Compressive Sensing for Reducing Feedback in MIMO Broadcast Channels

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***Abstract*—We propose a generic feedback channel model, and compressive sensing based opportunistic feedback protocol for feedback resource (channels) reduction in MIMO Broadcast Channels under the assumption that both feedback and downlink channels are noisy and undergo block Rayleigh fading. The feedback resources are shared and are opportunistically accessed by users who are *strong* (users above a certain fixed threshold). Strong users send same feedback information on all shared channels. They are identified by the base station via compressive sensing. The proposed protocol is shown to achieve the same sum-rate throughput as that achieved by dedicated feedback schemes, but with feedback channels growing only logarithmically with number of users.**

I. Introduction

Recently, it has been shown that dirty paper coding (DPC) achieves the sum-rate capacity of the multiple-input multiple-output (MIMO) broadcast channel. However, it requires a great deal of feedback as the base station (BS) needs perfect channel state information for all users and is computationally expensive [1]. Since then, many works have attempted to achieve the same sum-rate capacity with imperfect channel state information (reduced feedback load) at the BS. This was done by applying opportunistic communication in the downlink [2]-[5]. Components that are generally fed back are i) Channel Direction Information (CDI), e.g., beam index, channel quantization index etc. ii) Channel Quality Information (CQI), e.g., SNR, SINR etc. iii) User identity. User selection is based on either one or a combination of these components [2]-[7].

While the downlink is opportunistic in nature, almost all feedback schemes are non-opportunistic. Here, each user has a dedicated feedback channel. Since interaction or cooperation among the competing users is not allowed, hence defying opportunism, there is a linear increase in the feedback resources (channels) with the number of users. Even if thresholding is applied, there is no reduction in the number of feedback channels. This is because the channels are reserved even when users are not sending any CQI feedback information.

Recently, some works [6]-[7] have started to consider opportunistic feedback schemes where feedback resources (time slots) are shared and are opportunistically accessed by strong users. While the scheme proposed in [7] works for MIMO systems, the scheme proposed in [6] is limited to the single-input single-output (SISO) case. Feedback to the BS is successful if only one user is attempting to feed back in a slot otherwise collision occurs. Although the schemes requires only an integer feedback per slot, both these schemes require accurate timing-synchronization to avoid collisions which is difficult to achieve in practice. Moreover, the two schemes only work for digital feedback but not when the designer is interested in analog feedback. In all the feedback schemes discussed above, the feedback links were assumed ideal when the downlink was subjected to both fading and noise. This asymmetry in the way the two links are treated is unrealistic.

The paper proposes generic feedback channel model, and compressive sensing (CS) [8]-[9] based opportunistic feedback protocol for feedback resource reduction. We consider a broadcast scenario where the downlink and the feedback links are symmetric in the sense that they are both i) non-ideal and ii) opportunistic or shared. Thus, both links (downlink and feedback links) undergo Rayleigh fading and are subjected to additive Gaussian noise. Moreover, the channels in both links are shared and are opportunistic in the sense that channels are dominated by strong users.

The proposed protocol is shown to achieve the sum-rate capacity as that achieved by dedicated feedback schemes but with feedback channels growing only logarithmically with number of users. As the feedback links are noisy, so the BS backs off on the noisy CQI based on the variance of the noise. We obtain the optimum back off on the noisy CQI that maximizes the throughput. Also, note that our scheme is less sensitive to timing-synchronization errors, as the scheme will be affected only if out of synchronization user is the strongest among all strong users sending feedback information at that time (the probability of which is low).

The remainder of the paper is organized as follows. In Section II, generalized feedback model is introduced. In Section III we discuss the proposed feedback strategy. In Section IV, performance evaluation of the proposed feedback scheme is presented followed by numerical results and conclusions in Sections V and VI respectively.

