

AN ITERATIVE RECEIVER DESIGN FOR MIMO OFDM TRANSMISSION OVER TIME VARIANT SPATIALLY CORRELATED CHANNELS

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ABSTRACT

In this paper, we consider receiver design for space time block coded MIMO OFDM transmission over frequency selective block fading channels. The receiver employs the Expectation Maximization (EM) algorithm for joint channel and data recovery. It makes a collective use of the data and channel constraints that characterize the communication problem. The data constraints include pilots, the cyclic prefix, the finite alphabet constraint, space-time block coding, and the outer code. The channel constraints include the finite delay spread and frequency and time correlation as well as transmit and receive correlation. The channel estimation part of the algorithm (or the Maximization step) is obtained using a forward-backward (FB) Kalman filter.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a technique that enables high speed transmission over frequency selective channels with simple equalizers. It does this by creating a set of parallel, frequency-flat channels over which large constellation signals can be transmitted. For frequency flat fading channels, space-time codes provide diversity and coding gain benefits compared with single input single output (SISO) systems improving the BER performance of the system [2]. When multiple antennas MIMO are combined with OFDM, space-time codes can be used per tone, providing the benefit of multiple antennas with simple channel equalization.

For proper operation of an OFDM system, the receiver needs to estimate the channel and eliminate its effect. The estimation process can be carried out jointly or separately. There are many techniques for channel estimation which rely on a priori natural or artificial constraints of the communication problem. These techniques fall into three distinct classes: 1) training based, 2) semiblind and 3) blind methods. Training/pilot based methods transmit symbols which are already known at the receiver (pilots) to estimate the channel. Blind methods rely only on a priori constraints for channel estimation and data recovery. Semiblind methods make use of both pilots and additional channel/input data constraints to perform channel identification and equalization. These methods use pilots to obtain an initial channel estimate and improve the estimate by

using a variety of *a priori* information e.g. the time and frequency correlation [8] and the sub-space of the channel [9]. The channel estimate can also be improved iteratively in a data-aided fashion [6] or more rigorously by the expectation maximization (EM) approach [10], [11].

We propose a semiblind iterative receiver using the EM algorithm for joint channel and data recovery in OSTBC OFDM transmission over a frequency selective, time variant channel. We make a collective use of the structure of the communication problem (i.e. the constraints on the data and on the channel). The data constraints include the finite alphabet constraint [7], the cyclic prefix [5], pilots [3], [4] and the OSTBC. In addition, the receiver uses the following constraints on the channel: the finite delay spread, frequency and time correlation [12], and spatial correlation. The channel estimation and data detection as well as the exploitation of the system constraints is done in a semi-optimal manner through the Expectation-Maximization (EM) algorithm. This guarantees a relatively simple receiver structure.

In spite of the complexity of the problem that we address here and the many constraints we incorporate, our algorithm maintains its transparency. The maximization step is used for channel estimation and makes use of the channel constraints by employing a FB-Kalman Filter. The expectation step is used for data detection and makes use of the data constraints.

The paper is organized as follows. After introducing our notation, we give an overview of the transceiver in Section 3. Section 4 then derives the input/output equations for MIMO-OFDM with ST coding (the equations are needed for channel and data recovery). Channel estimation using the FB-Kalman is derived in Section 5 and Section 6 presents our simulations. We conclude the paper in Section 7.

2. NOTATION

In this paper, we denote scalars with small-case letters (e.g., x), vectors with small-case boldface letters (e.g., \mathbf{x}), and matrices with uppercase boldface letters (e.g., \mathbf{X}). Calligraphic notation (e.g. \mathcal{X}) is reserved for vectors in the frequency domain. A hat over a variable indicates an estimate of the variable (e.g., $\hat{\mathbf{h}}$ is an estimate of \mathbf{h}). We use $*$ to denote conjugate transpose, \otimes to denote Kro-

necker product, \mathbf{I}_N to denote the size $N \times N$ identity matrix, and $\mathbf{0}_{M \times N}$ to denote the all zero $M \times N$ matrix. Given a sequence of vectors $\mathbf{h}_{r_x}^{t_x}$ for $r_x = 1 \cdots R_x$ and $t_x = 1 \cdots T_x$, we define the following stack variables

$$\mathbf{h}_{r_x} = \begin{bmatrix} \mathbf{h}_{r_x}^1 \\ \vdots \\ \mathbf{h}_{r_x}^{T_x} \end{bmatrix} \quad \text{and} \quad \mathbf{h} = \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_{R_x} \end{bmatrix} \quad (1)$$

3. SYSTEM OVERVIEW

In this section, we give an overview of the communications system: transmitter, channel, and receiver.

3.1. Transmitter

A block diagram of the transmitter is shown in Figure 1. The bit sequence to be transmitted passes through a convolutional encoder that serves as an outer code for the system. The coded output is punctured to increase the code rate. The punctured sequence then passes through a random interleaver which rearranges the order of the bits according to a random permutation. The interleaved bit sequence is mapped to QAM symbols using gray coding and the QAM symbols are in turn mapped to the OFDM symbols with some tones reserved for the pilot symbols. The STBC encoder uses the OFDM symbols to construct the ST block by mapping the various OFDM symbols to a specific antenna and specific time slot depending on the ST code used. Each antenna performs an IFFT operation on the OFDM symbols to produce the time-domain OFDM symbols and adds a cyclic prefix to each prior to transmission.

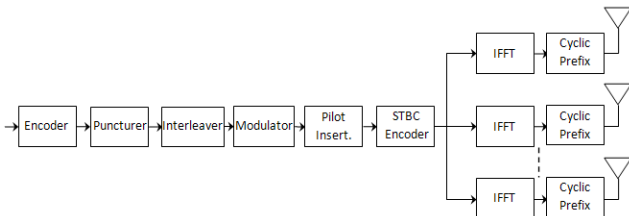


Fig. 1. Transmitter.

