Exploiting Error-Control Coding and Cyclic-Prefix in Channel Estimation for Coded OFDM Systems

Ghazi A. Al-Rawi, Tareq Y. Al-Naffouri, Ahmad Bahai, and John Cioffi

***Abstract*—OFDM systems typically use coding and interleaving across subchannels to exploit frequency diversity on frequency-selective channels. This letter presents a low-complexity iterative algorithm for blind and semi-blind joint channel estimation and soft decoding in coded OFDM systems. The proposed algorithm takes advantage of the channel finite delay-spread constraint and the extra observation offered by the cyclic-prefix. It converges within a single OFDM symbol and, therefore, has a minimum latency.**

***Index Terms*—Adaptive equalization, blind channel estimation, iterative detection, multicarrier transmission.**

I. Introduction

OFDM is an effective multicarrier modulation technique for mitigating intersymbol interference (ISI) on frequency-selective wireless channels. Estimating the channel at the receiver enables coherent detection, which saves 3 dB compared to differential detection and allows the use of more efficient multi-amplitude signaling. OFDM systems usually use coding and interleaving across subchannels to exploit frequency diversity in frequency-selective channels. It is natural then to attempt to use this coding information to aid in estimating the channel as in [1], in which hard estimates of the decoded symbols were used.

Blind channel estimation techniques allow higher data rates since they eliminate the training overhead. Most of the proposed blind estimation techniques for OFDM systems [2], [3], however, ignore the coding information and thus typically require a large number of OFDM symbols to achieve a sufficiently accurate estimate of the channel. This requirement not only introduces a significant latency in the system, but also limits these techniques to slowly time-varying channels.

This paper presents an iterative algorithm for joint soft decoding and channel estimation that provides an accurate blind or semi-blind channel estimate within a single OFDM symbol. The proposed channel estimation technique is based on the EM algorithm [4] and is performed in the time domain, allowing the receiver to exploit the channel-length constraint and the extra observation offered by the cyclic-prefix.

Various iterative blind channel estimation techniques that exploit coding information have recently been suggested [5], [6]. Most of these techniques target single-carrier systems and, consequently, have to deal with complicated time-domain equalization. On the other hand, because of the cyclic-prefix, equalization in multicarrier systems is trivial, making adaptive equalization techniques even more attractive in these systems. A semi-blind (training-based) iterative algorithm based on soft decoding was presented in [7] for the detection of space-time block codes in unknown channel and interference environments.

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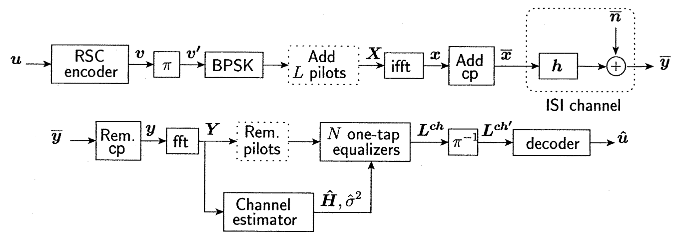

Fig. 1. Coded OFDM system model.

II. System Model

Fig. 1 shows the system model and the notation used in this paper. For simplification, we assume that each subchannel uses BPSK modulation. To minimize the complexity of the algorithm, the system uses 4-state recursive systematic convolutional (RSC) encoder with the generator matrix $G(D) = [1(1+D^2/1+D+D^2)]$, where D is a delay operator. To minimize the latency of the system, coding is assumed to be performed over a single OFDM symbol. The interleaver is assumed to be a random interleaver.

