

Exploiting Error-Control Coding and Cyclic-Prefix in Channel Estimation for Coded OFDM Systems

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Abstract— OFDM systems typically use coding and interleaving across subchannels to exploit frequency diversity on frequency-selective channels. This paper presents a low-complexity iterative algorithm for combined blind and semi-blind channel estimation and soft decoding in coded OFDM systems. Channel estimation is performed in the time domain using the expectation maximization (EM) algorithm to take advantage of the channel-length constraint and the extra observation offered by the cyclic-prefix. The proposed technique converges within a single OFDM symbol and, therefore, has a minimum latency and is suitable for fast time-varying channels.

I. INTRODUCTION

OFDM is an effective multicarrier modulation technique for mitigating intersymbol interference (ISI) in frequency-selective wireless channels. In wireless OFDM systems, the use of differential phase-shift keying (DPSK) eliminates the need for channel estimation at the receiver. This approach, however, limits the number of bits per symbol and results in a 3 dB loss in signal-to-noise ratio (SNR) [1]. If the channel is estimated at the receiver, coherent detection and hence more efficient multi-amplitude signaling schemes can be used. If the channel changes slowly, reference pilot symbols or decision-directed channel tracking techniques can be used [2]. On the other hand, if the channel state changes significantly from one symbol to the next due to a high Doppler frequency, channel estimation within a single OFDM symbol may be required. This can be achieved using L pilot tones equally spaced across the N subchannels in the frequency domain, where L is the number of active taps in the channel. OFDM systems usually use coding and interleaving across subchannels to exploit frequency diversity in frequency-selective channels. It is natural then to attempt to use this coding information to aid in estimating the channel as in [2], in which hard estimates of the decoded symbols are used.

Blind channel estimation techniques allow higher data rates since they eliminate the training overhead. Most of the proposed blind estimation techniques for OFDM systems [3], [4], [5], however, ignore the coding information and thus typically require a large number of OFDM symbols to achieve a sufficiently accurate estimate of the channel. This requirement not only introduces a significant latency in the system, but also limits these techniques to slowly varying channels. This paper presents an iterative algorithm for joint soft decoding and channel estimation that provides an accurate blind or semi-blind channel estimate within a single OFDM symbol. Therefore, this algorithm has a minimum latency and is more appropriate for

fast time-varying channels.

As with most hill-climbing techniques [6], in the blind case, the proposed iterative algorithm can potentially get trapped in local minima or stationary points. This problem can be alleviated by running the algorithm multiple times starting with different random initial conditions, and then adopting the results of the best trial. To increase robustness and accelerate convergence, a semi-blind approach can be adopted, wherein L pilot tones equally spaced among the N subchannels are used to obtain an initial channel estimate for the iterative algorithm.

Various iterative blind channel estimation techniques that exploit coding information have recently been suggested [7], [8], [9]. Most of these techniques target single-carrier systems and, consequently, have to deal with complicated time-domain equalization. On the other hand, because of the cyclic-prefix, equalization in multicarrier systems is trivial, making adaptive equalization techniques even more attractive in these systems.

The proposed channel estimation technique is based on the EM algorithm [10] and is performed in the time domain, allowing us to exploit the channel-length constraint, as well as the extra observation offered by the cyclic-prefix. The information contained in the cyclic-prefix observation has been used in the past for timing and frequency synchronization [11], and for channel tracking [5] in multicarrier modulation systems.

Section II introduces the system model and notation used in this paper. The proposed iterative channel estimation and decoding algorithm is presented in Section III. Section IV presents the simulation results, and concluding remarks are given in Section V.

II. SYSTEM MODEL

Fig. 1 shows the system model and the notation used in this paper. For simplification, we assume that each sub-channel uses BPSK modulation. The encoder in Fig. 1 is assumed to be a 4-state recursive systematic convolutional (RSC) encoder with the generator matrix $G(D) = [1 \ \frac{1+D^2}{1+D+D^2}]$, where D is a delay operator. The interleaver is assumed to be a random interleaver.