Notation: We use bold upper and lower case letters for matrices and vectors, respectively. $\mathbf{A}^*$ refers to Hermitian conjugate of $\mathbf{A}$. $\Re(\mathbf{A})$ & $\Im(\mathbf{A})$ represents real and imaginary part of $\mathbf{A}$ respectively. $\mathbb{E}(\cdot)$ denotes the expectation operator, and $\mathbb{P}[\ ]$ is the probability of a given event. The natural logarithm is referred to as $\log(\cdot)$, while the base 2 logarithm is denoted as $\log_2(\cdot)$. $f(x) = O(g(x))$ is equivalent to $f(x) = cg(x)$ where c is a constant. $|A|$ denotes the size of a set A.

II. SYSTEM MODEL

A. Downlink Transmission Model

We consider a single cell multi-antenna broadcast channel with p antennas at the BS (transmitter) and n users (receivers) each having one antenna. The channel is described by a propagation matrix which is constant during the coherence interval and is known completely at the receiver. Let $\mathbf{u} \in \mathbb{C}^{p \times 1}$ be the transmit symbol vector and let x_i be the received signal by the i-th user, the received signal by the i-th user can then be written as

$$x_i = \sqrt{\rho_i}\mathbf{h}_i\mathbf{u} + w_i, \qquad i = 1, \ldots, n \tag{1}$$

where $\mathbf{h}_i \in \mathbb{C}^{1 \times p}$ is the channel gain vector between the transmitter and the user, and w_i is the additive noise. The entries of $\mathbf{h}_i$ and w_i are i.i.d. complex Gaussian with zero mean and unit variance, $\mathcal{CN}(0, 1)$. Moreover, $\mathbf{u}$ satisfies an average transmit power constraint $\mathbb{E}\{\mathbf{u}^*\mathbf{u}\} = 1$ and ρ_i is the SNR of the i-th user. A homogeneous network is considered, in which all users have the same SNR, i.e. $\rho_i = \rho$ for $i = 1, \ldots, n$. We also assume that the number of users is greater than or equal to the number of transmit antennas, i.e., $n \geq p$, and that the BS selects p out of n users to transmit to.

B. Generic Multi-antenna Feedback Channel

We present here a generic model for the multiuser feedback channel with r feedback channels (possibly shared) among n users, in which users report channel quality information (CQI) to the base station in order to exploit multiuser diversity. As we shall soon see, this model encompasses the existing feedback models. The feedback channels are described by a propagation matrix $\mathbf{A}$ which is constant during the coherence interval and is assumed to be perfectly known at the BS (receiver), and are to be independent of the downlink channel. Let $\mathbf{v} \in \mathbb{C}^{n \times 1}$ be transmit feedback vector and let y_i be the signal received via the i-th feedback channel. The signal received through the i-th feedback channel is mathematically described as

$$\begin{bmatrix} y_1 \\ \vdots \\ y_r \end{bmatrix} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{r1} & \cdots & a_{rn} \end{bmatrix} \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} + \begin{bmatrix} w_1 \\ \vdots \\ w_r \end{bmatrix}$$

or

$$\mathbf{y} = [\mathbf{a}_1 \cdots \mathbf{a}_n]\mathbf{v} + \mathbf{w}$$

or equivalently

$$\mathbf{y} = \mathbf{A}\mathbf{v} + \mathbf{w} \tag{2}$$

where $r \ll n$. The entries of $\mathbf{A}$ are i.i.d. complex Gaussian with zero mean and unit variance, $a_{ij} \sim \mathcal{CN}(0, 1)$. Note that in contrast to the majority of existing feedback reduction techniques, a noisy feedback channel is assumed. The entries of $\mathbf{w}$ represent the additive noise and are assumed to be i.i.d. complex Gaussian with zero mean and variance σ^2, $w_i \sim \mathcal{CN}\left(0, \sigma^2\right)$.

