Quadrature Amp. Modulation

Pp. 174 - 175
Proakis

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Solution of the PAM efficiency problem

$$x_m(t) = A_{mc}\, g(t) \cos 2\pi f_c t - A_{ms}\, g(t) \sin 2\pi f_c t$$

info bearing signals

$$\varphi_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}}\, g(t) \cos 2\pi f_c t$$

$$\varphi_2(t) = -\sqrt{\frac{2}{\mathcal{E}_g}}\, g(t) \sin 2\pi f_c t$$

$$d_{mn} = \sqrt{\frac{1}{2}\mathcal{E}_g\left[(A_{mc}-A_{nc})^2 + (A_{ms}-A_{ns})^2\right]}$$

$$x_m = \begin{bmatrix} A_{mc}\sqrt{\frac{\mathcal{E}_g}{2}} \\ A_{ms}\sqrt{\frac{\mathcal{E}_g}{2}} \end{bmatrix}$$

Relate QAM to PAM & PSK

$$x_m(t) = V_m \cos(2\pi f_c t + \theta_m) \qquad V_m = \sqrt{A_{mc}^2 + A_{ms}^2}$$

$$\theta_m = \tan^{-1}\left(\frac{A_{ms}}{A_{mc}}\right)$$

$\Rightarrow$ QAM is a combined amplitude & phase modulation.

Any combination of M_1-level PAM $\quad M_1 = 2^m$
M_2-phase PSK $\quad M_2 = 2^n$

