

Optimal detection for a General vector channel

Math model for AWGN ch. is

$$r = s_m + n$$

This is what we expect to get when project $r(t)$ on $\phi_1(t), \phi_2(t), \dots, \phi_M(t)$.

Now r is a vector in the \mathbb{R}^N space. How do we construct our decision regions to minimize the prob. of error.

When $n(t)$ is Gaussian, n will be Gaussian & so will be m .

Consider the general case:



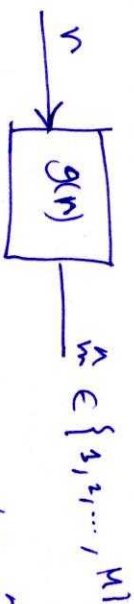
signal vectors $\{s_m\}$ are selected from $\{s_m \mid 1 \leq m \leq M\}$ according to a priori probabilities P_m & transmitted over channel.

Received Signal

received sig. r depends statistically on Tx vector through cond. prob. density function $P(r|s_m)$

Receiver

Observes r & based on this decides which message was transmitted



$g(r) = \hat{m}$ receiver decides that \hat{m} was transmitted

Prob. that decision is correct ~~is~~ probability that \hat{m} is the Tx message given that r is received

$$P[\text{correct decision} | r] = P[\hat{m} \text{ is sent} | r]$$

$$P[\text{correct decision}] = \int P[\text{correct} | r] p(r) dr$$

$$= \int P[\hat{m} \text{ sent} | r] p(r) dr$$

Opt. detector

Detector that minimizes prob. of error
 that max. prob. of correct decisi
 Detector

$$P[\text{correct decisi}] = \int P[\hat{m} \text{ sent} | r] p(r) dr$$

maximized if for each
 $\forall P[\hat{m} \text{ sent} | r]$
 is maximized.

$$\hat{m} = \underset{1 \leq m \leq M}{\text{arg max}} P[m | r]$$

Check all $P[m | r]$

largest.

$1 \leq m \leq M$ & select the

or

$$\hat{m} = \underset{1 \leq m \leq M}{\text{arg max}} P[s_m | r]$$

MAP & ML Receivers

Decision rule

$$\hat{m} = \underset{1 \leq m \leq M}{\text{arg max}} P[s_m | r]$$

(MAP)

is the max a posteriori probability rule

Can be equivalently written as

$$\hat{m} = \underset{1 \leq m \leq M}{\text{arg max}} \frac{P_m P(r|s_m)}{P(r)}$$

indep. of m for all m

⇒

$$\hat{m} = \underset{1 \leq m \leq M}{\text{arg max}} P_m P(r|s_m)$$

← Easier to use than *

because P_m & $P(r|s_m)$ are directly available

ML Receiver

If the messages are equiprobable

$$P_m = \frac{1}{M}$$

Optimal decision is

$$\hat{m} = \underset{1 \leq m \leq M}{\text{arg max}} P(r|s_m)$$

maximum likelihood Rx

ML receiver not optimal decoder unless messages are equiprobable.

Popular since exact probabilities also messages difficult to get (e.g. info comes from different sources)

Decision Regions

Detector partitions space \mathbb{R}^N into M regions D_1, D_2, \dots, D_M s.t.

if $r \in D_m$, then ~~output~~ $\hat{m} = g(r) = m$

D_m is the set of channel outputs r that are mapped into msg m by detector

For MAP

$$D_m = \{r \in \mathbb{R}^N : P(m|r) > P(m'|r) \forall \Delta \leq m' \leq M, m' \neq m\}$$

What happens if two messages are such that $P(m|r) = P(m'|r)$ for a given r

In that case, assign r to one of the 2 decision regions.

This will not affect

prob. of error.

Error Probability

An error occurs if s_m is transmitted, r is not in D_m

\Rightarrow symbol error prob. with decision regions $\{D_m, 1 \leq m \leq M\}$ is given by

$$P_e = \sum_{m=1}^M P_m P[r \notin D_m | s_m \text{ sent}] = \sum_{m=1}^M P_m P_{e|m}$$

$P_{e|m}$ = error prob. when msg m is tx

$$P_{e|m} = \int_{D_m^c} p(r|s_m) dr = \sum_{\substack{m'=1 \\ m' \neq m}}^M \int_{D_{m'}} p(r|s_m) dr$$