This model subsumes the non-opportunistic feedback model [2]-[5] as a special case. Here, each user is allocated its own feedback channel, thus the feedback channel matrix $\mathbf{A}$ becomes diagonal and of size n (equal to the number of users). If no fading is considered ($\mathbf{A}$ is deterministic), all entries of $\mathbf{A}$ are equal to a constant. Specifically, for the opportunistic models proposed in [6]-[7], the feedback channel matrix $\mathbf{A}$ becomes diagonal of size $r \times r$, where r is the number of feedback slots and is usually less than n. $\mathbf{v}$ represents feedback data in each slot, and when a collision in a particular slot takes place, the corresponding entry of $\mathbf{v}$ is not valid. In all these schemes, the additive noise $\mathbf{w}$ is set to zero.

We can interpret the system of equations (cf. (2)) by assuming that each single-antenna user is going to feedback the same information over r frequency bands shared with the other users. Thus, a_{ij} represents the gain of the j-th user in the i-th band (frequency feedback channels).

III. PROPOSED FEEDBACK STRATEGY

Before we discuss the proposed feedback strategy, we present important compressive sensing results used in our work.

A. Compressive Sensing Results

Compressive sensing refers to the recovery of sparse signals $\mathbf{v}$ from a small number of compressive measurements $\mathbf{y} = \mathbf{A}\mathbf{v}$. Two approaches for recovering the sparsity pattern S (with $|S| = s$) of signal $\mathbf{v} \in \mathbb{R}^n$ ($S = \{i \in \{1, \ldots, n\} | v_i \neq 0\}$) in the noisy setting (cf. (2)) are discussed here. These results were derived for the case when the entries of $\mathbf{A}$ and $\mathbf{w}$ are i.i.d. real Gaussians i.e., $a_{ij} \sim \mathcal{N}(0, 1)$ and $w_i \sim \mathcal{N}\left(0, \sigma^2\right)$.

1) Sparsity Pattern Recovery Results: A recent paper by Fletcher et. al. [8] shows that it is surprisingly possible to recover the sparsity pattern of signals accurately from limited measurements in a noisy setting using maximum correlation provided the number of measurements (or channels) satisfies $r \sim O(s \log(n - s))$. Similar results for sparsity pattern recovery from limited measurements in a noisy setting using LASSO are derived by Wainwright in [9].

2) Estimating or Refining Sparse Signal: Once the sparsity pattern S is known, least squares can be used to estimate or refine $\mathbf{v}_S$ (non-zero entries of $\mathbf{v}$) as follows [10]:

$$\mathbf{v}_S^{ls} = (\mathbf{A}_S^*\mathbf{A}_S)^{-1}\mathbf{A}_S^*\mathbf{y} \tag{3}$$

where $\mathbf{A}_S$ denotes the sub-matrix formed by the columns $\{\mathbf{a}_j | j \in S\}$, indexed by the sparsity pattern S.

B. General Strategy of Using Compressive Sensing for Feedback

Any feedback scheme has two components, a direction component and a magnitude component. The transmitter usually has certain pre-determined directions for which it seeks user feedback. Thus, the BS announces that it is seeking feedback for a particular direction. At this instant, the strong users, users whose channels lie at or are close to this direction, feedback their CQI. Now a limited number of users will feedback on the set of shared feedback channels according to input/output equation (2).

Thus the vector $\mathbf{v}$ in (2) is sparse with sparsity level s determined by the number of users who feedback. CS can now be used to recover the sparsity pattern of $\mathbf{v}$ [8]-[9] (i.e. which user prefer that particular direction) and could also recover the vector $\mathbf{v}$ itself [9] (i.e. users' feedback CQI). A factor that enhances the level of recovery is how sparse the vector $\mathbf{v}$ as compared to the number of feedback channels available. We need at least one strong user (i.e. $s \geq 1$) for each beam or direction in order to achieve full multiplexing gain which implies that small values of s are sufficient. To reduce s, we pursue a thresholding strategy where the user will feedback if his CQI is greater than a threshold ζ to be determined.