Let $\boldsymbol{X} = \boldsymbol{Q}\boldsymbol{x}$ be the BPSK modulated vector, where $\boldsymbol{Q}$ is the $N \times N$ DFT matrix. Let $\overline{\boldsymbol{y}}^T = [\,\underline{\boldsymbol{y}}^T \quad \boldsymbol{y}^T\,]$ be the output of the channel of length $N+\nu$, where $\underline{\boldsymbol{y}}$ is the cyclic-prefix observation of length ν, and $\boldsymbol{y}$ is the remaining part of length N, which can be obtained through the following cyclic convolution:

$$\boldsymbol{y} = \boldsymbol{h} \otimes \boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{h}$ is the channel impulse response, and $\boldsymbol{n}$ is a complex additive white Gaussian noise (AWGN) vector with the covariance matrix $\boldsymbol{R}_{nn} = \sigma^2 \boldsymbol{I}_N$. We can then write

$$\boldsymbol{Y} = \mathrm{diag}(\boldsymbol{H})\boldsymbol{X} + \boldsymbol{N} \tag{2}$$

where $\boldsymbol{Y} = \boldsymbol{Q}\boldsymbol{y}$, $\boldsymbol{N} = \boldsymbol{Q}\boldsymbol{n}$, and $\boldsymbol{H} = \boldsymbol{V}\boldsymbol{h}$, where $\boldsymbol{V}$ is an $N \times L$ matrix of the first L columns of $\boldsymbol{Q}$ scaled by $\sqrt{N}$. The notation $\mathrm{diag}(\boldsymbol{a})$ denotes the diagonal matrix formed by the vector $\boldsymbol{a}$ on the diagonal.

We assume that $\boldsymbol{h}$ can have up to L nonzero complex taps from 0 to $\nu = L - 1$ and that it is fixed over the period of a single OFDM symbol.

For the jth OFDM symbol, (2) can be rewritten as

$$\begin{aligned} \boldsymbol{Y}_j =& \mathrm{diag}(\boldsymbol{X}_j)\boldsymbol{H}_j + \boldsymbol{N}_j & (3)\\ =& \mathrm{diag}(\boldsymbol{X}_j)\boldsymbol{V}\boldsymbol{h}_j + \boldsymbol{N}_j. & (4) \end{aligned}$$

The cyclic-prefix observation of the jth OFDM symbol can be written as

$$\underline{\boldsymbol{y}}_j = \underline{\boldsymbol{xx}}_j \boldsymbol{h}_j + \underline{\boldsymbol{n}}_j \tag{5}$$

where $\underline{\boldsymbol{xx}}_j$ is the following toeplitz matrix of the cyclic-prefix parts of the current and previous transmitted OFDM symbols $\overline{\boldsymbol{x}}_j$ and $\overline{\boldsymbol{x}}_{j-1}$:

$$\underline{\boldsymbol{xx}}_j = \begin{bmatrix} x_0^j & x_{\nu-1}^{j-1} & x_{\nu-2}^{j-1} & \dots & x_0^{j-1} \\ x_1^j & x_0^j & x_{\nu-1}^{j-1} & \dots & x_1^{j-1} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x_{\nu-1}^j & x_{\nu-2}^j & \dots & x_0^j & x_{\nu-1}^{j-1} \end{bmatrix}. \tag{6}$$

Equations (4) and (5) can be combined as

$$\begin{bmatrix} \underline{\boldsymbol{y}}_j \\ \boldsymbol{Y}_j \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{xx}}_j \\ \mathrm{diag}(\boldsymbol{X}_j)\boldsymbol{V} \end{bmatrix} \boldsymbol{h}_j + \begin{bmatrix} \underline{\boldsymbol{n}}_j \\ \boldsymbol{N}_j \end{bmatrix} \tag{7}$$

which can be written in matrix form as

$$\boldsymbol{\mathcal{Y}}_j = \boldsymbol{A}_j \boldsymbol{h}_j + \boldsymbol{\mathcal{N}}_j. \tag{8}$$

