t) = L\frac{di_L}{dt} = -2e^{-\frac{1}{5}(t-2)}$; For $t > 2$

Q4 (a).



$$I_a = \frac{60}{3-j4} = \frac{60}{5\angle -53.13^\circ} = \frac{60}{5}\angle 53.13^\circ = 12\angle 53.13^\circ.$$

$$I_b = \frac{60}{j6} = 10\angle -90^\circ$$

$$I_c = \frac{60}{9+j12} = \frac{60}{15\angle 53.13^\circ} = 4\angle -53.13^\circ.$$

For branch a-a'. $\quad S = \underline{V}\,\underline{I_a}^* = (60\angle 0^\circ)(12\angle -53.13^\circ)$

$\Rightarrow S = 720\angle -53.13^\circ = 432 - j576$ VA

$\therefore P = 432\,W$; $Q = -576$ VAr ; $P.f = \cos(-53.13^\circ) = 0.6$

For branch b-b' $\quad S = \underline{V}\,\underline{I_b}^* = (60\angle 0^\circ)(10\angle -90^\circ)$

$\Rightarrow S = 600\angle -90^\circ = -j600$ VA.

$\therefore P = 0\,W$; $Q = -600$ VAr ; $P.f = \cos(-90) = 0$

For branch c-c' $\quad S = \underline{V}\,\underline{I_c}^* = (60\angle 0^\circ)(4\angle -53.13^\circ)$

$\Rightarrow S = 240\angle -53.13^\circ \Rightarrow S = 144 - j192$ VA

$\therefore P = 144\,W$; $Q = -192$ VAr ; $P.f = \cos(-53.13) = 0.6$

(b) $I = I_a + I_b + I_c \Rightarrow I = 12\angle 53.13^\circ + 10\angle -90^\circ + 4\angle -53.13^\circ$

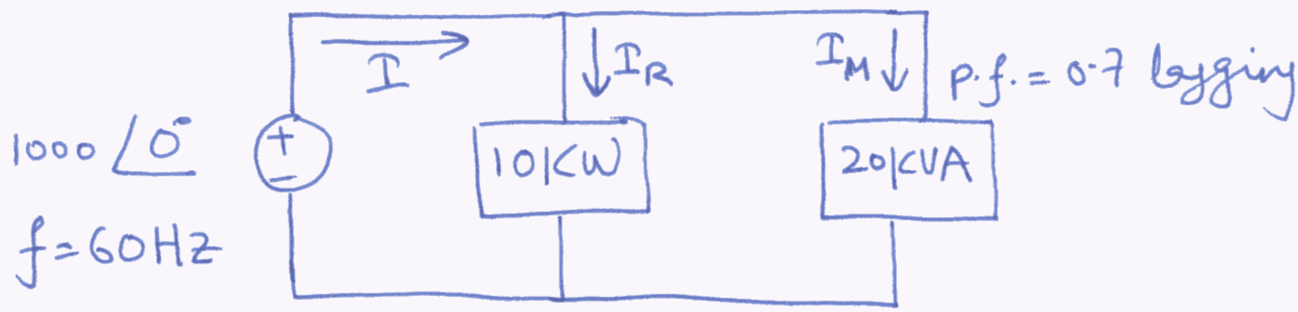
$\Rightarrow I = 7.2 + j9.6 - j10 + 2.4 - j3.2$

$\Rightarrow I = 9.6 - j3.6 = 10.25\angle -20.6^\circ.$

$\therefore P.f = \cos(\theta_V - \theta_I) = \cos(0 - (-20.6)) = 0.936$

Since $\theta_V - \theta_I = +ve$ $\therefore$ lagging $\therefore$ inductive.

Q5



(a) $P = VI\cos\theta \Rightarrow 10{,}000 = 1000\, I_R \cdot 1 \Rightarrow |I_R| = 10A$

$\therefore I_R = 10\angle 0°$. ($\because$ resistive $\therefore \theta_I = \theta_V$).

For motor $|S| = |V||I_M| = 20{,}000$

$\Rightarrow |I_M| = \dfrac{20{,}000}{1000} = 20A.$

$\therefore I_M = 20\angle -45.57$

$\therefore I = I_R + I_M = 10 + 20\angle -45.57° = 10 + 14 - j14.3$

$I = 24 - j14.3$ A.

(b) After connecting a capacitor in parallel

$I = (I_R + I_M) + I_C$

$= (24 - j14.3) + (j\omega C)(V) = (24 - j14.3) + (j \times 2 \times \pi \times 60\, C)(1000)$

$I = 24 - j14.3 + j376991.11C$

$\Rightarrow I = 24 + j(376991.11C - 14.3)$.

$\Rightarrow \theta_I = \tan^{-1}\left(\dfrac{376991.11C - 14.3}{24}\right)$.

Since we want $\cos(\theta_V - \theta_I) = 0.95$

$\Rightarrow \theta_V - \theta_I = +18.2° \Rightarrow \theta_I = -18.2°.$

$\therefore \tan^{-1}\left(\dfrac{376991.11C - 14.3}{24}\right) = -18.2°.$

$\Rightarrow \boxed{C = 17\mu F}$