Now consider a particular beam (CDI) (all beams will behave in an identical manner as the users are i.i.d. and the beams are equi-powered). Noting that the users' CQI (SINR) are i.i.d., we can choose ζ to produce a sparsity level s. This happen by requiring that

$$F(\zeta) = \arg \max_{u \in (0,1)} \binom{n}{s} u^s (1-u)^{n-s} \tag{4}$$

where $F(\zeta) = \mathbb{P}[\text{SINR} > \zeta] = \frac{\exp(-\zeta/\rho)}{(1+\zeta)^{p-1}}$, $\zeta \geq 0$ [2].

Lemma 1: Threshold ζ that maximizes (4) is given by $\zeta = F^{-1}\left(\frac{s}{n}\right)$

Proof: Let $\psi = \binom{n}{s} u^s (1-u)^{n-s}$. Differentiating ψ w.r.t. u and setting the derivative equal to 0, and solving for u yields $u = s/n$. Thus, $F(\zeta) = s/n$, or $\zeta = F^{-1}(s/n)$.

C. Feedback Protocol

In the analog feedback scenario, users above threshold feed back their analog CQI value. The CS strategy then allows the BS to recover all strong users who transmitted their CQI. Here, we assume that the probability the a user is strongest for more than one beam is negligible as the number of users are relatively much larger than the number of beams. It has been shown in [2] that this is a valid assumption under these conditions. The steps of the proposed compressive sensing based opportunistic feedback protocol are as follows:

1) **Threshold Determination:** BS decides on thresholding level ζ based on the sparsity level that can be recovered.
2) **User Feedback:** Repeat the following steps for each beam.
   - CQI Determination: Each user determines his best beam (corresponding to the highest CQI value).
   - CQI Feedback: Each user feeds back his CQI if it is higher than ζ on all shared channels according to (2). Otherwise, the user remains silent.
   - Compressive Sensing: BS finds the strong users (sparsity pattern) using CS.
   - Least-squares estimation/refining: BS use least-squares (3) to estimates or refines CQIs.
   - Optimum CQI Back off: BS backs off on the noisy CQI (SINR) based on the noise variance such that the throughput is maximized.
3) **User Selection:** Select users and schedule them to beams.

Remark 1: Once CQI has been determined for each beam direction, the base station can proceed to implement any of the various multiuser scheduling techniques, e.g. random beamforming (RBF) or semi-orthogonal user selection (SUS) algorithm (zero beamforming) proposed in [4].

D. Throughput in the RBF Case

The sum rate throughput achieved in the RBF case with dedicated ideal feedback links is given by [2]

$$\mathcal{R} \approx \mathbb{E}\{\sum_{m=1}^{p} \log_2(1 + \max_{1\leq i \leq n} \text{SINR}_{i,m})\} \tag{5}$$

Also, it is shown in [2] that (5) is equivalent to

$$\mathcal{R} \approx p \log_2(1 + \rho \log(n) - \rho(p-2) \log\log(n)) \tag{6}$$

As the SINRs fed back by the users are transmitted *as is* and the feedback links are noisy, so there is a need to back off the noisy received SINRs based on the noise variance as follows:

$$\begin{aligned} \mathbf{v}_S^{ls} &= (\mathbf{A}_S^* \mathbf{A}_S)^{-1} \mathbf{A}_S^* \mathbf{y} & (7)\\ &= (\mathbf{A}_S^* \mathbf{A}_S)^{-1} \mathbf{A}_S^* (\mathbf{A}\mathbf{v} + \mathbf{w}) & (8)\\ &= \mathbf{v}_S + (\mathbf{A}_S^* \mathbf{A}_S)^{-1} \mathbf{A}_S^* \mathbf{w} & (9)\\ &= \mathbf{v}_S + \mathbf{w}' & (10) \end{aligned}$$