3.2. Channel Model

We consider a time-variant and frequency selective MIMO channel. For a general MIMO system, the input/output time-domain relationship is described by

$$\mathbf{y}(m) = \sum_{p=0}^P \mathbf{H}(p) \mathbf{x}(m-p)$$

where $\mathbf{H}(p)$ is the $R_x \times T_x$ MIMO impulse response at tap p and where m represents the sample time index. The

taps $\mathbf{H}(p)$ usually incorporate the effect of the transmit filter and the effects of the transmit and receive correlation making $\mathbf{H}(p)$ correlated across space and tap. We will assume that the tap $\mathbf{H}(p)$ remains constant over a single ST block (and hence over the constituent OFDM symbols) and changes from the current block ($\mathbf{H}_t(p)$) to the next ($\mathbf{H}_{t+1}(p)$) according to a first order dynamical equation¹

$$\mathbf{H}_{t+1}(p) = \alpha(p) \mathbf{H}_t(p) + \sqrt{(1 - \alpha^2(p))} e^{-\beta p} \mathbf{U}_t(p) \quad (2)$$

Here, $\mathbf{U}_t(p)$ is an iid matrix with entries that are $\mathcal{N}(0, 1)$ and $\alpha(p)$ is related to the Doppler frequency $f_D(p)$ by $\alpha(p) = J_0(2\pi f_D T(p))$, where T is the time duration of one ST block. The variable β in (2) corresponds to the exponent of the channel decay profile while the factor $\sqrt{(1 - \alpha^2(p))} e^{-\beta p}$ ensures that each link maintains the exponential decay profile ($e^{-\beta p}$) for all time.

In the Appendix, we derive the dynamical equation for the MIMO impulse response that incorporates time, frequency, and spatial correlation and show that the concatenation of all impulse responses for the MIMO channel \mathbf{h} satisfies

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}) \mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G}) \mathbf{u}_t \quad (3)$$

where \mathbf{h}_{t+1} , \mathbf{h}_t , and \mathbf{u}_t are vectors of size $T_x R_x (P+1) \times 1$. The dynamical equation (3) shows explicit dependence on the space-time index t .

3.3. Receiver

This paper is concerned with designing a receiver for the system described above. For completeness and as an allusion to the developments further ahead, Figure 2 shows a block diagram of the proposed receiver. As we shall show, the receiver's core operation is based on the EM algorithm which performs joint channel and data recovery:

3.3.1. STBC Decoder/Data Detector (Estimation Step)

The STBC decoder/data detector calculates the conditional first and second moments of the transmitted data (soft estimate) to be used by the channel estimator.

3.3.2. Channel Estimator (Maximization Step)

Pilots are used to initialize channel estimation. The channel estimator then uses the soft data estimates together with the data and channel constraints to improve the channel estimate. These two processes (channel estimation and data detection) go on iteratively until a stopping criterion is satisfied.

4. I/O EQUATIONS FOR MIMO-OFDM

As pointed out in Subsection 3.3, the receiver performs two operations, channel estimation and data detection. As such, we need to derive two forms of the (I/O) equations: one that lends itself to *channel estimation* (i.e. treats the

¹We shall at times suppress the time dependence for notational convenience.

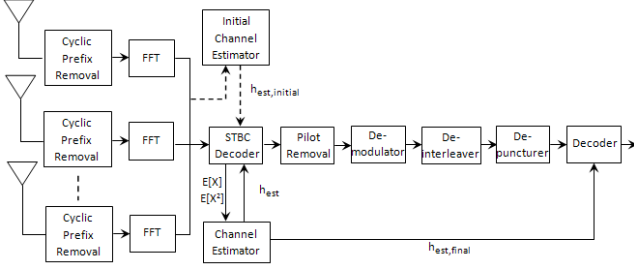


Fig. 2. Receiver.

channel impulse response as the unknown) and a dual version that lends itself to *data detection* (i.e. treats the input in its uncoded form as the unknown). To this end, Let \mathcal{X}_{t_x} be the OFDM symbol transmitted through antenna t_x which first undergoes an IDFT $\mathbf{x}_{t_x} = 1/N\mathbf{Q}\mathcal{X}_{t_x}$ where \mathbf{Q} is the $N \times N$ IDFT matrix. The system then appends a cyclic prefix before transmission. At the receiver end, the receiver strips the cyclic prefix to obtain the time domain symbol $\mathbf{y}_{r_x}^{t_x}$. The I/O equation of the OFDM system between transmit antenna t_x and receive antenna r_x is best described in the frequency domain

$$\mathbf{y}_{r_x}^{t_x} = \text{diag}(\mathcal{X}_{t_x}) \mathbf{Q}_{P+1}^* \mathbf{h}_{r_x}^{t_x} + \mathcal{N}_{r_x} \quad (4)$$

where $\mathbf{y}_{r_x}^{t_x}$, \mathcal{X}_{t_x} , $\mathcal{H}_{r_x}^{t_x}$, and $\mathcal{N}_{r_x}^{t_x}$ are the (length- N) DFT's of $\mathbf{y}_{r_x}^{t_x}$, \mathbf{x}_{t_x} , $\mathbf{h}_{r_x}^{t_x}$, \mathbf{n}_{r_x} , respectively, and where (4) follows from the fact that

$$\mathcal{H}_{r_x}^{t_x} = \mathbf{Q}^* \begin{bmatrix} \mathbf{h}_{r_x}^{t_x} \\ \mathbf{0}_{(N-P-1) \times 1} \end{bmatrix} = \mathbf{Q}_{P+1}^* \mathbf{h}_{r_x}^{t_x} \quad (5)$$