Equation (8) represents the overall input/output relation of the system as a function of the channel impulse response. We propose a low-complexity iterative algorithm for finding a good-quality sub-optimal solution to the following joint maximum likelihood channel/data estimation problem:

$$\left(\hat{\boldsymbol{X}}_j, \hat{\boldsymbol{h}}_j\right) = \arg \max_{\boldsymbol{X}_j, \boldsymbol{h}_j} \left\{ p\left(\boldsymbol{\mathcal{Y}}_j | \boldsymbol{X}_j, \boldsymbol{h}_j\right) \right\}. \tag{9}$$

III. Iterative Joint Decoding and Channel Estimation

Fig. 2 shows a block diagram of the proposed iterative algorithm briefly described in the steps below. Since the noise variance does not usually vary too fast, for simplicity, we will assume that σ^2 is known to the receiver, i.e., $\hat{\sigma}^2 = \sigma^2$.

- **Step 1)** Find an initial channel estimate $\hat{\boldsymbol{h}}^{(it=0)}$, which is simply random in the blind case that uses no pilots, and which in the semi-blind case can be obtained as

$$\hat{\boldsymbol{h}}^{(it=0)} = \frac{1}{\sqrt{L}} \boldsymbol{Q}_L^H \hat{\boldsymbol{H}}^{\boldsymbol{p}} \tag{10}$$

  where $\hat{\boldsymbol{H}}^{\boldsymbol{p}}$ is an $L \times 1$ vector of the gain estimates $(\hat{H}_i^p = Y_i^p / X_i^p)$ of uniformly spaced L pilot subchannels, $\boldsymbol{Q}_L$ is an $L \times L$ DFT matrix, and ^{H} denotes a conjugate transpose operation. Then, $\hat{\boldsymbol{H}}^{(it=0)} = \boldsymbol{V}\hat{\boldsymbol{h}}^{(it=0)}$.
- **Step 2)** Given $\hat{\boldsymbol{H}}^{(it)}$, equalize the received vector $\boldsymbol{Y}$ using N parallel single-tap equalizers and obtain the vector $\boldsymbol{L}^{\boldsymbol{ch}(it+1)}$ of the *extrinsic* channel log-likelihood ratios (LLR's) as

$$L_i^{ch\,(it+1)} = \log \frac{p\left(Y_i | \hat{H}_i^{(it)}, X_i = +1\right)}{p\left(Y_i | \hat{H}_i^{(it)}, X_i = -1\right)} \tag{11}$$

$$= \frac{2}{\sigma^2} \cdot Re\left\{ Y_i \left(\hat{H}_i^{(it)}\right)^* \right\} \tag{12}$$

  where $i = 0, 1, \dots, N-1$.
- **Step 3)** Perform soft MAP sequence estimation. This can be implemented using the Max-Log-Map algorithm [8], which is less complex than the Log-Map algorithm. More importantly, the Max-Log-Map algorithm provides optimal MAP sequence (or OFDM symbol) estimation, as opposed to the optimal MAP BPSK symbol estimation provided by the Log-Map algorithm. The *extrinsic* log-likelihood ratios for the coded bits $\boldsymbol{L}^{\mathbf{ext}}$ are

$$\boldsymbol{L}^{\mathbf{ext}(it+1)} = \boldsymbol{L}^{\boldsymbol{app}(it+1)} - \boldsymbol{L}^{\boldsymbol{ch}(it+1)} \tag{13}$$

  where $\boldsymbol{L}^{\boldsymbol{app}}$ is the interleaved version of the a posteriori LLR's vector for the coded bits $\boldsymbol{L}^{\boldsymbol{app}'}$ provided by the Max-Log-Map algorithm. The extrinsic probabilities of the BPSK symbols are then obtained as

$$p^{\mathrm{ext}}(X_i = 1) = \frac{e^{L_i^{\mathrm{ext}}}}{1 + e^{L_i^{\mathrm{ext}}}}, \quad p^{\mathrm{ext}}(X_i = -1) = \frac{1}{1 + e^{L_i^{\mathrm{ext}}}} \tag{14}$$