$M_1 M_2$ QAM representing $\log_2 M_1 M_2 = m+n$ bits

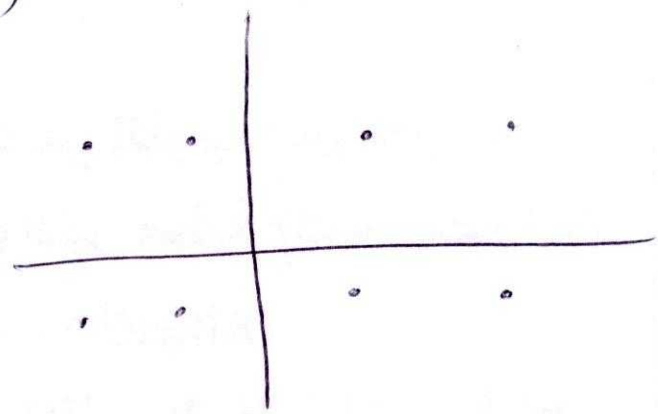
$\Rightarrow$ Bit rate $= R/(m+n)$

Special case

signal amplitudes takes discrete values

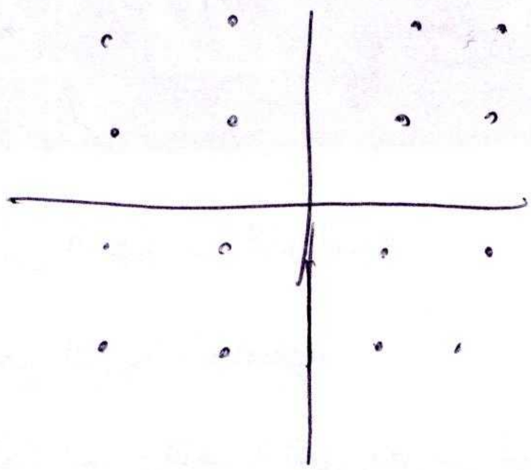
$$A_{mc} = (2m - 1 - M_1)\, d$$

$$A_{ms} = (2m - 1 - M_2)\, d$$



Another version of 8-QAM

Most famous are the square QAM



Probability of error pp. 276 – 280

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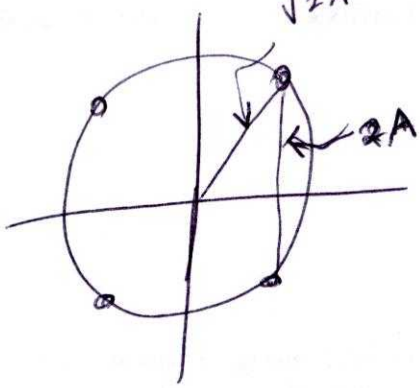
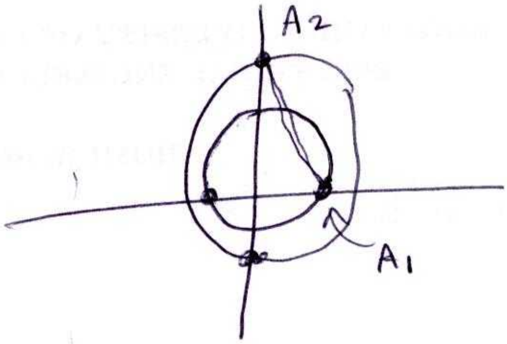
$M=4$

Two possibilities





Prob. of error dominated by ~~prob. of error~~ min distance

$$P_{av} = \frac{1}{4} 4 (\sqrt{2}A)^2$$
$$= 2A^2$$

To get minimum distance $d = 2A$ set $A_1 = A$

~~To maintain $P_{av} = 2A^2$~~

~~$A_2 = \sqrt{3}A$~~

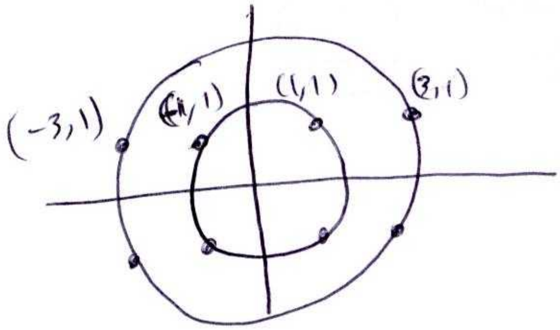
with $A_2 = \sqrt{3}A$, we get same Avg. power

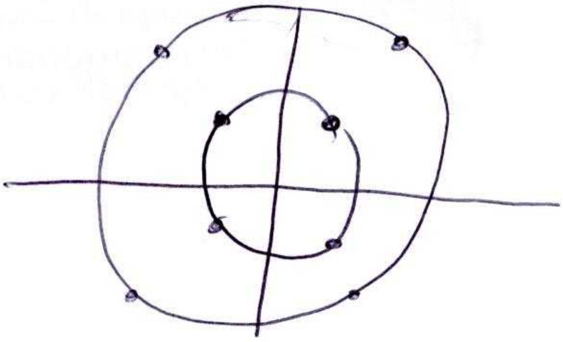
$\Rightarrow$

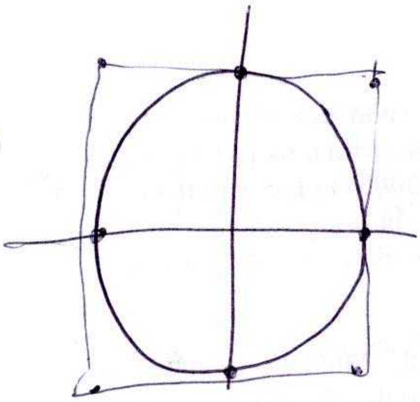
Same min dist $\Rightarrow$ same prob. of error

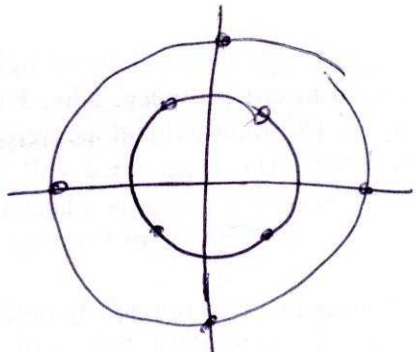
Same prob. of error (min distance wise)