The output of the encoder can be written as

$$[u(D) \ p(D)] = u(D) \cdot \left[1 \ \frac{1+D^2}{1+D+D^2} \right], \quad (1)$$

where $u(D)$ and $p(D)$ represent the sequences of systematic and parity bits, respectively. Each of these sequences has a length $K = r \cdot N$, where $r = 1/2$ is the code rate, and N is the number of subchannels. Let \mathbf{v} be the multiplexed

This research was cosponsored by National Semiconductor.

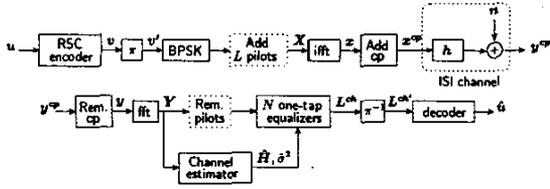


Fig. 1. Coded OFDM system model.

output vector of length N with $v_{2k} = u_k$, and $v_{2k+1} = p_k$, where $k = 0, 1, \dots, K-1$. Let the BPSK modulated vector be

$$\mathbf{X} = (2\mathbf{v}' - 1) = \mathbf{Q}\mathbf{x}, \quad (2)$$

where \mathbf{v}' is the interleaved version of \mathbf{v} , and \mathbf{Q} is the $N \times N$ Discrete Fourier Transform (DFT) matrix.

Let $\mathbf{y}^{cpT} = [\mathbf{y}^T \ \mathbf{y}^T]$ be the output of the channel of length $N + \nu$, where \mathbf{y} is the cyclic-prefix observation of length ν , and \mathbf{y} is the remaining part of length N , which can be obtained through the following cyclic convolution:

$$\mathbf{y} = \mathbf{h} \otimes \mathbf{x} + \mathbf{n}, \quad (3)$$

where \mathbf{h} is the channel impulse response, and \mathbf{n} is a complex additive white Gaussian noise (AWGN) vector with the covariance matrix $\mathbf{R}_{nn} = \sigma^2 \mathbf{I}_N$. We can then write

$$\mathbf{Y} = \text{diag}(\mathbf{H})\mathbf{X} + \mathbf{N}, \quad (4)$$

where $\mathbf{X} = \mathbf{Q}\mathbf{x}$, $\mathbf{Y} = \mathbf{Q}\mathbf{y}$, $\mathbf{N} = \mathbf{Q}\mathbf{n}$, and $\mathbf{H} = \mathbf{V}\mathbf{h}$, where \mathbf{V} is an $N \times L$ Vandermonde matrix with elements given by $V_{n,l} = e^{-j\frac{2\pi}{N}nl}$ for $n = 0, 1, \dots, N-1$ and $l = 0, 1, \dots, L-1$.

We assume that \mathbf{h} can have up to L non-zero complex taps from 0 to $\nu = L-1$ and that it is fixed over the period of a single OFDM symbol.

For the j th OFDM symbol, (4) can be rewritten as

$$\mathbf{Y}_j = \text{diag}(\mathbf{X}_j)\mathbf{H}_j + \mathbf{N}_j, \quad (5)$$

$$= \text{diag}(\mathbf{X}_j)\mathbf{V}\mathbf{h}_j + \mathbf{N}_j. \quad (6)$$

The cyclic-prefix observation of the j th OFDM symbol can be written as

$$\mathbf{y}_j = \mathbf{xx}_j\mathbf{h}_j + \mathbf{n}_j, \quad (7)$$

where \mathbf{xx}_j is the following toepplitz matrix of the cyclic-prefix parts of \mathbf{x}^{cp}_j and \mathbf{x}^{cp}_{j-1}

$$\mathbf{xx}_j = \begin{bmatrix} x_0^j & x_{\nu-1}^{j-1} & x_{\nu-2}^{j-1} & \dots & x_0^{j-1} \\ x_1^j & x_0^j & x_{\nu-1}^{j-1} & \dots & x_1^{j-1} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x_{\nu-1}^j & x_{\nu-2}^j & \dots & x_0^j & x_{\nu-1}^{j-1} \end{bmatrix}. \quad (8)$$