  where $i = 0, 1, \dots, N-1$.
- **Step 4)** Use the *extrinsic* soft output of the decoder to find the maximum likelihood (ML) estimate of $\boldsymbol{h}$. The extrinsic (rather than the A POSTERIORI) soft output of the decoder is used to maintain the independence between the elements of $\boldsymbol{X}_j$ and to prevent self-biasing which leads to premature convergence. From the input/output relation (8), the ML estimate of $\boldsymbol{h}$ for the jth OFDM symbol can be obtained as

$$\hat{\boldsymbol{h}}_j^{(it+1)} = \arg \min_{\boldsymbol{h}_j} \|\boldsymbol{\mathcal{Y}}_j - \boldsymbol{A}_j \boldsymbol{h}_j\|^2. \tag{15}$$

  However, since the input $\boldsymbol{A}_j$ (or $\boldsymbol{X}_j$) is not directly observable, an averaged form of the cost function is minimized, which is the essence of the expectation maximization (EM) algorithm [4]:

$$\hat{\boldsymbol{h}}_j^{(it+1)} = \arg \min_{\boldsymbol{h}_j} E\|\boldsymbol{\mathcal{Y}}_j - \boldsymbol{A}_j \boldsymbol{h}_j\|^2 \tag{16}$$

$$= \left(E\left[\boldsymbol{A}_j^H \boldsymbol{A}_j | \boldsymbol{\mathcal{Y}}_j, \hat{\boldsymbol{h}}_j^{(it)}\right]\right)^{-1} E\left[\boldsymbol{A}_j | \boldsymbol{\mathcal{Y}}_j, \hat{\boldsymbol{h}}_j^{(it)}\right]^H \boldsymbol{\mathcal{Y}}_j. \tag{17}$$

  By noting that $E[\boldsymbol{x}] = \boldsymbol{Q}^H E[\boldsymbol{X}]$, the terms $E\left[\boldsymbol{A}_j^H \boldsymbol{A}_j | \boldsymbol{\mathcal{Y}}_j, \hat{\boldsymbol{h}}_j^{(it)}\right]$ and $E\left[\boldsymbol{A}_j | \boldsymbol{\mathcal{Y}}_j, \hat{\boldsymbol{h}}_j^{(it)}\right]$ can be expressed in terms of $E[\boldsymbol{X}_{j-1}]$, $E\left[\boldsymbol{X}_j | \boldsymbol{\mathcal{Y}}_j, \hat{\boldsymbol{h}}_j^{(it)}\right]$, and $E\left[\boldsymbol{X}_j \boldsymbol{X}_j^H | \boldsymbol{\mathcal{Y}}_j, \hat{\boldsymbol{h}}_j^{(it)}\right]$ [9], which in turn can be easily computed using the extrinsic probabilities given in (14). Simulation results do not show significant error propagation as a result of using the final soft estimate of the previous OFDM symbol $(E[\boldsymbol{X}_{j-1}])$ in the detection of the current symbol.
- **Step 5)** Return to Step 2, and repeat until a stopping criterion is reached.

IV. Simulation Results and Discussion

The proposed iterative algorithm was simulated using a 4-state rate-1/2 convolutional code with $G(D) = [1(1 + D^2/1 + D + D^2)]$. It is assumed here that $N = 128$, $L = 16$, and that BPSK modulation is used on each of the subchannels. The actual channel taps were generated from independent complex Gaussian distributions with zero means and equal variances.

In the blind case, the maximum number of iterations was set to 10. As with most hill-climbing techniques, in the blind

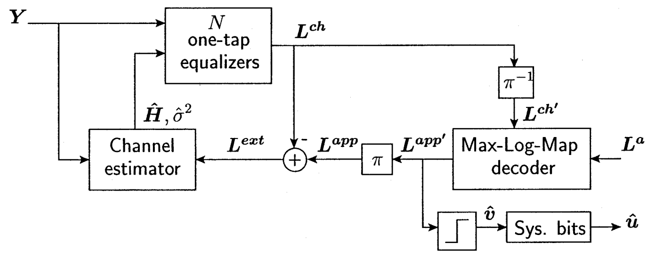

Fig. 2. Iterative decoding and channel estimation.