8 - QAM









fix the minimum distance &
Calculate avg. energy

Least energy for
a given min. distance

best possible
QAM constellation

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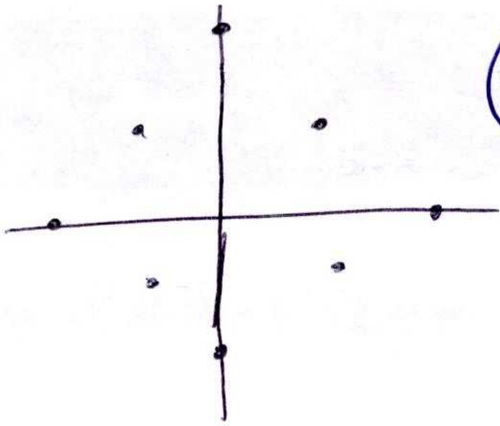
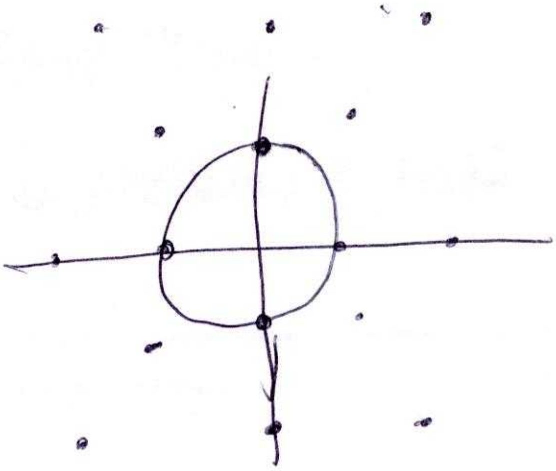


M=8



M=16

M QAM (M≥16):

Generalization of 8-QAM circular QAM is not the optimal 16 QAM.

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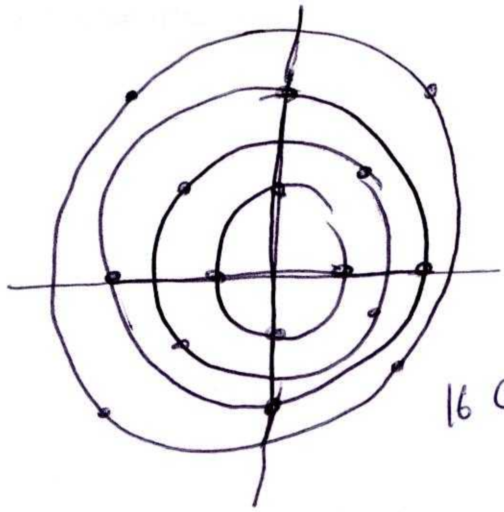
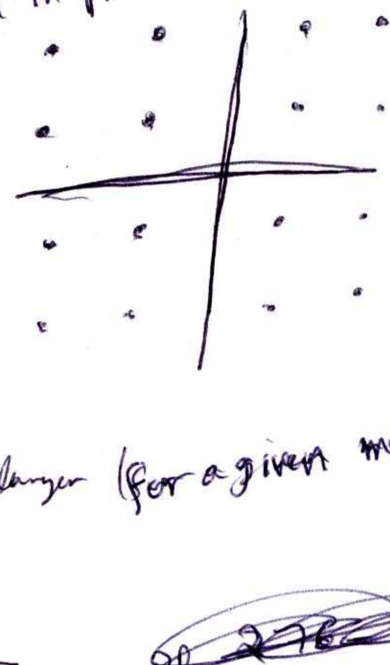