Equations (6) and (7) can be combined as

$$\begin{bmatrix} \mathbf{y}_j \\ \mathbf{Y}_j \end{bmatrix} = \begin{bmatrix} \mathbf{xx}_j \\ \text{diag}(\mathbf{X}_j)\mathbf{V} \end{bmatrix} \mathbf{h}_j + \begin{bmatrix} \mathbf{n}_j \\ \mathbf{N}_j \end{bmatrix}, \quad (9)$$

which can be written in matrix form as

$$\mathbf{y}_j = \mathbf{A}_j\mathbf{h}_j + \mathcal{N}_j. \quad (10)$$

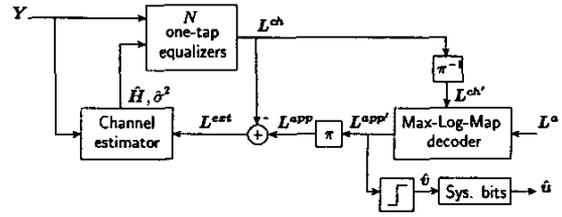


Fig. 2. Iterative decoding and channel estimation.

The objective is to solve the joint maximum likelihood (ML) channel/data estimation problem which can be stated as

$$(\hat{\mathbf{X}}_j, \hat{\mathbf{H}}_j) = \arg \max_{\hat{\mathbf{X}}_j, \hat{\mathbf{H}}_j} \left\{ p(\mathbf{y}_j | \hat{\mathbf{X}}_j, \hat{\mathbf{H}}_j) \right\}. \quad (11)$$

The optimal solution to this problem is overly complex for practical implementations. Therefore, we propose an iterative algorithm for finding a good-quality approximate solution.

III. ITERATIVE JOINT DECODING AND CHANNEL ESTIMATION

Fig. 2 shows a block diagram of the proposed iterative algorithm briefly described in the steps shown below. Since the noise variance does not usually vary too fast, for simplicity we will assume that σ^2 is known to the receiver, i.e., $\hat{\sigma}^2 = \sigma^2$. In practice, once the channel and data have been estimated for the j th OFDM symbol, $\hat{\sigma}^2$ can be calculated as

$$\hat{\sigma}_j^2 = \alpha \hat{\sigma}_{j-1}^2 + (1 - \alpha) \frac{1}{N} \sum_{i=0}^{N-1} |Y_i^j - \hat{H}_i^j \hat{X}_i^j|^2, \quad (12)$$

where α is an exponential smoothing factor. We can then use $\hat{\sigma}_j^2$ as an estimate of σ^2 for the next symbol.

• **Step 1.** Find the initial channel estimate $\hat{\mathbf{h}}^{(it=0)}$, which is simply random in the blind case that uses no pilots, and which in the semi-blind case can be obtained as

$$\hat{\mathbf{h}}^{(it=0)} = \frac{1}{\sqrt{L}} \mathbf{Q}_L^H \mathbf{H} \mathbf{P}, \quad (13)$$

where $\mathbf{H} \mathbf{P}$ is an $L \times 1$ vector of the gain estimates ($H_i^p = Y_i^p / X_i^p$) of the uniformly spaced L pilot subchannels, \mathbf{Q}_L is an $L \times L$ DFT matrix, and H denotes the conjugate transpose operation. Then $\hat{\mathbf{H}}^{(it=0)} = \mathbf{V} \hat{\mathbf{h}}^{(it=0)}$.

• **Step 2.** Given $\hat{\mathbf{H}}^{(it)}$, equalize the received vector \mathbf{Y} using N parallel single-tap equalizers and obtain the vector $\mathbf{L}^{ch(it+1)}$ of the *extrinsic* channel log-likelihood ratios (LLRs) as

$$L_i^{ch(it+1)} = \log \frac{p(\mathbf{Y} | \hat{\mathbf{H}}^{(it)}, X_i = +1)}{p(\mathbf{Y} | \hat{\mathbf{H}}^{(it)}, X_i = -1)}, \quad (14)$$

$$= \log \frac{p(Y_i | \hat{H}_i^{(it)}, X_i = +1)}{p(Y_i | \hat{H}_i^{(it)}, X_i = -1)}, \quad (15)$$

$$= \frac{2}{\sigma^2} |\hat{H}_i^{(it)}|^2 \cdot \text{Re} \left\{ \frac{Y_i}{\hat{H}_i^{(it)}} \right\}, \quad (16)$$

where $i = 0, 1, \dots, N - 1$.