Here \mathbf{Q}_{P+1} represents the first $P + 1$ rows of \mathbf{Q} . By superposition and using the stacking notation (1), we can express the I/O equation at receive antenna r_x as

$$\mathbf{y}_{r_x} = [\text{diag}(\mathcal{X}_1) \cdots \text{diag}(\mathcal{X}_{T_x})] (\mathbf{I}_{T_x} \otimes \mathbf{Q}_{P+1}^*) \mathbf{h}_{r_x} + \mathcal{N}_{r_x} \quad (6)$$

4.1. I/O Equations with Space Time Coding: Channel Estimation Version

Consider a set of N_u uncoded OFDM symbols $\{\mathcal{S}(1), \dots, \mathcal{S}(N_u)\}$ which we would like to transmit over T_x antennas and N_c time slots. Following [14], we can perform ST coding using the set of $T_x \times N_c$ matrices $\{\mathbf{A}(1), \mathbf{B}(1), \dots, \mathbf{A}(N_u), \mathbf{B}(N_u)\}$ which characterizes the ST code. We can now show that the OFDM symbol transmitted from antenna t_x at time n_c is given by

$$\mathbf{x}_{t_x}(n_c) = \sum_{n_u=1}^{N_u} a_{t_x, n_c}(n_u) \text{Re} \mathcal{S}(n_u) + j b_{t_x, n_c}(n_u) \text{Im} \mathcal{S}(n_u)$$

where $a_{t_x, n_c}(n_u)$ is the (t_x, n_c) element of $\mathbf{A}(n_u)$ and $b_{t_x, n_c}(n_u)$ is the (t_x, n_c) element of $\mathbf{B}(n_u)$. Thus, in the presence of ST coding, (6) reads

$$\mathbf{y}_{r_x}(n_c) = [\text{diag}(\mathcal{X}_1(n_c)) \cdots \text{diag}(\mathcal{X}_{T_x}(n_c))] (\mathbf{I}_{T_x} \otimes \mathbf{Q}_{P+1}^*) \mathbf{h}_{r_x} + \mathcal{N}_{r_x}(n_c)$$

This represents the I/O equation at antenna r_x at OFDM symbol n_c of a ST block. Collecting this equation for all such symbols yields

$$\mathbf{y}_{r_x} = \mathbf{X} \mathbf{h}_{r_x} + \mathcal{N}_{r_x} \quad (7)$$

where

$$\mathbf{y}_{r_x} = \begin{bmatrix} \mathbf{y}_{r_x}(1) \\ \vdots \\ \mathbf{y}_{r_x}(N_c) \end{bmatrix}$$

and

$$\mathbf{X} = \begin{bmatrix} \text{diag}(\mathcal{X}_1(1)) \cdots \text{diag}(\mathcal{X}_{T_x}(1)) \\ \text{diag}(\mathcal{X}_1(2)) \cdots \text{diag}(\mathcal{X}_{T_x}(2)) \\ \vdots \\ \text{diag}(\mathcal{X}_1(N_c)) \cdots \text{diag}(\mathcal{X}_{T_x}(N_c)) \end{bmatrix} \quad (8)$$

Now, by further collecting this relationship over all receive antennas, we obtain

$$\mathbf{y}_t = (\mathbf{I}_{R_x} \otimes \mathbf{X}_t) \mathbf{h}_t + \mathcal{N}_t \quad (9)$$

This equation captures the I/O relationship at *all* frequency bins, for *all* input and output antennas, and for *all* OFDM symbols of the t^{th} ST block. To perform initial channel estimation, we select those equations where the pilots are present. Let I_p denote the index set of the pilots bins. Then, the pilot/output equation takes the form

$$\mathbf{y}_{t_{I_p}} = (\mathbf{I}_{R_x} \otimes \mathbf{X}_{t_{I_p}}) \mathbf{h}_t + \mathcal{N}_{t_{I_p}} \quad (10)$$

4.2. I/O Equations with Space Time Coding: Data Detection Version

Signal detection in ST-coded OFDM is done on a tone-by-tone basis (i.e., as in SISO OFDM), except that the tones are collected for the whole ST block (i.e., for R_x receive antennas and over N_c time slots). From (4), we can construct the following I/O equation *at any tone* n belonging to the OFDM symbol n_c

$$\mathbf{y}_{r_x}(n_c) = [\mathcal{H}_{r_x}^1 \cdots \mathcal{H}_{r_x}^{T_x}] \begin{bmatrix} \mathcal{X}_1(n_c) \\ \vdots \\ \mathcal{X}_{T_x}(n_c) \end{bmatrix} + \mathcal{N}_{r_x}(n_c)$$

We suppress the dependence on n for notational convenience. Collecting this relationship for all receive antennas yields

$$\begin{bmatrix} \mathbf{y}_1(n_c) \\ \vdots \\ \mathbf{y}_{R_x}(n_c) \end{bmatrix} = \begin{bmatrix} \mathcal{H}_1^1 & \cdots & \mathcal{H}_1^{T_x} \\ \vdots & \cdots & \vdots \\ \mathcal{H}_{R_x}^1 & \cdots & \mathcal{H}_{R_x}^{T_x} \end{bmatrix} \begin{bmatrix} \mathcal{X}_1(n_c) \\ \vdots \\ \mathcal{X}_{T_x}(n_c) \end{bmatrix} + \begin{bmatrix} \mathcal{N}_1(n_c) \\ \vdots \\ \mathcal{N}_{R_x}(n_c) \end{bmatrix}$$

