Impulse noise estimation and cancelation in DSL using block compressive sensing

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One of the most severe problems that are encountered in DSL design is impulse noise. As its name suggests, impulse noise is a phenomenon that happens rarely, but when it occurs it almost completely destroys the DSL signal. Impulse noise occurs due to effects from electronic switching equipment in the telephone network (or nearby disturbances such as the starting of an automobile or vacuum cleaner). Impulse noise is an impulsive or a group of large individual impulses that take place in the time domain and then spreads out in the frequency domain to destroy the DSL signal.

It is difficult to design DSL systems to combat impulse noise because it happens rarely (and thus it is not economical to design the system for a worst case scenario). Impulse noise cannot be ignored also because when it takes place, it could devastate transmission and force the receiver to drop a few DSL symbols.

1 Signal model and problem formulation

The transmission model in a DSL system can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{e} + \mathbf{z} \tag{1.1}$$

where $\mathbf{y}$ and $\mathbf{x}$ are the time-domain OFDM receive and transmit signal blocks (after CP removal) and $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$. The vector $\mathbf{e}$ is an impulse noise process as specified above. Specifically, $\mathbf{e}$ is a random vector with support (set of the non zero components) $\mathcal{I}(\mathbf{e})$ uniformly distributed over all $\binom{m}{s}$ possible supports of cardinality $s \ll m$, and i.i.d. non-zero components $\sim \mathcal{CN}(0, I_0)$.

Due to the presence of the cyclic prefix, $\mathbf{H}$ is a circulant matrix describing the cyclic convolution of the channel impulse response with the block $\mathbf{x}$. Let $\mathbf{F}$ denote a unitary DFT matrix with (k, ℓ) element $[\mathbf{F}]_{k,\ell} = \frac{1}{\sqrt{n}} e^{-j2\pi k\ell/n}$ with $k, \ell \in \{0, \dots, n-1\}$. The time domain signal is related to the frequency domain signal by

$$\mathbf{x} = \frac{1}{\sqrt{n}} \mathbf{F}^{\mathsf{H}} \check{\mathbf{x}} \tag{1.2}$$

Furthermore, given a circulant convolution matrix $\mathbf{H}$ we have that

$$\mathbf{H} = \mathbf{F}^{\mathsf{H}} \mathbf{D} \mathbf{F} \tag{1.3}$$

where $\mathbf{D} = \text{diag}(\check{\mathbf{h}})$ and $\check{\mathbf{h}} = \sqrt{n}\mathbf{F}\mathbf{h}$ is the DFT of the channel impulse response (whose coefficients are found, by construction, on the first column of $\mathbf{H}$).

Demodulation amounts to computing the DFT

$$\begin{aligned} \check{\mathbf{y}} &= \mathbf{F}\mathbf{y} \\ &= \text{diag}(H_0, \dots, H_{m-1})\check{\mathbf{x}} + \mathbf{F}^{\mathsf{H}}\mathbf{z} + \mathbf{F}^{\mathsf{H}}\mathbf{e} \\ &= \mathbf{D}\check{\mathbf{x}} + \check{\mathbf{z}} + \mathbf{F}\mathbf{e} \end{aligned} \tag{1.4}$$

where $H_i = \sum_{k=0}^{L} h_\ell e^{j\frac{2\pi}{N}\ell i}$ are the DFT coefficients of the channel impulse response, and where $\check{\mathbf{z}} = \mathbf{F}\mathbf{z}$ has the same distribution of $\mathbf{z}$. Without impulsive noise, it is well-known that (1.4) reduces to a set of m parallel Gaussian channels $\check{y}_i = H_i \check{x}_i + \check{z}_i$, for $i = 1, \dots, m$.

In the presence of the impulsive noise, the performance of a standard OFDM demodulator may dramatically degrade since even a single impulse in an OFDM block may cause significant degradation to the whole block. This is because $\check{\mathbf{e}} = \mathbf{Fe}$ can have a large variance per component, and therefore it affects more or less evenly all symbols of the block.

In any DSL transmission, some carriers are left unutilized. These unutilized carriers come from guard-bands, or from carriers left because the user does not use all available bandwidth. For example, with longer DSL lines, we know that the available transmission bandwidth gets smaller and that leaves many carriers free. When impulsive noise attacks a DSL signal, it does so in the time domain. It is difficult to deal with impulsive noise in the time domain by clipping because the OFDM signal is itself impulsive in nature. However, in the frequency domain, the impulsive noise will appear alone in the free carriers. By partial observation of the impulsive noise in the free carriers, we can detect the presence of impulsive noise, estimate it and cancel it before proceeding to decoding. This method was used in [1] to deal with impulse noise using compressive sensing [2], [3].

There are different types of impulse noise but we will focus on REIN (Repetitive Electrical Impulse Noise) in this project. As its name suggests, this type of noise is repetitive in nature with frequency that is twice as much and the AC frequency (i.e. 120 Hz) with short duration 200 μsec to 1.2 msec (amounting to 1-8 DMT symbols). As it occurs in bundle (i.e. many impulses occur together), normal compressive sensing techniques cannot be utilized here. Instead, block compressive sensing techniques [4] - [9] might be used to estimate it and cancel its effect.

2 Objective

The main objective of this project is to estimate and cancel the effect of REIN in DSL systems using block compressive sensing.

References

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