• **Step 3.** Perform the soft MAP sequence estimation, which can be implemented using the Max-Log-Map algorithm [12] which is less complex and numerically more stable for practical implementation compared to the Log-Map algorithm. It is also more robust against channel estimation errors. Moreover, as demonstrated by simulation results, the Max-Log-Map algorithm results in better performance in our iterative algorithm because it provides optimal MAP sequence (or OFDM symbol) estimation, as opposed to optimal MAP BPSK symbol estimation provided by the Log-Map algorithm.

We obtain the *extrinsic* log-likelihood ratios for the coded bits L^{ext} as

$$L^{\text{ext}(it+1)} = L^{\text{app}(it+1)} - L^{\text{ch}(it+1)}, \quad (17)$$

where L^{app} is the interleaved version of the a posteriori LLRs vector for the coded bits $L^{\text{app}'}$, whose entries are obtained as

$$\begin{aligned} L_{2k}^{\text{app}'} &= \max_{(l', l) \in B(u_k=1)} [\bar{\alpha}_{k-1}(l') + \bar{\gamma}_k(l', l) + \bar{\beta}_k(l)] \\ &- \max_{(l', l) \in B(u_k=0)} [\bar{\alpha}_{k-1}(l') + \bar{\gamma}_k(l', l) + \bar{\beta}_k(l)], \\ L_{2k+1}^{\text{app}'} &= \max_{(l', l) \in B(p_k=1)} [\bar{\alpha}_{k-1}(l') + \bar{\gamma}_k(l', l) + \bar{\beta}_k(l)] \\ &- \max_{(l', l) \in B(p_k=0)} [\bar{\alpha}_{k-1}(l') + \bar{\gamma}_k(l', l) + \bar{\beta}_k(l)], \end{aligned} \quad (18)$$

where $k = 0, 1, \dots, K - 1$, (l', l) is the branch from state l' to state l , and $l', l = 0, 1, \dots, M_s - 1$, where M_s is the number of states in the trellis of the code. $B(u_k = 0(1))$ is the set of branches in the k th section of the trellis with $u_k = 0(1)$. Similarly, $B(p_k = 0(1))$ is the set of branches in the k th section of the trellis with $p_k = 0(1)$.

After appropriate normalizations, the branch metrics $\bar{\gamma}_k$ can be written in terms of the a priori and channel LLRs as

$$\bar{\gamma}_k(l', l) = u_k \cdot L_k^a + u_k \cdot L_{2k}^{\text{ch}'} + p_k \cdot L_{2k+1}^{\text{ch}'}, \quad (19)$$

where $k = 0, 1, \dots, K - 1$, and $L_k^a = \frac{p(u_k=1)}{p(u_k=0)}$ is the a priori LLR of u_k and is assumed to be zero unless some a priori information for u_k is available such as that provided by another code in serial or parallel concatenation.

$$\bar{\alpha}_k(l) = \max_{l'} \{\bar{\alpha}_{k-1}(l') + \bar{\gamma}_k(l', l)\}, \quad (20)$$

$$\bar{\beta}_k(l) = \max_{l'} \{\bar{\beta}_{k+1}(l') + \bar{\gamma}_{k+1}(l', l)\}, \quad (21)$$

where $k = 0, 1, \dots, K - 2$, and $\bar{\alpha}_{-1}(l = 0) = \bar{\beta}_{K-1}(l = 0) = 0$, $\bar{\alpha}_{-1}(l) = \bar{\beta}_{K-1}(l) = -\infty$ for $l \neq 0$.

