**KING ABDULLAH UNIVERSITY OF SCIENCE & TECHNOLOGY**

**EE 353** **ELECTRICAL ENGINEERING DEPARTMENT** Feb. 26, 2010

# HOMEWORK #2

Due Date: Mar. 3rd, 2010

**Q1 (Independent random variables)** Let $x$ and $y$ be two independent variables. Find the least mean square error (l.m.s.e) estimate of $x$ given $y$ and find the resulting minimum mean square error.

**Q2 (Nested expectations)** In class, we used the fact that

$$E[E[x|y]] = E[x]$$

Prove this by using the formal definition of expectation, i.e.

$$E[z] = \int_{-\infty}^{\infty} z f_z(z) dz$$

**Q3 (Jointly Gaussian random variables)** Let $x$ and $y$ be two jointly Gaussian random variables with means $\bar{x}$ and $\bar{y}$, respectively, and autocovariance matrix

$$R = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix}$$

Here, $\sigma_{xy} \triangleq E(x - \bar{x})(y - \bar{y})$. Find the least mean square error (m.m.s.e) estimate of $x$ given $y$ and find the resulting minimum mean square error.

**Q4 (MSE vs. MAP)** Consider Example 2.1 in the textbook.

1. Prove that the l.m.s.e estimate of $x$ is given by

$$\hat{x} = \tanh[y(0) + y(1)]$$

2. Find the MAP estimate of $x$ given $y(0)$ and $y(1)$ (hint: draw the constellation diagram and draw the decision regions using the minimum Euclidean distance principle)