

HOMEWORK #3

Due Date: Mar. 17, 2010

Q1 (Matrix manipulations of a scalar quantity) Consider the expression

$$z(\mathbf{x}) = \sigma + \mathbf{b}^* \mathbf{x} + \mathbf{x}^* \mathbf{c} + \mathbf{x}^* \mathbf{A} \mathbf{x}$$

where σ , \mathbf{b} , \mathbf{c} , \mathbf{A} , and \mathbf{x} are in general complex valued

1. Write $z(\mathbf{x})$ in the form $\mathbf{a}^* \mathbf{Q} \mathbf{a}$ for some \mathbf{a} and \mathbf{Q} that you should specify.
2. What are the conditions on σ , \mathbf{b} , \mathbf{c} , and \mathbf{A} for z to be real.

Q2 (Positive definiteness) Let \mathbf{A} and \mathbf{B} be two Hermitian positive definite matrices. Define the matrix

$$\mathbf{Z} = \begin{bmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{O} & \mathbf{B} \end{bmatrix}$$

Prove that \mathbf{Z} is positive definite if and only if both \mathbf{A} and \mathbf{B} are positive definite.

Q3 Solve problem II.11 of the text book.

Q4 Solve problem II.12 of the text book.

Q5 Solve problem II.14 of the text book.