Advantage of rectangular of QAM

- easily generated
- Two PAM signals impressed on in phase & quad carrier



- Avg. power is only slightly larger (for a given min. dist.) than best M-QAM

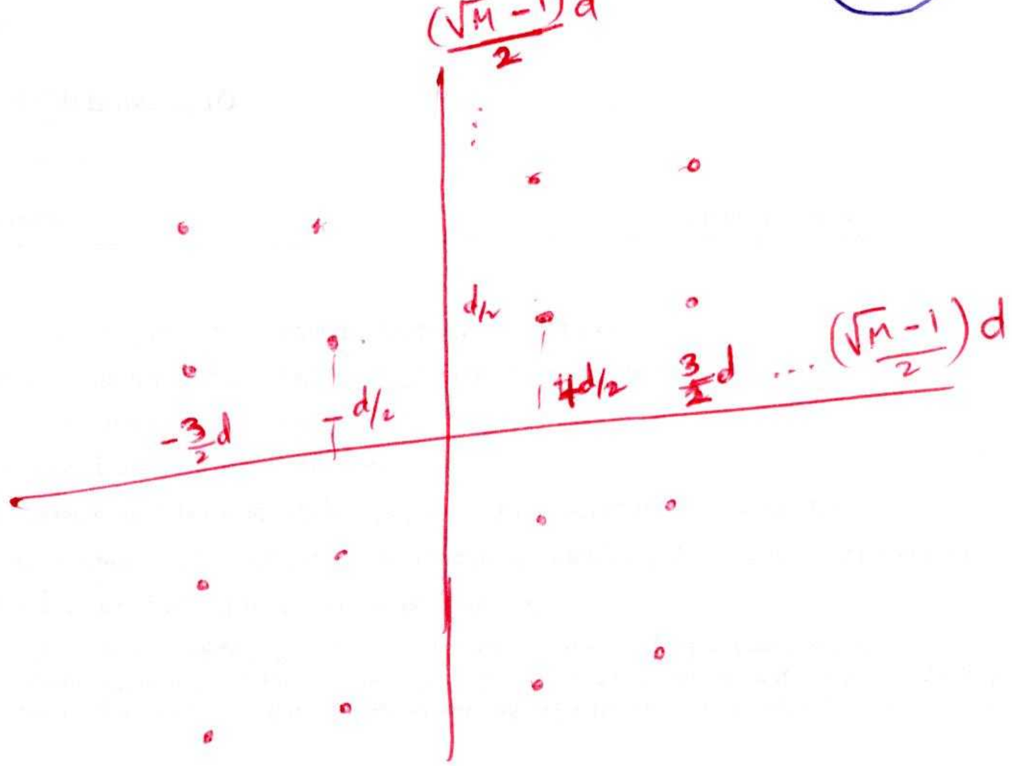
~~pp 276~~ 280

Prob. of error for rect. constellations

- $M = 2^k$ k is even

⇒ Two QAM constellations on quad carriers each with $\sqrt{M} = 2^{k/2}$ symbols

QAM Square constellation



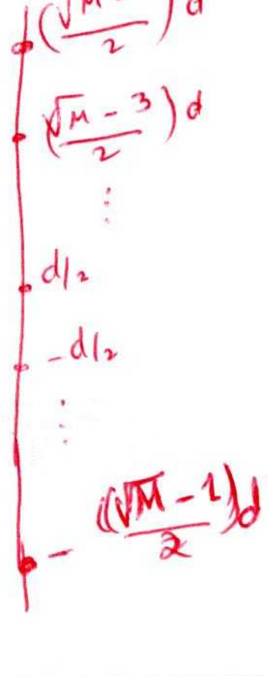
$$\mathcal{E}_{M-QAM} = \frac{1}{M}\sum_{i=1}^{\sqrt{M}}\sum_{j=1}^{\sqrt{M}} x_i^2 + x_j^2$$

$$= \frac{1}{M}\left[\sqrt{M}\sum_{i=1}^{\sqrt{M}} x_i^2 + \sqrt{M}\sum_{j=1}^{\sqrt{M}} x_j^2\right]$$

$$= \frac{2}{\sqrt{M}}\sum_{i=1}^{\sqrt{M}} x_i^2$$

$$= 2\,\mathcal{E}_{\sqrt{M}-PAM}$$

$$= d^2\left(\frac{M-1}{6}\right)$$



Prob of error for rectangular constellations

$M = 2^k$ $\quad$ k is even

$\Rightarrow$ Two QAM constellations on quad carriers each with $\sqrt{M} = 2^{k/2}$ symbols