Or, more succinctly,

$$\mathbf{y}(n_c) = \mathcal{H} \mathcal{X}(n_c) + \mathcal{N}(n_c)$$

By further concatenating this relationship for $n_c = 1, \dots, N_c$, we can show that the following relationship holds (see [14])

$$\mathbf{y} = \mathbf{C} \begin{bmatrix} \text{Re } \mathcal{S} \\ \text{Im } \mathcal{S} \end{bmatrix} + \mathcal{N} \quad (11)$$

where $\mathbf{y} = \begin{bmatrix} \mathcal{Y}(1) \\ \vdots \\ \mathcal{Y}(N_c) \end{bmatrix}$, $\mathcal{S} = \begin{bmatrix} \mathcal{S}(1) \\ \vdots \\ \mathcal{S}(N_u) \end{bmatrix}$ and

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_a & \mathbf{C}_b \end{bmatrix}$$

with $\mathbf{C}_a = \begin{bmatrix} \text{vec}(\mathcal{H}\mathbf{A}(1)) & \dots & \text{vec}(\mathcal{H}\mathbf{A}(N_u)) \end{bmatrix}$ and $\mathbf{C}_b = \begin{bmatrix} \text{vec}(\mathcal{H}\mathbf{B}(1)) & \dots & \text{vec}(\mathcal{H}\mathbf{B}(N_u)) \end{bmatrix}$. We finally note that the STBC code is orthogonal if and only if the matrix \mathbf{C} satisfies [14]

$$\text{Re}[\mathbf{C}^* \mathbf{C}] = \|\mathcal{H}\|^2 \mathbf{I}_{2N_u} \quad \forall \mathcal{H} \quad (12)$$

This property is essential to perform data detection. We stress that the relationships (11) through (12) apply at a particular tone n and that this dependence has been omitted for notational convenience.

5. THE EM ALGORITHM FOR JOINT CHANNEL AND DATA ESTIMATION

5.1. The EM Algorithm

Consider the I/O equation (9), reproduced here for convenience

$$\mathbf{y}_t = (\mathbf{I}_{R_x} \otimes \mathbf{X}_t) \mathbf{h}_t + \mathcal{N}_t \quad (13)$$

Ideally, we estimate \mathbf{h}_t by maximizing some log-likelihood function, e.g.,

$$\hat{\mathbf{h}}_t^M \text{ AP} = \max_{\mathbf{h}_t} \ln p(\mathbf{y}_t | \mathbf{X}_t, \mathbf{h}_t) + \ln p(\mathbf{h}_t)$$

In our case, however, the input \mathbf{X}_t is not available. Thus, we use the EM algorithm and maximize instead an averaged form of the log-likelihood function. Specifically, starting from an initial estimate $\hat{\mathbf{h}}_t^{(0)}$, the estimate $\hat{\mathbf{h}}_t$ is calculated iteratively, with the estimate at the j^{th} iteration given by

$$\hat{\mathbf{h}}_t^{(j)} = \arg \max_{\mathbf{h}_t} E_{\mathbf{X}_t | \mathbf{y}_t, \hat{\mathbf{h}}_t^{(j-1)}} \ln p(\mathbf{y}_t | \mathbf{X}_t, \mathbf{h}_t) + \ln p(\mathbf{h}_t)$$

For example, when the channel obeys the I/O relationship (13) and \mathbf{h}_t is $\mathcal{N}(\mathbf{0}, \mathbf{\Pi})$, the EM-based estimate (at the j^{th} iteration) is given by²

$$\hat{\mathbf{h}}_t^{(j)} = \arg \min_{\mathbf{h}_t} \|\mathbf{y}_t - (\mathbf{I}_{R_x} \otimes E[\mathbf{X}_t]) \mathbf{h}_t\|_{\frac{1}{\sigma_n^2}}^2 + \|\mathbf{h}_t\|_{\mathbf{I} \otimes \text{Cov}[\mathbf{X}_t^*]}^2 + \|\mathbf{h}_t\|_{\mathbf{\Pi}^{-1}}^2 \quad (14)$$

where the two moments of \mathbf{X}_t are taken given the output \mathbf{y}_t and the most recent channel estimate $\hat{\mathbf{h}}_t^{(j-1)}$. We now derive the EM algorithm for the time-variant case.

²The weighted norm notation $\|\mathbf{h}\|_{\mathbf{A}}^2$ stands for $\mathbf{h}^* \mathbf{A} \mathbf{h}$.

5.2. The EM-Based Forward-Backward Kalman

Consider the OFDM system of this paper, essentially described by the state-space model

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}) \mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G}) \mathbf{u}_t \quad (15)$$

$$\mathbf{y}_t = (\mathbf{I}_{R_x} \otimes \mathbf{X}_t) \mathbf{h}_t + \mathcal{N}_t \quad (16)$$

with $\mathbf{h}_0 \sim \mathcal{N}(\mathbf{0}, \mathbf{\Pi})$ and $\mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_u)$. The MMSE estimate of the channel sequence given the input and output sequences \mathbf{X}_0^T and \mathbf{y}_0^T is obtained by the forward-backward (FB) Kalman filter. We will consider two cases.

