Kalman Filter

We have received $y_0, y_1, \dots, y_{i-1}$

& we would like to estimate x_i

Consider ~~the~~ $y_i = H_i x_i + v_i$

$$y_{i|i-1} = H_i x_{i|i-1} + v_{i|i-1}$$

what does this mean ?

What can you say about $v_{i|i-1}$?

$v_i \perp y_j$ for $j \leq i-1$

because v_i is uncorrelated with the observations $\{y_0, y_1, \dots, y_{i-1}\}$

Why?

Because y_j is a linear combination of the variables

$x_0, n_0, v_0, n_1, v_1, \dots, n_{j-1}, v_{j-1}, v_j$

all of which are uncorrelated with v_i for $j < i$

So

$$\hat{y}_{i|i-1} = H_i \hat{x}_{i|i-1} + \cancel{v_{i|i-1}} \rightarrow 0$$

$$= H_i x_{i|i-1}$$

What is the value of $\hat{y}_{i|i-1}$?

It gives us the innovations

$$e_i = y_i - \hat{y}_{i|i-1}$$

$$= y_i - H_i \hat{x}_{i|i-1}$$

So, to find innovations e_i, we need to find $\hat{x}_{i|i-1}$ as opposed to finding $\hat{y}_{i|i-1}$

Now, we would like to obtain a recursion for $\hat{x}_{i|i-1}$

We have that

$$\hat{x}_{i+1|i} = \hat{x}_{i+1|i-1} + x_{i+1|e_i}$$

$$= \hat{x}_{i+1|i-1} + E[x_{i+1} e_i^*] R_{e,i}^{-1} e_i$$

$$= \underbrace{\hat{x}_{i+1|i-1}} + E[x_{i+1} e_i^*] R_{e,i}^{-1} (y_i - H_i \hat{x}_{i|i-1})$$

Where to get this ?

$$\hat{x}_{i+1} = F_i x_i + G_i n_i$$

$$\Rightarrow \quad \hat{x}_{i+1|i-1} = F_i x_{i|i-1} + G_i n_{i|i-1}$$

what is this ?

Can we see that $n_i \perp y_j \quad j \leq i-1$?

So

$$\hat{x}_{i+1|i-1} = F_i \, \hat{x}_{i|i-1} + G_i \, \hat{n}_{i|i-1}$$

$$= F_i \, \hat{x}_{i|i-1}$$

So

$$\hat{x}_{i+1|i} = \underbrace{\hat{x}_{i+1|i-1}}_{F_i \, \hat{x}_{i|i-1}} + \left(\hat{x}_{i+1|e_i}\right) \overset{?}{=} E\left[x_{i+1} e_i^*\right] R_{e,i}^{-1} e_i$$

So

$$\hat{x}_{i+1|i} = F_i \, \hat{x}_{i|i-1} + E[x_{i+1} e_i^*] \, R_{e,i}^{-1} \, e_i$$

$$= F_i \, \hat{x}_{i|i-1} + \underbrace{K_{p,i}}_{\text{need to find } K_{p,i}} (y_i - H_i \, \hat{x}_{i|i-1})$$

This is a self contained recursion for $x_{i|i-1}$

What is the initial condition $x_{0|-1}$?