• **Step 4.** Use the *extrinsic* soft output of the decoder to find the ML estimate of H . To take advantage of the cyclic-prefix observation and channel-length constraint, channel estimation is performed in the time domain.

From (10), the ML estimate of h for the j th OFDM symbol can be obtained through the maximization step of the EM algorithm [10] as

$$\hat{h}_j^{(it+1)} = (E[A_j^H A_j | \mathcal{Y}_j, \hat{h}_j^{(it)}])^{-1} E[A_j | \mathcal{Y}_j, \hat{h}_j^{(it)}]^H \mathcal{Y}_j, \quad (22)$$

$$\hat{H}_j^{(it+1)} = \mathbf{V} \hat{h}_j^{(it+1)}. \quad (23)$$

Note that $E[A_j^H A_j | \mathcal{Y}_j, \hat{h}_j^{(it)}]$ can be viewed as an estimate of the a posteriori autocorrelation matrix of the transmitted sequence, which is approximately proportional to the identity matrix in the case of constant modulus modulation. Therefore, in this case, the matrix inversion in (22) can be avoided, and we can instead use the following approximation:

$$\hat{h}_j^{(it+1)} = \frac{1}{N + \nu} E[A_j | \mathcal{Y}_j, \hat{h}_j^{(it)}]^H \mathcal{Y}_j. \quad (24)$$

where

$$E[A_j | \mathcal{Y}_j, \hat{h}_j^{(it)}] = \begin{bmatrix} E[\underline{x}_j | \mathcal{Y}_j, \hat{h}_j^{(it)}] \\ \text{diag}(E[X_j | \mathcal{Y}_j, \hat{h}_j^{(it)}]) \mathbf{V} \end{bmatrix}, \quad (25)$$

where

$$E[X_j | \mathcal{Y}_j, \hat{h}_j^{(it)}] = \tanh^{-1} \left(\frac{1}{2} L^{\text{ext}(it+1)} \right), \quad (26)$$

and

$$E[\underline{x}_j | \mathcal{Y}_j, \hat{h}_j^{(it)}] = \mathbf{Q}^H E[X_j | \mathcal{Y}_j, \hat{h}_j^{(it)}], \quad (27)$$

$$E[x_{j-1}] = \mathbf{Q}^H E[X_{j-1}]. \quad (28)$$

Equation (28) indicates that the final soft estimate of the previous OFDM symbol is used during the iterative channel/data estimation of the current symbol. Simulation results show that error propagation has a negligible effect, which is taken into account in the results presented in the next Section.

• **Step 5.** Return to Step 2, and repeat until a stopping criterion is reached.

IV. SIMULATION RESULTS AND DISCUSSION

The proposed iterative algorithm was simulated using a 4-state rate 1/2 CC with $G(D) = [1 \frac{1+D^2}{1+D+D^2}]$. It is assumed here that $N = 128$, $L = 16$, and that BPSK modulation is used on each of the subchannels. The actual channel taps were generated from independent complex Gaussian distributions with zero means and equal variances. The maximum number of iterations was set to 15.

For the blind estimation case, 10 trials were used, starting with different random initial channel states. The results of the best trial, defined as the one corresponding to the largest average of a posteriori LLRs $\bar{L}^{\text{app}} = 1/N \cdot \sum_{i=0}^{N-1} L_i^{\text{app}}$ after convergence, were then chosen. It is worth mentioning that if hard decisions are used in the iterations as in [2], the algorithm does not converge in the blind case, which underlines the importance of exchanging soft extrinsic information between the iterative modules. In

the semi-blind case, L pilot tones, uniformly spaced among the N subchannels, were used to obtain an initial estimate of the channel.

