FREQUENCY DOMAIN ESTIMATION OF MULTIPLE ACCESS OFDM CHANNELS

Tareq Y. Al-Naffouri and Ashfaq Ahmed Mukaddam

Electrical Engineering Dept., King Fahd University of Petroleum and Minerals, Saudi Arabia

ABSTRACT

OFDM modulation combines the advantages of high achievable rates and relatively easy implementation. However, for proper recovery of the input, the OFDM receiver needs accurate channel information. Most algorithms proposed in literature perform channel estimation in the time domain which increases the computational complexity in multiaccess situations where the user is only interested in part of the spectrum. In this paper, we propose a frequency domain algorithm for channel estimation in OFDM. The algorithm performs eigenvalue decomposition of the channel autocorrelation matrix and approximates the channel frequency response seen by each user by the first few dominant eigenvectors. This reduces the parameter estimation space and allows estimation using a few number of pilots.

***Index Terms*—** Estimation, Orthogonal

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is an effective technique for high bit rate transmission. It has found widespread applications and is already part of many standards including digital audio and video broadcasting (DAB DVB), high speed transmission over digital subscriber line (DSL), and wireless local area network (WLAN) standards (e.g., IEEE 802.11a/b/g and HIPERLAN/2). It has recently been proposed for broadband wireless metropolitan area networks (like WiMAX).

For proper operation of an OFDM system, the receiver needs an accurate estimate of the channel state information. There has been a lot of work on channel estimation for OFDM. One way of classifying this work is according to whether channel estimation is performed in time or frequency domain. In the following, we contrast the two approaches. We set the stage, however, by introducing the system model.

1.1. System model

Consider the length-N symbol $\boldsymbol{\mathcal{X}}$ that we wish to transmit. The symbol $\boldsymbol{\mathcal{X}}$ undergoes an IFFT operation to produce the time domain symbol $\boldsymbol{x} = \sqrt{N}\boldsymbol{Q}^*\boldsymbol{\mathcal{X}}$, where $\boldsymbol{Q}$ is the $N \times N$ FFT matrix for which $q_{l,m} = e^{-j\frac{2\pi(l-1)(m-1)}{N}}$. The transmitter then appends a cyclic prefix (CP) $\underline{\boldsymbol{x}}$ of length P to $\boldsymbol{x}$ resulting finally in the super-symbols $\overline{\boldsymbol{x}}$ of length $N+P$. We assume that the channel $\underline{\boldsymbol{h}}$ (of maximum length $P+1$) remains fixed over a given OFDM symbol (and its associated CP). At the channel output, we obtain the time-domain super-symbol $\overline{\boldsymbol{y}}$, which after stripping the cyclic prefix $\underline{\boldsymbol{y}}$, produces the time-domain symbol $\boldsymbol{y}$. The input/output (I/O) relationship of the OFDM system is best described in the frequency domain

$$\begin{aligned} \boldsymbol{\mathcal{Y}} &= \text{diag}(\boldsymbol{\mathcal{X}})\boldsymbol{\mathcal{H}} + \boldsymbol{\mathcal{N}} \quad (1) \\ &= \text{diag}(\boldsymbol{\mathcal{X}})\boldsymbol{Q}_P\underline{\boldsymbol{h}} + \boldsymbol{\mathcal{N}} \quad (2) \end{aligned}$$

where $\boldsymbol{\mathcal{N}} \sim \mathcal{N}(\boldsymbol{0}, \sigma_n^2\boldsymbol{I})$ is the additive noise and $\boldsymbol{\mathcal{Y}}$ and $\boldsymbol{\mathcal{H}}$ are the FFT's of y and $\underline{h}$ respectively. The second equality in (2) follows from the FFT relationship $\boldsymbol{\mathcal{H}} \triangleq \boldsymbol{Q}\begin{bmatrix}\underline{\boldsymbol{h}} \\ \boldsymbol{0}\end{bmatrix} = \boldsymbol{Q}_P\underline{\boldsymbol{h}}$, where $\boldsymbol{Q}_P$ consists of the first P columns of $\boldsymbol{Q}$. Alternatively, with $\boldsymbol{X} \triangleq \text{diag}(\boldsymbol{\mathcal{X}})\boldsymbol{Q}_P$, we can write

$$\boxed{\boldsymbol{\mathcal{Y}} = \boldsymbol{X}\underline{\boldsymbol{h}} + \boldsymbol{\mathcal{N}}} \quad (3)$$