5.2.1. Channel estimation–Known input case

Consider the state-space model (15)–(16). Given the input and output sequences \mathbf{X}_0^T and \mathbf{y}_0^T , the MAP (or equivalently MMSE) estimate of \mathbf{h}_0^T is obtained by applying the following (forward-backward Kalman) filter to the state-space model (15)–(16)

Forward run: For $i = 1, \dots, T$, calculate

$$\mathbf{R}_{e,t} = \sigma_n^2 \mathbf{I}_{T_x R_x N} + (\mathbf{I}_{R_x} \otimes \mathbf{X}_t) \mathbf{P}_{t|t-1} (\mathbf{I}_{R_x} \otimes \mathbf{X}_t^*)$$

with $\mathbf{P}_{0|-1} = \mathbf{\Pi}_0$

$$\mathbf{K}_t = \mathbf{P}_{t|t-1} (\mathbf{I}_{R_x} \otimes \mathbf{X}_t^*) \mathbf{R}_{e,t}^{-1}$$

$$\hat{\mathbf{h}}_{t|t} = (\mathbf{I}_{T_x R_x (P+1)} - \mathbf{K}_t (\mathbf{I}_{R_x} \otimes \mathbf{X}_t)) \hat{\mathbf{h}}_{t|t-1} + \mathbf{K}_t \mathbf{y}_t,$$

$$\hat{\mathbf{h}}_{t+1|t} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}) \hat{\mathbf{h}}_{t|t}, \quad \mathbf{h}_{0|-1} = \mathbf{0}$$

$$\mathbf{P}_{t+1|t} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}) (\mathbf{P}_{t|t-1} - \mathbf{K}_t \mathbf{R}_{e,t} \mathbf{K}_t^*) (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}^*) + \mathbf{G} \mathbf{R}_u \mathbf{G}^*$$

Backward run: Starting from $\lambda_{T+1|T} = \mathbf{0}$ and for $t = T, T-1, \dots, 0$, calculate

$$\lambda_{t|T} = (\mathbf{I}_{P+N} - (\mathbf{I}_{R_x} \otimes \mathbf{X}_t^*) \mathbf{K}_t^*) (\mathbf{I} \otimes \mathbf{F}^*) \lambda_{t+1|T} + (\mathbf{I} \otimes \mathbf{X}_t) \mathbf{R}_{e,t}^{-1} (\mathbf{y}_t - (\mathbf{I} \otimes \mathbf{X}_t) \hat{\mathbf{h}}_{t|t-1})$$

$$\hat{\mathbf{h}}_{t|T} = \hat{\mathbf{h}}_{t|t-1} + \mathbf{P}_{t|t-1} \lambda_{t|T}$$

The desired estimate is $\hat{\mathbf{h}}_{t|T}$. For a proof, see problem 10.9 in [16].

5.2.2. Channel estimation–Unknown input case

Consider the state-space model (15)–(16) and assume that the receiver does not have access to the transmitted data \mathbf{X}_0^T . The channel estimate at the j^{th} iteration $\hat{\mathbf{h}}_0^{T(j)}$ of the EM algorithm is obtained by applying the forward-backward Kalman described above to the following state-space model

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}) \mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G}) \mathbf{u}_t \quad (17)$$

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{0}_{T_x R_x (P+1) \times 1} \end{bmatrix} = \begin{bmatrix} \mathbf{I}_{R_x} \otimes E[\mathbf{X}_t] \\ \mathbf{I}_{R_x} \otimes \text{Cov}[\mathbf{X}_t^*]^{1/2} \end{bmatrix} \mathbf{h}_t + \begin{bmatrix} \mathcal{N}_t \\ \mathbf{n}_t \end{bmatrix} \quad (18)$$

where \mathbf{n}_t is virtual noise that is not physically present and that is independent of the physical noise \mathcal{N}_t .

To fully implement the EM algorithm, we need to initialize the algorithm and calculate the first and second moments of the input, which we do next.

5.3. Initial Channel Estimation

We obtain the initial channel estimate from the pilot/output equation (10) together with the dynamical channel model (3). Specifically, we do this by applying the FB Kalman to the following state-space model

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}) \mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G}) \mathbf{u}_t \quad (19)$$

$$\mathcal{Y}_{t_{I_p}} = (\mathbf{I}_{R_x} \otimes \mathbf{X}_{t_{I_p}}) \mathbf{h}_t + \mathcal{N}_{t_{I_p}} \quad (20)$$

i.e., by applying the FB Kalman filter with the following substitution

$$\mathcal{Y}_t \longrightarrow \mathcal{Y}_{t_{I_p}}, \mathbf{X}_t \longrightarrow \mathbf{X}_{t_{I_p}}, \text{ and } \mathbf{I}_{T_x R_x N} \longrightarrow \mathbf{I}_{T_x R_x | I_p}$$

5.4. Data Detection

To detect the data, we use the data detection version of the I/O equation (11). Upon multiplying both sides by \mathbf{C}^* and taking the real part, we obtain

$$\tilde{\mathcal{Y}} = \|\mathcal{H}\|^2 \begin{bmatrix} \text{Re } \mathcal{S} \\ \text{Im } \mathcal{S} \end{bmatrix} + \tilde{\mathcal{N}} \quad (21)$$

where $\tilde{\mathcal{Y}}$ and $\tilde{\mathcal{N}}$ are $2N_u \times 1$ vectors defined by $\tilde{\mathcal{Y}} = \text{Re } \mathbf{C}^* \mathcal{Y}$ and $\tilde{\mathcal{N}} = \text{Re } \mathbf{C}^* \mathcal{N}$. Since \mathbf{C} is orthogonal, the noise $\tilde{\mathcal{N}}$ remains white, and the input can be detected on an element-by-element basis. We will now demonstrate how to detect the elements of $\text{Re } \mathcal{S}$ (the imaginary part can be treated similarly). So let $\mathcal{R} = \{r_1, \dots, r_{|\mathcal{R}|}\}$ denote the alphabet set from which the elements of $\text{Re } \mathcal{S}$ take their values. The conditional pdf $f(\text{Re } \mathcal{S}(n_u) | \tilde{\mathcal{Y}}(n_u))$ is given by

$$f(r_i | \tilde{\mathcal{Y}}(n_u)) = \frac{e^{-\frac{|\tilde{\mathcal{Y}}(n_u) - \|\mathcal{H}\|^2 r_i|^2}{2\sigma_n^2}}}{\sum_{i=1}^{|\mathcal{R}|} e^{-\frac{|\tilde{\mathcal{Y}}(n_u) - \|\mathcal{H}\|^2 r_i|^2}{2\sigma_n^2}}} \quad (22)$$

