What to do when the data is <u>nonstationary</u> ?

We should give less weight ~~to~~ earlier data

Let $0 \ll \lambda < 1$ be a constant (we usually call forgetting factor)

So give a weights of λ^{N} to data at $i=0$

" " λ^{N-1} to " " $i=1$

$\vdots$

" " $\lambda^{N-(N)} = 1$ to " " $i=N$

$\lambda = 1$ is called the growing memory

~~Also~~

Now, the LS prob we solve is

$$\min_{w} \|w\|^2_{\Pi} + \|y_N - H_N w\|^2_{W_N}$$

where $W_N = \begin{bmatrix} \lambda^N & & & \\ & \lambda^{N-1} & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$

Can we directly deduce the RLS soln. from the soln. of the unweighted case?

We have another variation of the problem

$$\min \quad \lambda^{N+1} \|w\|_{\Pi}^{2} + \|y_N - H_N w\|_{W_N}^{2} \qquad W_N = \begin{bmatrix} \lambda^N & & & \\ & \lambda^{N-1} & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

Why the λ^{N+1} factor?

$$w_i = w_{i-1} + g_i \, e(i)$$

$$e(i) = d(i) - u_i w_{i-1}$$

$$g_i = \frac{1}{\lambda} P_{i-1} u_i^* \, \gamma(i)$$

$$\gamma(i) = \frac{1}{1 + \frac{1}{\lambda} \|u_i\|^2_{P_{i-1}}}$$

$$P_i = \frac{1}{\lambda} P_{i-1} + \frac{g_i g_i^*}{\gamma(i)}$$