The system performance using the proposed iterative algorithm for the blind and semi-blind cases was compared to that of an ideal coded system that has perfect channel state information (CSI) at the receiver. It was also compared to that of a non-iterative system that uses L pilot tones to estimate the channel and then performs MAP soft decoding with no iterations. Fig. 3 shows the bit error rate (BER) curves for the various systems, including the uncoded system with perfect CSI as a reference. Since these systems have different rates, the BER is plotted against E_b/N_0 instead of SNR to account for that fact. For a given E_b/N_0 , the SNR for each of the systems is given by

$$SNR = 2 \cdot R_{eff} \cdot \frac{E_b}{N_0}, \quad (29)$$

where R_{eff} is $\frac{N}{N+\nu}$ for the uncoded system with no pilots, $\frac{r \cdot N - m}{N + \nu}$ for the coded system with no pilots, and $\frac{r \cdot (N - L) - m}{N + \nu}$ for the coded system with L pilots, where $r = 1/2$ and $m = 2$ are the rate and memory of the code, respectively. We observe that the performance degradation for the proposed blind system is about 0.25 dB relative to the ideal coded system at the BER of 10^{-3} . The error floor at 10^{-4} is caused by the occasional misconvergence of the iterative algorithm or convergence to local minima and can be easily eliminated by using an outer code.

On the other hand, the semi-blind system using the proposed iterative algorithm has a degradation of about 0.75 dB relative to the ideal coded system at the BER of 10^{-3} . Most of this degradation is due to the rate loss caused by the transmission of L pilot tones, which is about 0.6 dB in this case. The Figure also shows that using the proposed iterative algorithm with the semi-blind system results in about a 3.25 dB gain with respect to the semi-blind system with no iterations. This significant gain is achieved with only a small increase in complexity because the iterative algorithm usually converges within a few iterations in the semi-blind case, particularly in the E_b/N_0 range of practical interest. This can clearly be seen in Fig. 4, which shows the average number of iterations required for convergence of the iterative algorithm in the blind and semi-blind cases. As expected, in the semi-blind case, the algorithm requires significantly fewer iterations to converge compared to the blind case.

Fig. 5 shows the average normalized channel estimation mean-square error $MSE = \|\mathbf{h} - \hat{\mathbf{h}}\|^2 / \|\mathbf{h}\|^2$ versus E_b/N_0 for the blind and semi-blind systems. This Figure indicates that by exploiting the coding information through the proposed iterative algorithm, the same level of channel estimation fidelity (MSE) can be achieved at a 4 to 5 dB lower E_b/N_0 compared to the non-iterative approach that ignores this information.

The requirement for multiple trials of the iterative algorithm in the blind case to avoid stationary points and convergence to local minima might raise some concerns about complexity. However, simulations indicate that fewer than 3 trials are needed on average to obtain zero bit errors at the E_b/N_0 range of practical interest, as shown in Fig. 6.

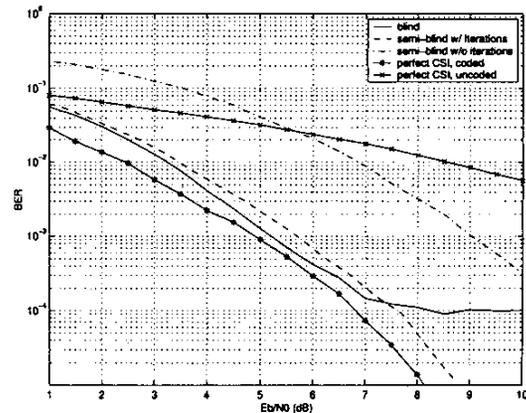


Fig. 3. BER versus E_b/N_0 for the various systems.

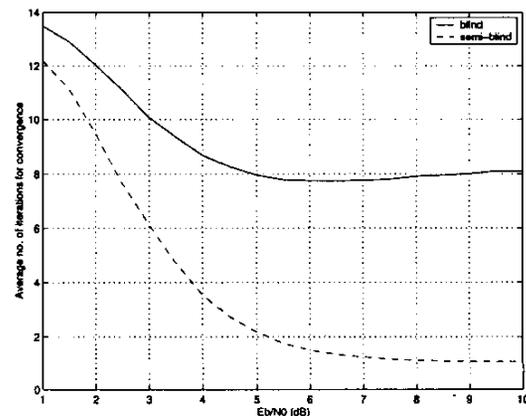


Fig. 4. Average no. of iterations for convergence versus E_b/N_0 for the blind and semi-blind cases.