We can use this pdf to calculate conditional expectation of $\text{Re } \mathcal{S}(n_u)$ and its second moment given the output $\tilde{\mathcal{Y}}(n_u)$

$$\begin{aligned} E[\text{Re } \mathcal{S}(n_u) | \tilde{\mathcal{Y}}(n_u)] &= \frac{\sum_{i=1}^{|\mathcal{R}|} r_i e^{-\frac{|\tilde{\mathcal{Y}}(n_u) - \|\mathcal{H}\|^2 r_i|^2}{2\sigma_n^2}}}{\sum_{i=1}^{|\mathcal{R}|} e^{-\frac{|\tilde{\mathcal{Y}}(n_u) - \|\mathcal{H}\|^2 r_i|^2}{2\sigma_n^2}}} \\ E[|\text{Re } \mathcal{S}(n_u)|^2 | \tilde{\mathcal{Y}}(n_u)] &= \frac{\sum_{i=1}^{|\mathcal{R}|} r_i^2 e^{-\frac{|\tilde{\mathcal{Y}}(n_u) - \|\mathcal{H}\|^2 r_i|^2}{2\sigma_n^2}}}{\sum_{i=1}^{|\mathcal{R}|} e^{-\frac{|\tilde{\mathcal{Y}}(n_u) - \|\mathcal{H}\|^2 r_i|^2}{2\sigma_n^2}}} \end{aligned} \quad (23)$$

We can similarly calculate the two moments of the imaginary part. Now equation (23), just like (11)–(12), apply at a certain frequency tone n . So collecting (23) for all tones ($n = 1, \dots, N$) produces the two moments of the uncoded OFDM symbols. Specifically, we can calculate

$$\begin{aligned} &E[\text{Re } \mathcal{S}(n_u)], \quad E[\text{Im } \mathcal{S}(n_u)], \\ &E[\text{diag}(\text{Re } \mathcal{S}(n_u))^2], \quad \text{and } E[\text{diag}(\text{Im } \mathcal{S}(n_u))^2] \end{aligned} \quad (24)$$

These moments are enough to characterize the first and second moments $E[\mathbf{X}]$ and $E[\mathbf{X}^* \mathbf{X}]$, which are needed for channel estimation.

5.5. Modification: Kalman- (Forward-Only) Based Estimation

One disadvantage of the FB Kalman (summarized in Subsection E above) is the storage and latency involved. The algorithm needs to wait for all $T + 1$ symbols before it can execute the backward run and hence obtain the channel estimate. One way around this is to reduce the window size T . Alternatively, we can run the filter in the forward direction only for both the initial estimation and the EM iteration. The algorithm then collapses to the Kalman-based filter proposed in [17] where the data and channel are recovered within one ST symbol.

6. SIMULATION RESULTS

The transmitter and receiver illustrated in Figures 1 and 2 were implemented. The outer encoder is a rate 1/2 convolutional encoder and the coded bits are mapped to 16-QAM symbols using gray coding. We use the OSTBC commonly known as the Alamouti code with number of time slots $N_s = 2$ and number of transmitters $T_x = 2$ [18].

We use two MIMO channel models. One is spatially white while the other one is spatially correlated with transmit and receive correlation matrices

$$\mathbf{T}(p) = \begin{bmatrix} 1 & \zeta \\ \zeta & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{R}(p) = \mathbf{I}$$

where $\zeta = 0.8$. All other parameters are same in both channel models i.e. $\alpha = 0.8$, $\beta = 0.2$, and $P = 16$.

Packets are transmitted at each SNR value until a minimum number of errors (five in this case) occur. Each packet consists of six ST blocks. The first ST block comprises of 16 pilots while the number of pilots in subsequent blocks remains fixed at 12.

Figure 3 compares the Kalman and the FB-Kalman over spatially white channel. The results are shown for two scenarios. In one, hard estimate of data is used while in the other soft estimate is used. The figure clearly illustrates that the FB-Kalman is better than the Kalman for both scenarios. It can also be observed from this figure that the FB-Kalman using soft estimate of data outperforms the one using hard estimate.

The performance of Kalman and the FB-Kalman is compared in Figure 4 over the two channel models i.e. spatially correlated and spatially white channel model. In both cases, soft estimate of data is used. It can also be observed from Figure 9 that both Kalman and FB-Kalman perform closer to the perfect channel in the case of spatially correlated channel. So, spatial correlation (practical scenario) makes the performance of both Kalman and FB-Kalman better as compared to the case of spatially white channel model.

7. CONCLUSION

In this paper, we have proposed a receiver for MIMO-OFDM transmission over time-variant channels. While

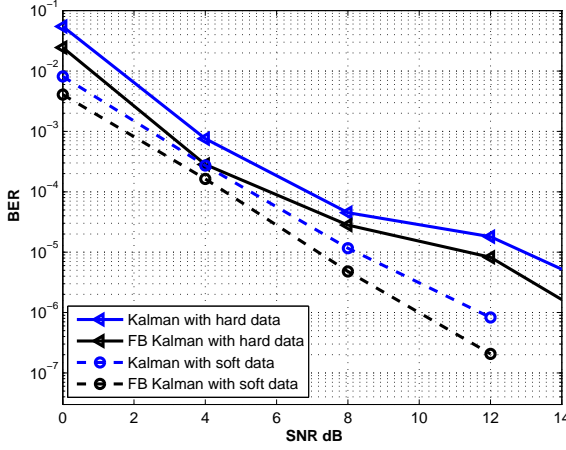


Fig. 3. BER performance of Kalman and FB-Kalman using Hard and Soft estimate of data.

