King Abdullah University of Science & Technology
Electrical Engineering Department
Fall 2009

Performance Analysis of Outage Models

Prepared for
Dr. Tareq Al-Naffouri

By
Emad Al-Hemyari
Ahmed Dehwah

Due

Sat, 12 Dec. 2009

The Outline

I. Introduction

Outage models are applied in wireless communication systems, which are subjected to fading and other interfering signals, to predict their performance. The performance of a given system is considered to be unacceptable when the Signal to Interference plus Noise Ratio (SINR) is below a given threshold. [4]

Different analysis ways to approach this problem can be followed depending on the way an outage model is defined:

A. The simplest definition is the definition adopted in this paper. [4]
B. Interference-limited model: where the performance is accepted when the SIR is above a defined power protection ratio, neglecting the receiver background noise and the thermal noise caused by the electrostatic discharge and the movement of electrons in the hardware. [1]
C. Noise-Limited model: where noise is treated as co-channel interference. Hence the outage model can be defined as the likelihood that the desired signal strength drops below the total interference and noise powers by power protection margins. [1]
D. Minimum Signal Power Constraint: the desired signal power must exceed the total interference power by a protection ratio and a minimum power level, simultaneously, in the presence of thermal noise.

II. The motivation behind this study of outage models:

- No closed form expression of the probability of outage for a sum of K multiple interfering signals with different powers of the same distribution.[1]
- Previous work either considered single number of interferers, limited number of interferers or numerical approximations.[1]
- Different communication systems require different conditions on the definition of the outage to achieve certain reliability. [4]
- Performance analysis of BPSK system through a Rayleigh fading channel in terms of the probability of error given the probability of an outage is done, rather than just studying the probability of outage. [3,4]

Back to our simple definition of the outage which states that: The performance of a given system is considered to be unacceptable when the Signal to Interference plus Noise Ratio (SINR) is below a given threshold. This threshold is a predefined constant that depends on system design requirements such as the type of modulation used, the coding rate…, etc [4]. Wireless communication systems are subjected to fading caused by the multipath and shadowing effects. They are also subjected to co-channel interference which is the crosstalk between two radio transmitters with the same frequency. Such reliability study of outage is conducted in terms of probability error given that the SINR is above the given threshold z_0.

$$\Pr(Error \mid SINR > z_0)$$

III. System Model

The signal at the receiver front is of the following form:

$$r(t) = s_0(t) + \sum_{k=1}^{K} s_k\left(t - \tau_k\right) + n(t)$$

Where

- $n(t)$: is a Gaussian noise of two sided power spectral density N_0/T_b.
- τ_k: is a uniform time offset of $S_k(t)$ over $[0, T_b]$.

The kth interfering signal $S_k(t)$ (k=0, 1, …., K) is given by:

$$s_k(t) = \pm\sqrt{2S_k}X_k\psi(t)\cos\left(\omega_c t + \phi_k\right), \quad 0 < t \le T_b$$

Where

- S_k: is the average received power
- X_k: is the Rayliegh fading level of the kth signal with $E[X_k^2] = 1$.
- $\emptyset_k$: is uniform over $[0,2\pi]$ and $\emptyset_0 = 0$.
- X_k and $\emptyset_k$ are mutually independent random variables
- $\psi(t)$ is the shaping waveform

Now the decision variable at the output of the correlator is given by:

$$r = \pm\sqrt{2S_0}X_0 + \sum_{k=1}^{K} I_k + \eta$$

Where

- η: is Gaussian Random variable with variance N_0/T_b.
- I_k: is the multiple access interference component due to the kth signal given by:

$$I_k = \sqrt{2S_k}X_k \cos\phi_k \left[(1-\tau_k)\, b_k^{(-1)} + b_k^{(0)}\tau_k\right]$$

Where

- $b_k^{(-1)} and\ b_k^{(0)} \in \{-1,1\}$ denote the consecutive bits overlapping with the reference bit.
- $X_k cos\emptyset_k$: is zero mean Gaussian random variable with variance 1/2.

We define the signal to noise plus interference ratio as follows in order to find it's distribution:

$$Z = \frac{2S_0X_0^2}{\sum_{k=1}^{K} v_k X_k^2 + N_0/T_b}$$

Where:

- X_0 and X_k are Gaussian random variables with zero means and variances equal to ½.
- K is the number of interfering signals.
- $v_k = S_k\left[(1-\tau_k)\, b_k^{(-1)} + \tau_k b_k^{(0)}\right]^2$

The conditional CDF

$$F_Z(z|v_1, v_2, \dots, v_K) = \Pr(Z > z|v_1, v_2, \dots, v_K) = \frac{A\, e^{-\frac{N_0 z}{T_b S_0}}}{2S_0\ |\Lambda|}$$

Where

$\Lambda = diag([\frac{1}{2}, \dots, \frac{1}{2}])$ (K+1)X(K+1) diagonal matrix

$$A = \frac{2S_0}{2^{(K+1)}\ \prod_{k=1}^{K}(1 + \frac{zv_k}{2S_0})}$$

On removing the condition on $\{v_1, v_2, \dots, v_K\}$, we get:

$$\Pr(Z > z) = \frac{e^{-\frac{N_0 z}{T_b S_0}}}{2^{(K+1)}\ |\Lambda|} \left[\mathrm{E}\left[1/(1 + \frac{zv_1}{2S_0})\right]\right]^K$$

Finding the expected value in this expression gives us a closed form CDF. From this CDF, we get the PDF by differentiating the CDF.

Now we can get the conditional PDF of the SINR given that the SINR is above a given threshold. And from this, we are left to find:

$$\Pr(bit\ or\ packet\ Error\ |\ SINR > z_0)$$

Based on our interest.

IV. Conclusion

Hopefully, we have given a full picture of the outage model reaching a conclusion of the study of their performance analysis. Although, we didn't go deep into the math of actually calculating the bit or symbol probability of error while being on the safe side without an outage happening.

V. References:

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2. D. Dardari, V. Tralli, and R. Verdone, .On the capacity of slotted Aloha with Rayleigh fading: the role played by the number of interferers,. IEEE Commun. Lett., vol. 4, pp. 155-157, May 2000. http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?arnumber=846496

3. K. A. Hamdi, . Exact probability of error of BPSK communication links subjected to asynchronous interference in Rayleigh fading environment,. IEEE Trans. Commun., vol. 50, pp. 1577 - 1579, Oct. 2002 http://ieeexplore.ieee.org/xpl/freeabs_all.jsp?arnumber=1044597

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