1.2. Pilot/output relationships

In general, the receiver needs pilots to obtain a channel estimate. Let the index set $I_\text{p} = \{i_1, i_2, \ldots, i_{L_\text{p}}\}$ denote the pilot locations within the OFDM symbol. Also, let $\boldsymbol{X}_{I_\text{p}}$ denote the matrix $\boldsymbol{X}$ pruned of the rows that do not belong to I_p. Then, the pilot/output equation can be derived from the I/O relationship (3) as

$$\boxed{\boldsymbol{\mathcal{Y}}_{I_p} = \boldsymbol{X}_{I_p}\underline{\boldsymbol{h}} + \boldsymbol{\mathcal{N}}_{I_p}} \quad (4)$$

1.3. Disadvantages of channel estimation in the time domain

Most channel estimation algorithms for OFDM transmission perform estimation in the time domain (instead of the frequency domain) [1], [2], [3], [4]. By performing estimation in the time domain, one can decrease the degrees of freedom from N, the number of frequency bins, to $P+1$, the number of (time-domain) channel taps. This is a drastic reduction since the number of channel taps is usually less than the cyclic prefix which is usually designed to be less than $\frac{N}{4}$. The reduction in the parameter estimation space in turn results in improved estimation accuracy.

This gain, however, does not come for free. By performing the estimation in the time domain, we loose the diagonal structure of the channel. Thus, instead of the frequency domain relationship (1) in which the various equations are decoupled, we employ the time-frequency relationship (3) which is no more diagonal (decoupled). This loss in transparency in return complicates channel estimation and makes it more computationally complex. For example, while the estimation in (1) is performed on a bin by bin basis according to

$$\boxed{\hat{\mathcal{H}}(l) = \frac{\mathcal{Y}(l)}{\mathcal{X}(l)} \quad l = 1, 2, \cdots, N} \tag{5}$$

channel estimation in (3) requires size $P+1$ matrix inversion

$$\boxed{\underline{\hat{\boldsymbol{h}}} = (\boldsymbol{X}^{*}\boldsymbol{X})^{-1}\boldsymbol{X}^{*}\boldsymbol{\mathcal{Y}}} \tag{6}$$

Moreover, since data detection is best performed in the frequency domain, estimating the channel in the time domain makes it necessary to perform an extra IFFT operation (to obtain the frequency domain estimate $\hat{\boldsymbol{\mathcal{H}}}$ from the time domain estimate $\underline{\hat{\boldsymbol{h}}}$ and use it for data detection). Thus, for data-aided channel estimation techniques, each channel estimation step would require one such IFFT operation.

Apart from the computational complexity, performing channel estimation in the time domain might be oversolving a problem. For example, in multiple access OFDM systems, like WiMAX, which we consider in this paper, users are not interested in the whole frequency spectrum, but only in that part of the spectrum in which they are operating. Moreover, even if some users were interested in estimating the whole spectrum, many standards would not be able to support that as there are not enough pilots to do so.

1.4. Can we perform channel estimation reliably in the frequency domain?

The problem with channel estimation in the frequency domain is the increase in the number of parameters to be estimated [5]. If we can reduce the parameter estimation space, then we can avoid the one disadvantage of frequency domain estimation as compared to time domain estimation. The frequency response of the channel is inherently limited by the degrees of freedom of the time domain impulse response. How does this limited degree of freedom manifests itself in the frequency domain? Figure 1 demonstrates the length 64 frequency response resulting from a 16 tap channel with exponential decay profile similar to the one we employ in our simulations. Note that within a narrow enough band of spectrum, the spectrum looks linear. As such, we employ model reduction in this paper to estimate the spectrum, thereby reducing the number of parameters to be estimated.