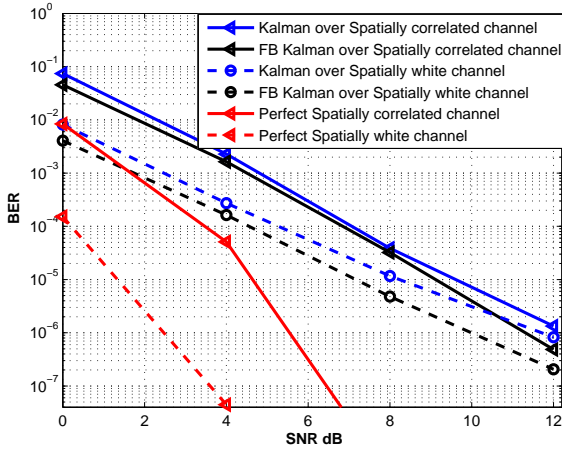


Fig. 4. BER performance of Kalman and FB-Kalman using Soft data over spatially white and correlated channel models.

the paper assumed the channel to be constant within any ST block, it is allowed to vary from one block to the next. This makes the receiver suitable for operation in high-speed environments.

The receiver employs the EM algorithm to achieve channel and data recovery. Specifically, the data recovery (or the expectation step) is as simple as decoding a space-time block code. Channel recovery (or the maximization step) is performed using a forward-backward Kalman filter. We also suggested a relaxed (forward-only) version of the algorithm that is able to perform recovery with no latency and hence avoid the delay and storage shortcomings of the FB-Kalman.

When compared with other MIMO receivers, our receiver makes the most use of the underlying structure. Specifically, the algorithm makes use of the finite alphabet constraints (23), the data in its soft form (17)–(18), pilots (19)–(20), finite-delay spread (in that channel estimation is done in the time domain), frequency- and time-

correlation (3), spatial correlation in (31), and space-time coding. It is also straightforward to incorporate the effect of an outer code and sparsity (see [15]). Our simulations show the favorable behavior of the two Kalman filters over both spatially white and spatially correlated channel models.

8. APPENDIX: CHANNEL MODEL IN THE PRESENCE OF SPATIAL CORRELATION

In what follows, we present the transmit correlation case, and then generalize our results to deal with the general (transmit and receive) correlation case. In the transmit correlation case, $\mathbf{H}(p)$, the MIMO impulse response at tap p , is given by

$$\mathbf{H}(p) = \mathbf{W}(p)\mathbf{T}^{1/2}(p) \quad (25)$$

where $\mathbf{T}^{1/2}(p)$ is the transmit correlation matrix (of size T_x) at tap p and where $\mathbf{W}(p)$ consists of iid elements. The matrix $\mathbf{W}(p)$ remains constant over a single ST block and varies from one ST block to the next according to³

$$\mathbf{W}_{t+1}(p) = \alpha(p)\mathbf{W}_t(p) + \sqrt{(1 - \alpha^2(p))e^{-\beta p}}\mathbf{U}_t(p) \quad (26)$$

where $\alpha(p)$, β , and $\mathbf{U}_t(p)$ are as defined in Subsection 3.2.

We would like to construct a recursion for the tap $h_{r_x}^{t_x}(p)$ and subsequently scale it up for the SISO and MIMO cases. Now since $h_{r_x}^{t_x}(p)$ is the (r_x, t_x) element of $\mathbf{H}(p)$, we deduce from (25) that it is the inner product of the r_x row of $\mathbf{W}(p)$ and the t_x column of $\mathbf{T}^{1/2}$, i.e.

$$h_{r_x}^{t_x}(p) = \mathbf{w}_{r_x}(p)\mathbf{t}^{t_x}(p) \quad (27)$$

Moreover, from (26), we have the following recursion for $\mathbf{w}_{r_x}(p)$

$$\mathbf{w}_{r_x,t+1}(p) = \alpha(p)\mathbf{w}_{r_x,t}(p) + \sqrt{(1 - \alpha^2(p))e^{-\beta p}}\mathbf{u}_{r_x,t}(p)$$

Post-multiplying both sides by $\mathbf{t}^{t_x}(p)$ yields

$$\mathbf{w}_{r_x,t+1}(p)\mathbf{t}^{t_x}(p) = \alpha(p)\mathbf{w}_{r_x,t}(p)\mathbf{t}^{t_x}(p) + \sqrt{(1 - \alpha^2(p))e^{-\beta p}}\mathbf{u}_{r_x,t}(p)\mathbf{t}^{t_x}(p)$$

This means that $h_{r_x}^{t_x}(p)$ satisfies the dynamical equation

$$h_{r_x,t+1}^{t_x}(p) = \alpha(p)h_{r_x,t}^{t_x}(p) + \sqrt{(1 - \alpha^2(p))e^{-\beta p}}ut_{r_x,t}^{t_x}(p) \quad (28)$$

where $ut_{r_x}^{t_x}$ is defined by

$$ut_{r_x}^{t_x}(p) = \mathbf{u}_{r_x}(p)\mathbf{t}^{t_x}(p)$$

Concatenating (28) for $p = 1, 2, \dots, P$ yields a dynamic equation for the impulse response

$$\mathbf{h}_{r_x}^{t_x} = \begin{bmatrix} h_{r_x}^{t_x}(0) \\ \vdots \\ h_{r_x}^{t_x}(P) \end{bmatrix} = \begin{bmatrix} \mathbf{w}_{r_x}(0)\mathbf{t}^{t_x}(0) \\ \vdots \\ \mathbf{w}_{r_x}(P)\mathbf{t}^{t_x}(P) \end{bmatrix}$$

which can be expressed in a more compact form as

$$\mathbf{h}_{r_x,t+1}^{t_x} = \mathbf{F}\mathbf{h}_{r_x,t}^{t_x} + \mathbf{G}ut_{r_x,t}^{t_x} \quad (29)$$