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Why energy of $\sqrt{M}$ PAM should be half energy of ~~M~~ QAM :

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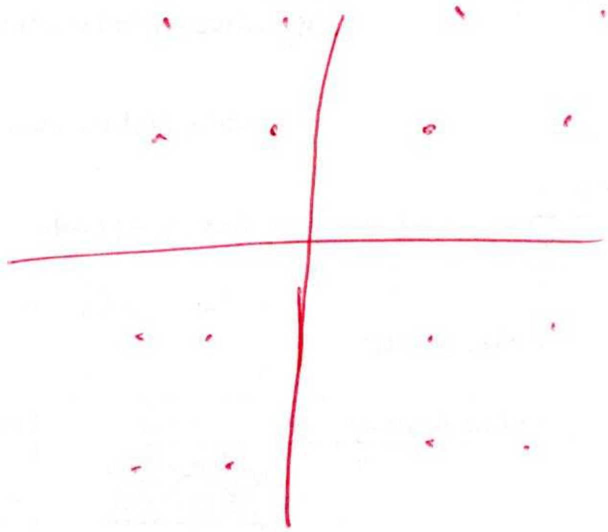

~~Energy of QAM =~~

$$\mathcal{E}_{av}(QAM) = \frac{1}{M}\sum_{j=1}^{\sqrt{M}}\left(\left(\sum_{k=1}^{\sqrt{M}} a_k^2\right) + b_j^2\right)$$

$$\cancel{= \frac{1}{M}\sum_{j=1}^{\sqrt{M}} a_k^2 + \sqrt{\ }}$$

$$= \frac{1}{M}\sum_{j=1}^{\sqrt{M}}\left\{\left(\sum_{k=1}^{\sqrt{M}} a_k^2\right) + \sqrt{M}\, b_j^2\right\}$$

$$= \frac{\sqrt{M}}{M}\sum_{k=1}^{\sqrt{M}} a_k^2 + \frac{\sqrt{M}}{M}\sum_{j=1}^{\sqrt{M}} b_j^2$$

$$= \underbrace{\frac{1}{\sqrt{M}}\sum_{k=1}^{\sqrt{M}} a_k^2}_{\frac{\mathcal{E}_{av}}{2}} + \underbrace{\frac{1}{\sqrt{M}}\sum_{j=1}^{\sqrt{M}} b_j^2}_{\frac{\mathcal{E}_{av}}{2}}$$

$$= \mathcal{E}_{av}$$

In phase & quad components can be separated at demod, can determine prob. of error for QAM from PAM

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$$P_c = \cancel{1-P_A}\left(P_{c\,PAM}\right)^2$$
$$= \left(1 - P_{\sqrt{M}}\right)^2$$

$\cancel{P_{\sqrt{A}}}$ $P_{\sqrt{M}}$ is prob. of error of $\sqrt{M}$ PAM with 1/2 average energy of the ~~equivalent~~ QAM

We know that for PAM

$$P_{\sqrt{M}} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\,\mathcal{E}_{av}}{(M^2-1)N_0}}\right)$$

Replace M by $\sqrt{M}$
$\mathcal{E}_{av}$ by $\mathcal{E}_{av}/2$

$$\Rightarrow P_{\sqrt{M}} = 2\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{\mathcal{E}_{av}}{N_0}}\right)$$

$$\Rightarrow P_{M\,(QAM)} = 1 - \left(1 - 2\left(1-\frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3}{M-1}\frac{\mathcal{E}_{av}}{N_0}}\right)\right)^2$$
$$\simeq 1 - \left(1 - 2Q\left(\sqrt{\frac{3}{M-1}\frac{\mathcal{E}_{av}}{N_0}}\right)\right)^2$$
$$\leq 4Q(\sqrt{\ }) - Q^2(\sqrt{\ })$$
$$\leq 4Q\left(\sqrt{\frac{3}{M-1}\frac{\mathcal{E}_{av}}{N_0}}\right)$$

B.