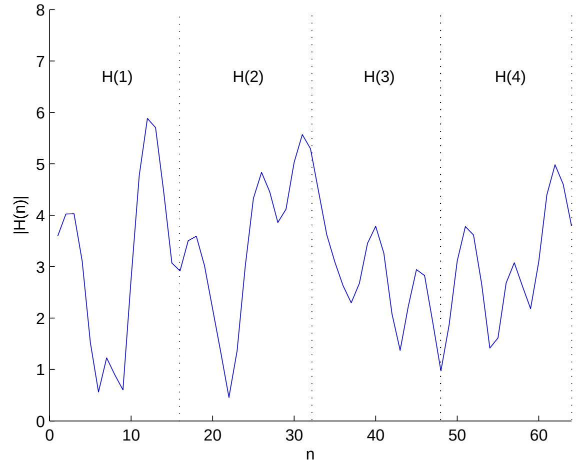

Figure 1. Frequency response of the channel ($L_f = 16$).

2. FREQUENCY DOMAIN CHANNEL ESTIMATION

In this section we derive the frequency domain based channel estimation algorithm. Our starting point is to partition the frequency response into a number of sections each of length L_f producing a total of $\lceil \frac{N}{L_f} \rceil$ sections[1]. Denote the j^{th} section of the frequency response by $\underline{\boldsymbol{\mathcal{H}}}^{(j)}$. Then the input/output equation that involves this section is given by

$$\boxed{\underline{\boldsymbol{\mathcal{Y}}}^{(j)} = \text{diag}(\underline{\boldsymbol{\mathcal{X}}}^{(j)})\underline{\boldsymbol{\mathcal{H}}}^{(j)} + \underline{\boldsymbol{\mathcal{N}}}^{(j)}} \tag{7}$$

where $\underline{\boldsymbol{\mathcal{Y}}}^{(j)}$, $\underline{\boldsymbol{\mathcal{X}}}^{(j)}$, $\underline{\boldsymbol{\mathcal{H}}}^{(j)}$ and $\underline{\boldsymbol{\mathcal{N}}}^{(j)}$ are the j^{th} section of $\boldsymbol{\mathcal{Y}}$, $\boldsymbol{\mathcal{X}}$, $\boldsymbol{\mathcal{H}}$ and $\boldsymbol{\mathcal{N}}$, respectively. Henceforth, we will suppress the dependence on j for notational convenience. Now let I_p denote the pilot locations within the j^{th} section, then the pilot/output equations are given by

$$\boxed{\underline{\boldsymbol{\mathcal{Y}}}_{I_p} = \text{diag}(\underline{\boldsymbol{\mathcal{X}}}_{I_p})\underline{\boldsymbol{\mathcal{H}}} + \underline{\boldsymbol{\mathcal{N}}}_{I_p}} \tag{8}$$

Obviously, the pilots are not enough to estimate the elements of $\underline{\boldsymbol{\mathcal{H}}}$. So we resort to model reduction starting from the autocorrelation function of $\underline{\boldsymbol{\mathcal{H}}}$, $\mathbf{R}_{\underline{\boldsymbol{\mathcal{H}}}}$. To this end, consider the eigenvalue decomposition of $\mathbf{R}_{\underline{\boldsymbol{\mathcal{H}}}}$,

$$\mathbf{R}_{\underline{\boldsymbol{\mathcal{H}}}} = \sum_{i=1}^{L_f} \lambda_i \mathbf{v}_i \mathbf{v}_i^T,$$

where $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_{L_f}$ are the (ordered) eigenvalues of $\mathbf{R}_{\underline{\boldsymbol{\mathcal{H}}}}$ and $\mathbf{v}_1, \ldots, \mathbf{v}_{L_f}$ are the corresponding eigenvectors. We can use this decomposition to represent $\underline{\boldsymbol{\mathcal{H}}}$ as

$$\underline{\boldsymbol{\mathcal{H}}} = \sum_{i=1}^{L_f} \alpha_i \mathbf{v_i}$$

[1]In an OFDM system, we can choose the section length to be the number of carriers allocated to each user. However, the sections need not have equal length over the frequency response.