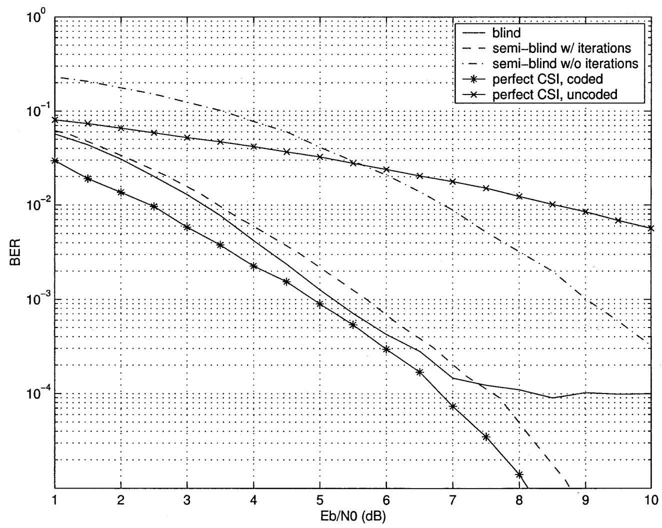

Fig. 3. BER versus E_b/N_0 for the various systems.

case, the proposed iterative algorithm can potentially become trapped in local minima or stationary points. Therefore, 3 trials (re-initializations of the iterative algorithm) were used starting with different random initial channel states. Simulations showed that this number of trials is enough for convergence with high probability in the *SNR* range of practical interest. The results of the best trial, defined as the one with the largest average of magnitudes of a posteriori LLR's at the output of the decoder, $\overline{|L^{app}|} = 1/N \cdot \sum_{i=0}^{N-1} |L_i^{app}|$, after 10 iterations, were then chosen. In the semi-blind case, L uniformly spaced pilot tones were used to initialize the iterative algorithm. The maximum number of iterations was set to 3 in this case.

The system performance using the proposed iterative algorithm for the blind and semi-blind cases was compared to that of an ideal coded system that has perfect channel state information (CSI) at the receiver. It was also compared to that of a non-iterative system that uses L pilot tones to estimate the channel and then performs MAP soft decoding with no iterations. Fig. 3 shows the bit error rate (BER) curves for the various systems, including the uncoded system with perfect CSI as a reference. Since these systems have different data rates, the BER is plotted against E_b/N_0 instead of *SNR*.

We observe that the performance degradation for the proposed blind system is about 0.25 dB relative to the ideal coded system at the BER of 10^{-3}. The error floor at 10^{-4} is caused by the occasional misconvergence of the iterative algorithm or convergence to local minima and can be easily eliminated by using an outer code. On the other hand, the semi-blind system using the proposed iterative algorithm has a degradation of about 0.75 dB relative to the ideal coded system at the BER of 10^{-3}. Most of this degradation is due to the rate loss caused by the transmission of L pilot tones, which is about 0.6 dB in this case. The Figure also shows that using the proposed iterative algorithm with the semi-blind system results in about a 3.25 dB gain with respect to the case with no iterations. This significant gain is achieved with only a small increase in complexity because the iterative algorithm converges within a few (<3) iterations in the semi-blind case.

V. Conclusion

Coding is typically used in OFDM systems to exploit frequency diversity. This paper presented a new low-complexity iterative algorithm that exploits this coding information. The proposed algorithm also takes advantage of the channel delay-spread constraint and the extra observation offered by the cyclic-prefix. This algorithm can be used to blindly estimate the channel within a single OFDM symbol or to significantly enhance the channel estimate obtained by L pilot tones.

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