In terms of bits,

$$\mathcal{E}_{M-QAM} = d^2\left(\frac{M-1}{6}\right) \Rightarrow d = \sqrt{6\frac{\mathcal{E}_{M-QAM}}{M-1}}$$

$$\cancel{d^2 \frac{M-1}{6}}$$

$$= d^2 \frac{2^k - 1}{6}$$

$$= d^2 \frac{2 \cdot 2^{k-1} - 2 + 1}{6}$$

$$\cancel{= 2 d^2 \cdot 2^{k-1}}$$

$$= 2 d^2 \frac{2^{k-1} - 1}{6} + \frac{d^2}{6}$$

$$\mathcal{E}_{2^k-QAM} = 2\,\mathcal{E}_{2^{k-1}-QAM} + d^2/6$$

$\Rightarrow$ avg. energy increases by 3 dB for each additional bit.

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~~When~~ For $M = 2^k$ k is odd, no equivalent PAM system exists

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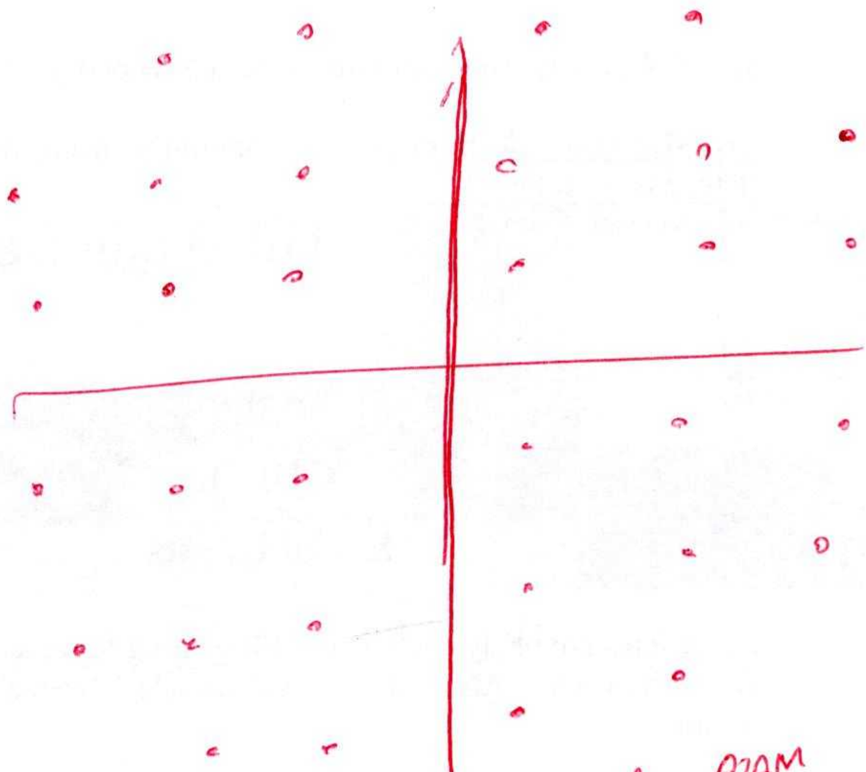
$M = 32$