Note that the noise represented by $\mathbf{w}'$ is gaussian as linear operations preserve the gaussian noise distribution, however the noise is no longer independent. For simplicity, we assume noise to be independent for back off calculation which results in a larger back off. Thus, in a sense, a lower bound on the throughput is obtained. Thus, from (10) we can write (in an approximate sense)

$$\text{SINR}' = \text{SINR} + w' \tag{11}$$

where the actual and noisy SINRs are denoted by SINR and SINR$'$ respectively and w' represents gaussian noise. Now, if we decide to back off the received SINRs by an amount $\triangle$, then the back-off efficiency (η) i.e. the probability that this backed off SINR is less than or equal to the actual SINR is given as follows:

$$\begin{aligned} \eta &= \mathbb{P}[\text{SINR}' - \triangle \leq \text{SINR}] \\ &= \mathbb{P}[w' \leq \triangle] = 1 - \mathbb{P}[w' > \triangle] \\ &= 1 - Q\left(\frac{\triangle}{\sigma_{w'}}\right) \end{aligned}$$

where Q represents the Q-function. Thus, the effective throughput (with back-off on noisy SINR) can be written as:

$$\mathcal{R}_{eff} \approx \left(1 - Q\left(\frac{\triangle}{\sigma_{w'}}\right)\right) p \log_2(\beta - \triangle) \tag{12}$$

where $\beta = 1 + \rho \log(n) - \rho(p-2) \log\log(n)$.

Differentiating $\mathcal{R}_{eff}$ w.r.t. $\triangle$ and setting it equal to 0, yields

$$Q\left(\frac{\triangle}{\sigma_{w'}}\right) + \left(\frac{\beta - \triangle}{\sqrt{2\pi}\sigma_{w'}}\right) \exp\left(-\frac{\triangle^2}{2\sigma_{w'}^2}\right) \log(\beta - \triangle) = 1 \tag{13}$$

The value of $\triangle$ (which can be written as $O(\sigma_{w'})$) that satisfies the above equation maximizes the effective throughput.

IV. Performance Evaluation

We consider following metrics for the performance evaluation of the proposed feedback scheme.

A. Feedback Resources Reduction

There is a significant reduction in number of feedback channels required for carrying feedback information. The proposed scheme requires only $O(\log(n))$ feedback channels (shown in the Lemma given below) as opposed to n feedback channels required in the dedicated feedback case.

Lemma 2: The number of multiple access feedback channels required for our scheme is $\frac{c}{2}(s\log(n))$, *where* c *is a constant.*

Proof: Specifically, let's assume that there are r channels shared between users over which feedback can take place. We can represent these channels using the system of equations (2). As already mentioned, (2) is similar to ones considered in [8]-[9], except that in our case the measurement matrix $\mathbf{A}$, and the noise vector $\mathbf{w}$ are complex instead of real. So, we replace the complex-valued model in (2) by its real-valued equivalent which upon simplification can be written as

$$\begin{bmatrix} \Re(\mathbf{y}) \\ \Im(\mathbf{y}) \end{bmatrix} = \begin{bmatrix} \Re(\mathbf{A}) \\ \Im(\mathbf{A}) \end{bmatrix} \begin{bmatrix} \mathbf{v} \end{bmatrix} + \begin{bmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{bmatrix}.$$

or

$$\underline{\mathbf{y}} = \underline{\mathbf{A}}\mathbf{v} + \underline{\mathbf{w}} \tag{14}$$

The entries of $\underline{\mathbf{A}}$ are i.i.d. $\mathcal{N}(0, 1/2)$, and the entries of $\underline{\mathbf{w}}$ are i.i.d. $\mathcal{N}\left(0, \sigma^2/2\right)$. The above model (14) gives us $2r$ real measurements, so the sparsity pattern recovery techniques discussed in Section III-A can be applied. Also, from Section III-B, we know that small values of s are sufficient, therefore

$$2r = O(s\log(n-s) \approx O(s\log(n)) = cs\log(n) \tag{15}$$

$$\Rightarrow r = \frac{c}{2}(s\log(n)). \tag{16}$$

Lemma 3: In the RBF case when $n \to \infty$, *the minimum number of multiple access feedback channels required is* $p(\log\log\log(n))\log(n)$.