³We suppress the time dependence at times for notational convenience.

where

$$\mathbf{F} = \begin{bmatrix} \alpha(0) & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \alpha(P) \end{bmatrix} \text{ and}$$

$$\mathbf{G} = \begin{bmatrix} \sqrt{1 - \alpha^2(0)} & & & \\ & \ddots & & \\ & & \ddots & \\ & & & \sqrt{(1 - \alpha^2(P))e^{-\beta P}} \end{bmatrix}$$

For complete characterization of the dynamical model, we need to specify the covariance of $\mathbf{u}t_{r_x}^{t_x}$. We have

$$E[\mathbf{u}t_{r_x}^{t_x}\mathbf{u}t_{r_x}^{t_x*}] = \begin{bmatrix} \mathbf{t}t_{t_x}^{t_x}(0) & & & \\ & \mathbf{t}t_{t_x}^{t_x}(1) & & \\ & & \ddots & \\ & & & \mathbf{t}t_{t_x}^{t_x}(P) \end{bmatrix}$$

$$\triangleq \text{diag}(\mathbf{t}t_{t_x}^{t_x})$$

where

$$\mathbf{t}t_{r_x}^{t_x} = \begin{bmatrix} \mathbf{t}^{r_x*}(0)\mathbf{t}^{t_x}(0) \\ \mathbf{t}^{r_x*}(1)\mathbf{t}^{t_x}(1) \\ \vdots \\ \mathbf{t}^{r_x*}(P)\mathbf{t}^{t_x}(P) \end{bmatrix} = \begin{bmatrix} \mathbf{t}_{r_x}(0)\mathbf{t}^{t_x}(0) \\ \mathbf{t}_{r_x}(1)\mathbf{t}^{t_x}(1) \\ \vdots \\ \mathbf{t}_{r_x}(P)\mathbf{t}^{t_x}(P) \end{bmatrix}$$

and where the first line follows from the fact that $\mathbf{t}^{r_x*}(p) = \mathbf{t}_{t_x}^{r_x}(p)$ since $\mathbf{T}^{1/2}(p)$ is conjugate symmetric. In general, we can show that

$$E[\mathbf{u}t_{r_x}\mathbf{u}t_{r_x'}^*] = \begin{bmatrix} \text{diag}(\mathbf{t}t_1^1) & \text{diag}(\mathbf{t}t_1^2) & \cdots & \text{diag}(\mathbf{t}t_1^{T_x}) \\ \text{diag}(\mathbf{t}t_2^1) & \text{diag}(\mathbf{t}t_2^2) & \cdots & \text{diag}(\mathbf{t}t_2^{T_x}) \\ \vdots & \vdots & \cdots & \vdots \\ \text{diag}(\mathbf{t}t_{T_x}^1) & \text{diag}(\mathbf{t}t_{T_x}^2) & \cdots & \text{diag}(\mathbf{t}t_{T_x}^{T_x}) \end{bmatrix}$$

for $r_x = r_x'$ and is zero otherwise. Alternatively, we can write this as

$$E[\mathbf{u}t_{r_x}\mathbf{u}t_{r_x'}^*] = \begin{cases} \sum_{p=0}^P \mathbf{T}(p) \otimes (\underline{\mathbf{I}}^p \mathbf{B} \bar{\mathbf{I}}^p) & \text{for } r_x = r_x' \\ \mathbf{0} & \text{otherwise} \end{cases}$$

where

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \underline{\mathbf{I}} = \begin{bmatrix} 0 & & & \\ 1 & 0 & & \\ & 1 & \ddots & \\ & & \ddots & 0 \\ & & & 1 & 0 \end{bmatrix}$$

$$\text{and } \bar{\mathbf{I}} = \begin{bmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & \ddots & 1 \\ & & & 0 & 1 \\ & & & & 0 \end{bmatrix}$$

Collecting (29) for all transmit and receive antennas yields

$$\mathbf{h}_{t+1} = (\mathbf{I}_{T_x R_x} \otimes \mathbf{F}) \mathbf{h}_t + (\mathbf{I}_{T_x R_x} \otimes \mathbf{G}) \mathbf{u}t_t \quad (30)$$

where

$$E[\mathbf{u}\mathbf{u}^*] = \mathbf{I}_{R_x} \otimes E[\mathbf{u}t_{r_x}\mathbf{u}t_{r_x}^*]$$

$$= \sum_{p=0}^P \mathbf{I}_{R_x} \otimes \mathbf{T}(p) \otimes (\underline{\mathbf{I}}^p \mathbf{B} \bar{\mathbf{I}}^p) \quad (31)$$

When the channel exhibits both transmit and receive correlation, the IR \mathbf{h} continues to satisfy the dynamical equation (30) except that the correlation of the innovation \mathbf{u} is now given by

$$E[\mathbf{u}\mathbf{u}^*] = \sum_{p=0}^P \mathbf{R}(p) \otimes \mathbf{T}(p) \otimes (\underline{\mathbf{I}}^p \mathbf{B} \bar{\mathbf{I}}^p)$$

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