where

$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_{L_f}]^T$$

is a parameter vector to be estimated, having zero mean and autocorrelation matrix

$$\mathbf{R}_\alpha = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{L_f}) \quad (9)$$

$$\triangleq \mathrm{diag}(\boldsymbol{\lambda}) \quad (10)$$

Using model reduction, we can represent $\underline{\boldsymbol{\mathcal{H}}}$ using the dominant eigenvalue and treat the rest as modeling noise[2], i.e.

$$\boxed{\underline{\boldsymbol{\mathcal{H}}} = \mathbf{V}_d\boldsymbol{\alpha}_d + \mathbf{V}_n\boldsymbol{\alpha}_n} \quad (11)$$

Upon Substituting (11) in (7), we obtain

$$\underline{\boldsymbol{\mathcal{Y}}} = \mathrm{diag}(\underline{\boldsymbol{\mathcal{X}}})\mathbf{V}_d\boldsymbol{\alpha}_d + (\underline{\boldsymbol{\mathcal{N}}} + \mathrm{diag}(\underline{\boldsymbol{\mathcal{X}}})\mathbf{V}_n\boldsymbol{\alpha}_n) \quad (12)$$

$$= \mathbf{A}\boldsymbol{\alpha}_d + \boldsymbol{\mathcal{N}}' \quad (13)$$

where $\mathbf{A} = \mathrm{diag}(\underline{\boldsymbol{\mathcal{X}}})\mathbf{V}_d$ and $\boldsymbol{\mathcal{N}}' = \underline{\boldsymbol{\mathcal{N}}} + \mathrm{diag}(\underline{\boldsymbol{\mathcal{X}}})\mathbf{V}_n\boldsymbol{\alpha}_n$ which we consider to be zero mean white gaussian noise with autocorrelation

$$\mathbf{R}_{\boldsymbol{\mathcal{N}}'} = \mathbf{R}_{\boldsymbol{\mathcal{N}}} + \mathrm{diag}(\underline{\boldsymbol{\mathcal{X}}})\mathbf{V}_n\mathrm{diag}(\boldsymbol{\lambda}_n)\mathbf{V}_n^*\mathrm{diag}(\underline{\boldsymbol{\mathcal{X}}})^*$$

We can now use the pilots to estimate the dominant interpolation parameters $\boldsymbol{\alpha}_d$. Starting from the I/O equation (13), we can construct a pilot/output equation similar to (4) by pruning $\underline{\boldsymbol{\mathcal{Y}}}$, $\mathbf{A}$ and $\underline{\boldsymbol{\mathcal{N}}}$ in (13) from the rows that don't belong to the pilot index I_p, to get

$$\underline{\boldsymbol{\mathcal{Y}}}_{I_p} = \mathbf{A}_{I_p}\boldsymbol{\alpha}_d + \boldsymbol{\mathcal{N}}'_{I_p}$$

We can now obtain the estimate $\boldsymbol{\alpha}_d$ that minimizes the mean square error (MSE) as

$$\boxed{\hat{\boldsymbol{\alpha}}_d = \mathbf{R}_{\alpha_d}\mathbf{A}_{I_p}^*[\mathbf{R}_{\boldsymbol{\mathcal{N}}'_{I_p}} + \mathbf{A}_{I_p}\mathbf{R}_{\alpha_d}\mathbf{A}_{I_p}^*)]^{-1}\underline{\boldsymbol{\mathcal{Y}}}_{I_p}} \quad (14)$$

The resulting mean square error is given by

$$\mathbf{R}_e = [\mathbf{R}_{\alpha_d}^{-1} + \mathbf{A}_{I_p}^*\mathbf{R}_{\boldsymbol{\mathcal{N}}'_{I_p}}^{-1}\mathbf{A}_{I_p})]^{-1}$$

The estimate of the j^{th} section of the spectrum is then given by

$$\hat{\underline{\boldsymbol{\mathcal{H}}}} = \mathbf{V}_d\hat{\boldsymbol{\alpha}}_d$$

The concatenation of all $\lceil \frac{N}{L_f} \rceil$ sections produces the frequency domain based estimate of the frequency response $\hat{\boldsymbol{\mathcal{H}}}^{(FD)}$.

