Recursive Least Squares:

In recursive least squares, we attempt to solve the prob.

$$\min_{w} \left[w^* \Pi w + \| y_N - H_N w \|^2 \right]$$

in a recursive manner

The normal equations yield

$$w_N = (\Pi + H_N^* H_N)^{-1} H_N^* y_N$$

(How do we remember this formula?)

We can express w_N in terms of w_{N-1} & the new data as

$$w_N = w_{N-1} + \frac{P_{N-1}}{1 + \|u_N\|^2_{P_{N-1}}} u_N^* \left(d(N) - u_N w_{N-1} \right)$$

$$= w_{N-1} + \frac{P_{N-1}}{1 + \|u_N\|^2_{P_{N-1}}} u_N^* e(N)$$

How is this similar to LMS?

How is it similar to ϵ-NLMS?

What is the update for P_{N-1}?

$$P_N = P_{N-1} - \text{a perturbation}$$

$$= P_{N-1} - \frac{\text{outer product}}{1 + \text{inner product}}$$

$$= P_{N-1} - \frac{P_{N-1} u_N^* u_N P_{N-1}}{1 + \|u_N\|^2_{P_{N-1}}}$$

$a^* b$

$a b^*$

Inner prod: $\|u_N\|^2_{P_{N-1}} = u_N P_{N-1} u_N^*$

Outer prod: $(u_N P_{N-1})^* (P_{N-1} u_N^*)^* = P_{N-1}^* u_N^* u_N P_{N-1}^*$

$= P_{N-1} u_N^* u_N P_{N-1}$

Prove that P_{N-1} is Hermitian

Prove that P_{N-1} is positive definite