Proof: From Lemma 2, we have $r = \frac{c}{2}(s\log(n))$ and for $n \to \infty$, $c = 2$ [9]. For RBF systems with large number of users ($n \to \infty$), the minimum value of s (the number of users who should feedback) required to achieve the sum-rate capacity is given by $p\log\log\log(n)$ [11]. Substituting these value of c and s in $r = \frac{c}{2}s\log(n)$, the desired result is achieved.

B. Feedback Noise Reduction

The other important benefit of this scheme is the feedback noise reduction (which eventually results in better throughput). This is because the feedback data of each user is carried over all shared channels instead of one as in the case of dedicated feedback. We analyze the error covariance matrix, as it will allow us to identify the optimum amount of back off required on the noisy SINR which depends on the noise variance. Error covariance matrix (ECM) after the sparsity pattern is identified and LS is applied is given by [12]

$$\text{ECM} = [\mathbf{R_v}^{-1} + \mathbf{A}_S^*\mathbf{R_w}^{-1}\mathbf{A}_S]^{-1} \tag{17}$$

where $\mathbf{R_v} = \mathbb{E}[\mathbf{v}_S\mathbf{v}_S^*] = \sigma_v^2\mathbf{I}$, and $\mathbf{R_w} = \mathbb{E}[\mathbf{w}_S\mathbf{w}_S^*] = \sigma_w^2\mathbf{I}$. $\mathbf{v}_S$ and $\mathbf{w}_S$ refers to the entries of $\mathbf{v}$ and $\mathbf{w}$ corresponding to S. Substituting these values in (17), yields

$$\mathbb{E}[\text{ECM}] = \mathbb{E}_{\mathbf{A}_S^*\mathbf{A}_S}\left[\left(\frac{1}{\sigma_v^2}\mathbf{I} + \frac{1}{\sigma_w^2}\mathbf{A}_S^*\mathbf{A}_S\right)^{-1}\right] \tag{18}$$

$$\overset{(a)}{\approx} \left(\frac{\sigma_w^2}{\frac{\sigma_w^2}{\sigma_v^2} + r}\right)\mathbf{I} \tag{19}$$

$$\overset{(b)}{\approx} \frac{\sigma_w^2}{r}\mathbf{I} \tag{20}$$

where (a) follows because for fixed s and large r, $\mathbb{E}[\mathbf{A}_S^*\mathbf{A}_S] \to r\mathbf{I}$ [13], and (b) follows because for large r and high SNR $(\frac{\sigma_w^2}{\sigma_v^2} + r) \to r$. From the above, it is clear that the noise variance, after the sparsity pattern is identified and LS is applied, is reduced by a factor of r. Now, the back off on the SINR is $O(\frac{\sigma_w}{\sqrt{r}})$ as opposed to $O(\sigma_w)$ in the case of dedicated feedback channel.

C. Feedback Load Reduction

In addition to the feedback resources reduction, there is a reduction in the amount of feedback. In RBF scheme with dedicated feedback channel, n real values and n integer values, as there are n users in the system whereas in our case, only rp real values are feedback. Note that the feedback load reduction is more dominant in systems with large number of users, as $r \sim O(s\log(n))$ and the values of s and p are small.

V. Numerical Results

In this Section, we present numerical results for the proposed feedback scheme by applying it in RBF context. We consider $p = 4$ base station antennas, SNR = 10 dB (both downlink and feedback link), and use optimum back off on noisy SINRs received by the BS for calculating the sum rate throughput.