[2]The cutoff between the parameters that are considered dominant and the ones that are considered as part of the modeling noise depends on the relative values of the $\lambda_j' s$. In our simulations, we use the condition $\frac{\lambda_{j+1}}{\lambda_j} > 5$ to place our cutoff.

2.1. Estimating the channel in the time domain

For a fair comparison, we need to compare the estimate $\hat{\boldsymbol{\mathcal{H}}}^{(FD)}$ obtained above with the time domain based estimate. This is done by using the pilot/output equation (4) to estimate the channel impulse response $\underline{\mathbf{h}}$ that minimizes the mean-square error given by

$$\boxed{\hat{\underline{\mathbf{h}}} = \mathbf{R}_h\mathbf{X}_{I_p}[\mathbf{R}_{\boldsymbol{\mathcal{N}}_{I_p}} + \mathbf{X}_{I_p}\mathbf{R}_h\mathbf{X}_{I_p}^*]^{-1}\underline{\boldsymbol{\mathcal{Y}}}_{I_p}} \quad (15)$$

We subsequently use (15) to obtain the frequency domain based estimate

$$\hat{\boldsymbol{\mathcal{H}}}^{(TD)} = \mathbf{Q}_P\hat{\underline{\mathbf{h}}}$$

3. SIMULATIONS AND RESULTS

We consider an OFDM system that transmits a symbol with 64 carriers and a cyclic prefix of length $P = 15$. The input data is 16 QAM mapped from a binary bit stream through Gray coding. The channel impulse response consists of 16 complex taps (the maximum length possible). It has an exponential delay profile $E[|\underline{h}_0(k)|^2] = e^{-0.2k}$ and remains fixed over any OFDM symbol. In what follows, we compare the performance of frequency domain based channel estimation and time domain based estimation. We perform estimation in both domains using $64, 32, 16$ and 8 pilots. We compare the performance of the two approaches in terms of the mean square error (MSE) and the BER.

3.1. Comparing the MSE of the time-domain channel estimate with the frequency-domain estimate

In Figure 2, we compare the error in estimating channel in the time domain with the error in estimating the channel in frequency domain. The time domain channel estimation consistently outperforms the frequency domain based estimation. This is not surprising as the time domain estimate makes use of all the pilots while frequency domain estimation of a given band makes use of those pilots in that band only. Nevertheless, we note that when the number of pilots is larger than 16, the two approaches give comparable results.

3.2. BER vs SNR comparison for channel estimation in time vs frequency domain

In Figure 3, we compare the the two estimates, i.e. the channel estimate in time and frequency domain, in terms of the BER. We again see that the time domain estimate is better than the frequency domain estimate, but the margin of error between the two cases is very close, especially for moderate number of pilots.

3.3. Determining the optimal number of frequency parameters

In Figure 4, we try to find the optimal number of frequency parameters needed to estimate the channel using the frequency domain approach. We fix the SNR to be 12 dB. It

