Estimation of time-varying multipath channel parameters in OFDM signal proposed by Drs. Habti Abeida and Tareq Y. Al-Naffouri

1 OFDM signal and channel model

Consider an OFDM system with N sub-carriers, and a cyclic prefix length N_g. The duration of an OFDM symbol is $T = N_t T_s$, where T_s is the sampling time and $N_t = N + N_g$. Let $\mathbf{x}_{(n)} = [x_{(n)}[-\frac{N}{2}], x_{(n)}[-\frac{N}{2}+1], \ldots, x_{(n)}[\frac{N}{2}-1]]^T$ be the *nth* transmitted OFDM symbol, where $\{x_{(n)}[b]\}$ are the BPSK or QAM symbols. After transmission over a multi-path Rayleigh channel, the *nth* received OFDM symbol $\mathbf{y}_{(n)} = \left[y_{(n)}[-\frac{N}{2}], y_{(n)}[-\frac{N}{2}+1], \ldots, y_{(n)}[\frac{N}{2}-1]\right]^T$ is given by:

$$\mathbf{y}_{(n)} = \mathbf{H}_{(n)}\mathbf{x}_{(n)} + w_{(n)} \tag{1.1}$$

where $\mathbf{w}_{(n)} = \left[w_{(n)}[-\frac{N}{2}], w_{(n)}[-\frac{N}{2}+1], \ldots, w_{(n)}[\frac{N}{2}-1]\right]^T$ is a white complex Gaussian noise vector with covariance matrix $\sigma_w^2 \mathbf{I}_N$ and $H_{(n)}$ is a $N \times N$ channel matrix with elements given by:

$$[H_{(n)}]_{k,m} = \frac{1}{N}\sum_{l=1}^{L}\left[e^{-j2\pi(\frac{m-1}{N}-\frac{1}{2})\tau_l}\sum_{q=0}^{N-1}\alpha_l^{(n)}(qT_s)e^{j2\pi\frac{m-k}{N}q}\right] \tag{1.2}$$

where L is the total number of propagation paths, α_l is the *lth* complex gain of variance $\sigma_{\alpha_l}^2$ and $\tau_l \times L$ is the *lth* delay (τ_l is not necessarily an integer, but $\tau_L < N_g$). The L individual elements of $\{\alpha_l^{(n)}(qT_s) = \alpha_l(qT_s+nT)\}$ are uncorrelated with respect to each other. They are wide-sense stationary (WSS), narrow-band complex Gaussian processes, with the so-called Jakes' power spectrum of maximum Doppler frequency f_d[1] (i.e., $\mathrm{E}(\alpha_l(q_1T_s)\alpha_l^*(q_2T_s) = \sigma_{\alpha_l}^2 \mathrm{J}_0(2\pi f_d T_s(q_1-q_2))))$ [3, 4]. The average energy of the channel is normalized to one, i.e., $\sum_{l=1}^{L}\sigma_{\alpha_l}^2 = 1$.

We note that for slowly time varying channel, we have $(\alpha(0) = \alpha(T_s) = \alpha(2T_s) = \ldots = \alpha((N-1)T_s))$ and consequently the $\mathbf{H}_{(n)}$ matrix becomes diagonals with elements are given by

$$[H_{(n)}]_{k,k} = \sum_{l=1}^{L}\left[\alpha_l^{(n)}e^{-j2\pi(\frac{k-1}{N}-\frac{1}{2})\tau_l}\right]. \tag{1.3}$$

Using (1.3), the observation model in (1.1) for the the *nth* OFDM symbol can be re-written as

$$\mathbf{y}_{(n)} = \mathrm{Diag}\{\mathbf{x}_{(n)}\}\mathbf{F}(\boldsymbol{\tau})\boldsymbol{\alpha}_{(n)} + w_{(n)} \tag{1.4}$$

where $\boldsymbol{\alpha}_{(n)} \stackrel{\text{def}}{=} \left[\alpha_1^{(n)}, \ldots, \alpha_L^{(n)}\right]^T$, $\boldsymbol{\tau} \stackrel{\text{def}}{=} [\tau_1, \ldots, \tau_L]^T$ and $\mathbf{F}$ is defined by

$$(\mathbf{F})_{k,l} = e^{-j2\pi(\frac{k-1}{N}-\frac{1}{2})\tau_l}$$

The objective is to jointly estimate the path delay parameter $\boldsymbol{\tau}$ and the state $\{\boldsymbol{\alpha}_{(n)}\}_n$ using the set of received signals $\{\mathbf{y}_{(n)}\}_n$.

[1] $\mathrm{J}_0(.)$ is the first kind 0th-order Bessel function

2 Objective

The main objective of this project is to extend the algorithm proposed in [2] by combining the expectation-maximization (EM) algorithm with Kalman smoother algorithm to yield time-varying complex gains estimation and ML estimate of the paths delay for OFDM signal. In [1] the EM algorithm has been adapted for joint channel and data recovery in fast fading environments. It should be interesting to extend this work for joint estimation parameters (i.e., time-varying complex gains and the paths delay parameters).

To evaluated the performance of this estimator it should be interesting to calculate the hybrid Cramér Rao-bound basing on the work presented in [5].

References

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