In Fig. 1, we present the sum rate throughput versus number of shared channels used for feedback. We set the threshold according to the sparsity level s, and use the maximum correlation technique for compressive sensing as this is much more computationally efficient than LASSO. Each point in the figure represents the sum-rate throughput achieved by the proposed scheme for a given number of shared channels (determined by $c/2$ & s according to (16)). We note that for small values of s the throughput is low. This is because the threshold works well for systems with large number of users but for systems with moderate number of users, we may have more or less number of users above the threshold than desired. So, if we set s low, then the probability that a beam has no strong user (which results in a multiplexing loss) is relatively higher than the case when s is large. However, large values of s requires

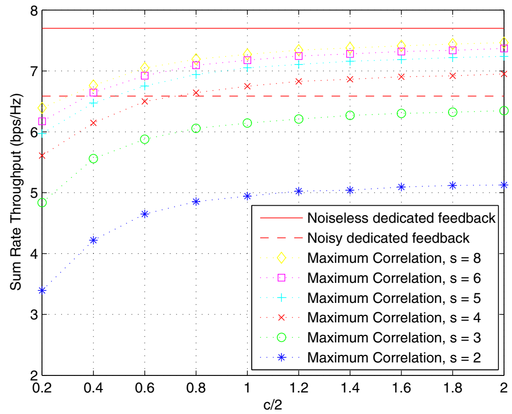

Figure 1. Throughput versus c/2 for RBF with shared channel feedback, $p = 4$ base station antennas, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for different values of s.

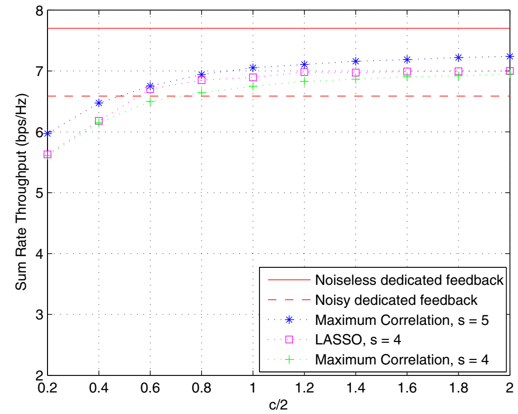

Figure 2. Throughput versus c/2 for RBF with shared channel feedback, $p = 4$ base station antennas, $n = 100$ and SNR = 10 dB (both downlink & feedback link) for LASSO and maximum correlation methods.

more feedback channels. For comparison purposes, we also plot the maximum possible sum rate throughput achieved in noiseless and noisy dedicated feedback scenarios. Note that in the noiseless dedicated feedback case, no back off on the SINR is applied as the feedback links are ideal i.e., there is no noise on the feedback links.

From Fig. 1, we see that the number of shared channels required to achieve the maximum possible throughput obtained in a noisy dedicated feedback scenario is 11 (corresponds to $c/2 = 0.4$ and $s = 6$). Also, it worth mentioning that the proposed scheme comes close to achieving the throughput obtained in a noiseless dedicated feedback scenario due to feedback noise reduction. Note that 90% of throughput in noiseless dedicated feedback case is achieved by 19 shared channels (corresponds to $c/2 = 0.8$ and $s = 5$).

In Fig. 2, we plot the the sum rate throughput versus number of shared channels used for feedback for different sparsity pattern recovery methods namely LASSO and maximum correlation. Results show that LASSO method performs marginally better than maximum correlation method as LASSO requires slightly less number of feedback channels than required by maximum correlation to achieve the same throughput.

VI. Conclusions

In this paper, a generic feedback channel model and compressive sensing based opportunistic feedback scheme are proposed. We have shown that the proposed opportunistic feedback scheme achieves the same sum rate throughput as that achieved by dedicated feedback schemes, but with feedback channels growing only logarithmically with number of users. Also, it has been shown that in a noisy scenario, the proposed scheme comes close to achieving the throughput obtained in the case of noiseless dedicated feedback.

Although the results presented here only show the performance of the the proposed scheme for analog feedback in the RBF case, the scheme can easily work with digital feedback and other beamforming methods.

VII. Acknowledgment

This work was supported by King Abdul Aziz City for Science and Technology (KACST), Project no. AR-29-79.

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