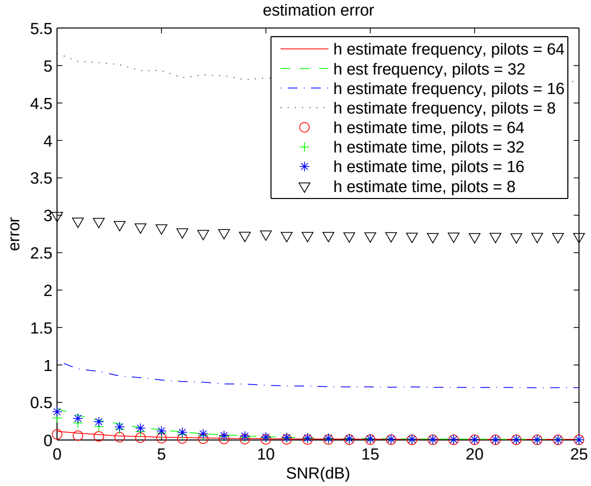

Figure 2. Error in estimating channel in time domain vs frequency domain

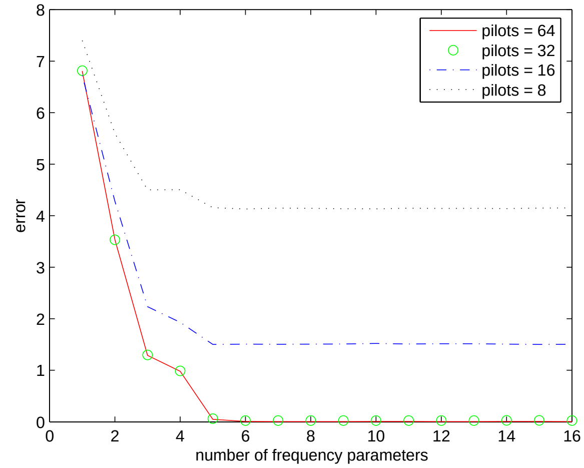

Figure 4. error in estimating the channel varying the number of frequency parameters, at SNR = 12dB .

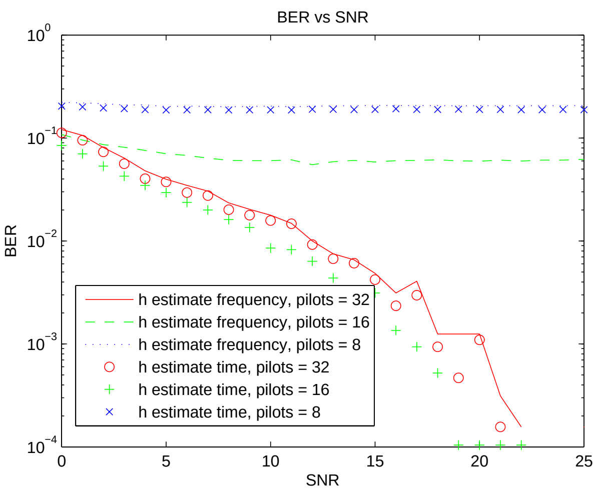

Figure 3. BER vs SNR.

is evident from the graph that we are able to estimate the channel optimally in the frequency domain using only 5 parameters (i.e. $\alpha_1, \ldots, \alpha_5$). Any further increase in the estimation space beyond that does not improve the estimation accuracy. Hence, we can safely justify model reduction, where we keep the dominant eigenvalues and treat the rest as modeling noise, to estimate the channel using the frequency domain approach.

4. CONCLUSION

In this paper, we proposed an algorithm for channel estimation in multiple access OFDM. Here, users are interested in a particular band of OFDM in which they are operating, not in the whole spectrum. Thus as opposed to the time domain approach which estimates the whole spectrum, we proposed a frequency domain approach in which the user estimates that part of the spectrum in which he operates. The advantage of this is a reduction in the computational cost incurred by each user. To counter the blow up in the number of parameters in frequency based estimation, we resorted to model reduction. Though the optimal time domain based estimate consistently outperforms the frequency based estimate, the performance of the two approaches is comparable for high enough number of pilots (i.e. if the number of pilots is 16 or larger). Data-aided estimation which we are currently investigating should close the gap between the two approaches.

5. REFERENCES

[1] G. Leus and M. Moonen, “Semi-blind channel estimation for block transmissions with non-zero padding,” *Proc. Asilomar Conf. on Signals, Syst., and Computers*, Nov. 2001, pp. 762–766.

[2] Xiaowen Wang and K. J. Ray Liu, “Adaptive channel estimation using cyclic prefix in multicarrier modulation system,” *IEEE Commun. Lett.*, vol. 3, no. 10, pp. 291–293, Oct. 1999.

[3] T. Y. Al-Naffouri, Adaptive algorithms for wireless channel estimation, Department of Electrical Engineering, Stanford University, Jan. 2005.

[4] T. Y. Al-Naffouri, “An EM-Based Forward-Backward Kalman Filter for the Estimation of Time-Variant Channels in OFDM” to appear in IEEE Trans. Sig. Process.

[5] O. Edfors, M. Sandell, J. van de Beek, K. S. Wilson, and P. O. Brjesson, “OFDM channel estimation by singular value decomposition,” *IEEE Trans. Signal Proc.*, vol. 46, no. 7, pp. 931–939, Jul. 1998.