Cross Rectangular ~~R~~QAM

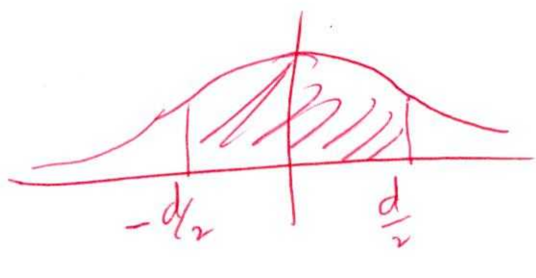
Bounding Prob. of error

$$P_{c|\text{any point}} = P\left(-\frac{d}{2} < n_1 \le \frac{d}{2}\right) P\left(-\frac{d}{2} < n_2 \le \frac{d}{2}\right)$$

$$= \left(1 - 2Q\left(\frac{d/2}{\sqrt{\frac{N_0}{2}}}\right)\right)^2$$

inner point.

$=$



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But $d = \sqrt{\frac{6\,\mathcal{E}_{M-QMAM}}{M-1}} = \sqrt{\frac{6\,\mathcal{E}_{av}}{M-1}}$

$\Rightarrow$

$P_c|_{\text{inner point}} = \left(1 - 2Q\left(\sqrt{\frac{3\,\mathcal{E}_{av}}{M-1}}\right)\right)^2$

$\Rightarrow P_e|_{\text{inner pt}} = 1 - \left(\quad\right)^2$

$\Rightarrow P_c|_{\text{outer point}} > \left(1 - 2Q\left(\sqrt{\frac{3\,\mathcal{E}_{av}}{M-1}}\right)\right)^2$

$\Rightarrow P_e|_{\text{outer point}} < 1 - \left(\quad\right)^2$

$\Rightarrow$

$P_e \leq 1 - \left(1 - 2Q\left(\sqrt{\frac{3\,\mathcal{E}_{av}}{M-1}}\right)\right)^2$

$\leq 4Q\left(\sqrt{\frac{3\,\mathcal{E}_{av}}{M-1}}\right)$

$\mathcal{E}_{bav} = \frac{\mathcal{E}_{av}}{k}$ (average energy per bit)

$\Rightarrow P_e \leq 4Q\left(\sqrt{\frac{3k\,\mathcal{E}_{bav}}{(M-1)N_0}}\right)$

Non rectangular QAM signals constellations

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Nearest neighbor bound

$$P_M < (M-1)\, Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$

$(M-1)$: # of neibours

For rectangular constelation

$$d_{min} = \sqrt{\frac{6\,\mathcal{E}_{avg}}{M-1}}$$

$$\Rightarrow P_M < (M-1)\, Q\left(\sqrt{\frac{3\,\mathcal{E}_{av}}{(M-1)N_0}}\right)$$

very loose compared with other bond.

Approximate P_M by the largest # of nearest neibourghrs (M_n)

$$P_M \simeq M_n\, Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right)$$

For rectangular constellation $M_n = 4$

$$\Rightarrow P_M \simeq 4\, Q\left(\sqrt{\frac{3\,\mathcal{E}_{av}}{(M-1)N_0}}\right)$$

(similar to the bound we get by approximation ~~the exact~~ the exact error probability).

Prob. of error dominated by argument of

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$$P_{e\,QAM} = 4\,Q\left(\sqrt{\frac{3k\,\mathcal{E}_{bav}}{M-1\,N_0}}\right)$$

$$P_{e\,PSK} = 2\,Q\left(\sqrt{2k\gamma_b \sin\frac{\pi}{M}}\right)$$

Ratio of two arguments $R_M = \dfrac{3/M-1}{2\sin^2(\pi/M)}$

$M=4 \Rightarrow R_M = 1$ (Same perf.) (as expected)

$M>4$ $\quad R_M > 4$ QAM better than PSK

32-QAM is 7dB better than PSK

It is expected for QAM to be better than PSK. In PSK, we are unecessarily ~~constraining~~ constraining the amplitude to be constant. This should eventually have a hit on the error probability

. Adv. of PSK is that it is immune to nonlinearities