To verify the fact that the Max-Log-Map decoding algorithm leads to better performance in the proposed iterative blind algorithm compared to the Log-Map decoding algorithm, we simulated the performance of the iterative algorithm using each of these algorithms for comparison. The same OFDM symbol received at $SNR = 3$ dB was processed using 100 trials in each case, with the initial channel state for each trial being randomly generated. In each trial, the number of bit errors after 15 iterations was recorded. The histograms in Fig. 7 show the number of trials versus the number of bit errors in each case. This Figure reveals that about 80% of the trials result in 0 bit errors when the Max-Log-Map algorithm is used, whereas most of the trials converge to local minima or become trapped in stationary points with the Log-Map algorithm.

V. CONCLUSION

Coding is typically used in OFDM systems to exploit frequency diversity. This paper presented a new low-complexity iterative algorithm that exploits this coding information. The proposed algorithm also takes advantage

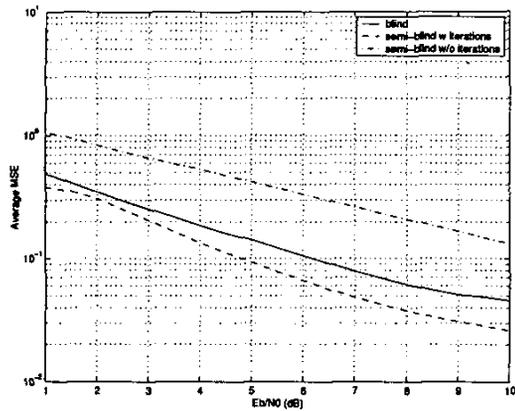


Fig. 5. Average MSE versus E_b/N_0 for the blind and semi-blind cases.

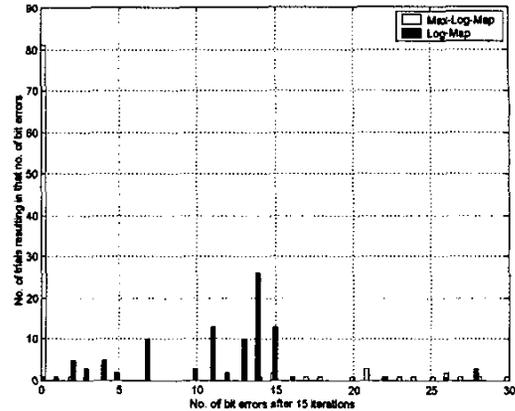


Fig. 7. Comparison of the performance of proposed blind iterative algorithm using the Max-Log-Map and the Log-Map decoding algorithms.

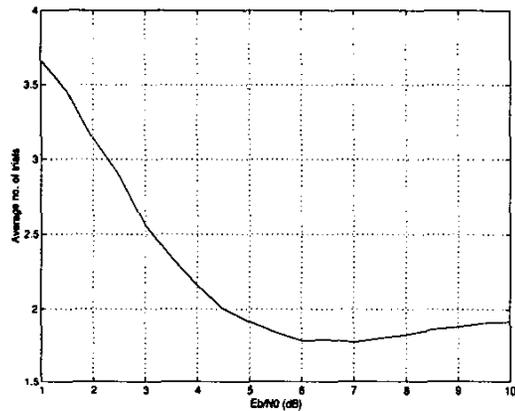


Fig. 6. Average no. of trials needed to obtain zero bit errors in the blind case versus E_b/N_0 .

of the channel spread constraint and the extra observation offered by the cyclic-prefix. It can be used to blindly estimate the channel within a single OFDM symbol or to significantly enhance the channel estimate obtained by L pilot tones. Compared to an ideal coded system with perfect channel knowledge, the blind system using the proposed algorithm has a degradation of less than 0.5 dB in the BER range of interest. In the semi-blind case, with only a small increase in complexity, the proposed algorithm results in a significant gain with respect to the traditional approach which ignores the coding information during channel estimation. The proposed iterative algorithm uses the Max-Log-Map algorithm for soft decoding which, in this case, is not only more convenient for practical implementation, but also leads to better performance.

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