We need a more general method for finding the orthonormal basis because (21)

① We would like to have the least # of $\phi_n(t)$'s

② Finding the $\phi_n(t)$'s might not always be intuitive

To do this, we employ the Gram Shmidt Procedure.

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Gram_Schmidt procedure:

Consider the set of waveforms $\{x_i(t), i = 1, 2, \ldots, M\}$

We would like to construct an orthonormal basis that spans this set.

1) Choose $\varphi_1(t) = \dfrac{x_1(t)}{\sqrt{\mathcal{E}_1}}$

where $\mathcal{E}_1 = \int x_1^2(t)\, dt$ (energy of $x_1(t)$)

Normalization makes sure that $\varphi_1(t)$ will have unit norm

② We use $x_2(t)$ to construct $\varphi_2(t)$. How?

first project $x_2(t)$ onto $\varphi_1(t)$

$$c_1^2 = \int x_2(t)\, \varphi_1(t)$$

~~then subtract~~ then form

$$\varphi_2'(t) = x_2(t) - c_1^2 \varphi_1(t)$$

and normalize it by its energy to get

$$\varphi_2(t) = \frac{\varphi_2'(t)}{\sqrt{\mathcal{E}_2}}$$

$\leftarrow$ energy of $\varphi_2'(t)$.

In general

$$\varphi_k(t) = \frac{\varphi_k'(t)}{\sqrt{E_k}}$$ (28)

where

$$\varphi_k'(t) = x_k(t) - \sum_{i=1}^{k-1} c_i^k \varphi_i(t)$$

where c_i^k is the projection of $x_k(t)$ on $\varphi_i(t)$.

The number of φ_k's will be less than or equal to the number of $x_i(t)$'s.

(See Proakis 163 - 164)

Why are we doing this?

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We want to do something similar for our digital waveforms

Let $x_0(t), x_1(t), \ldots, x_{M-1}(t)$ be M possible waveforms that we transmit. We want to represent each waveform as

$$x_i(t) = \sum_{n=1}^{N} x_n \varphi_n(t)$$

Have we seen something similar before?

Yes. Let $g(t)$ be a periodic signal with period T.

then

$$g(t) = \sum a_n \cos\left(\frac{2\pi}{T} n t\right) + b_n \sin\left(\frac{2\pi}{T} n t\right)$$

~~Demodulate~~

We can use the fact that (23)

$$x_i(t) = \sum_n x_n^i \varphi_n(t)$$

to build a demodulator. ~~Specifically~~

Let's first accept that it is enough to deal with x_n^i instead of $x_i(t)$.

Now note that

$$x_n^i = \int_0^T x_i(t)\,\varphi_n(t)$$

How do we implement this?

It is $x_i(t) * \varphi(T-t)\Big|_{t=T}$

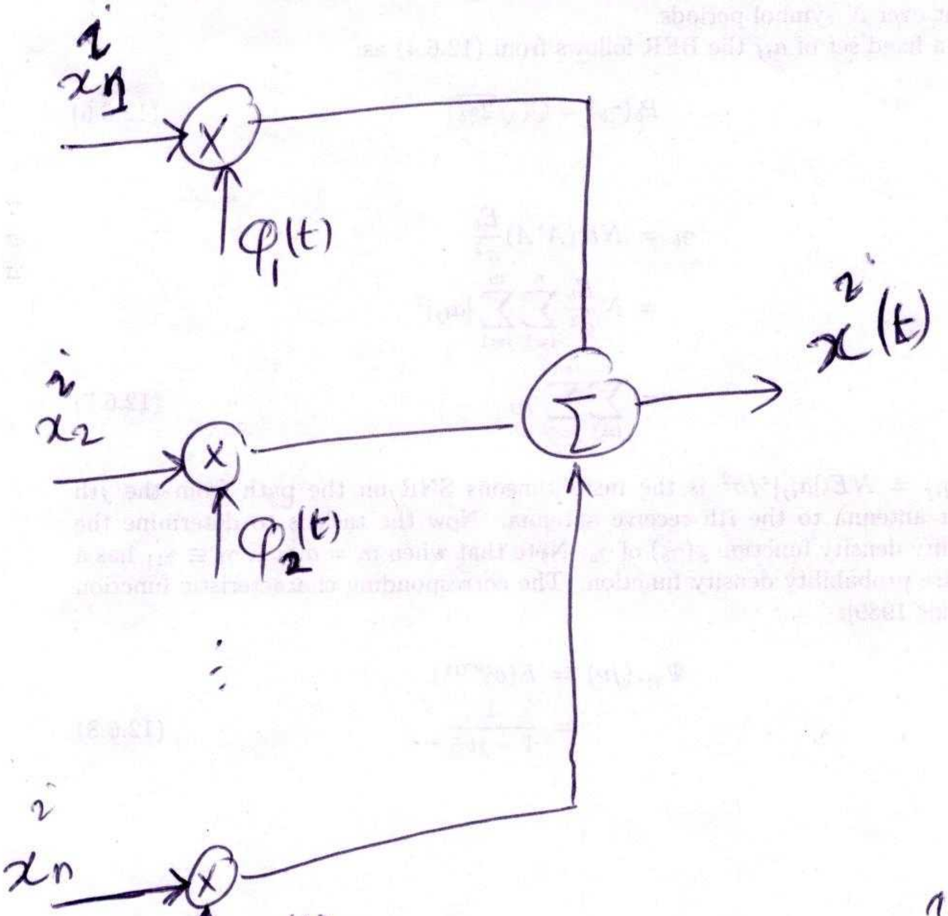
$$x_i(t) * \varphi(T-t) = \int_0^t x_i(\tau)\,\varphi(T-(t-\tau))\,d\tau$$

~~Now we can~~

We can also use the basis expansion

$$x^i(t) = \sum_n x_n^i \, \varphi_n(t)$$

to generate the waveforms i.e. to build the modulator



Total # of multipliers $= N$ (# of φ_n's)

$$\Rightarrow x_i(t) * \varphi(T-t)\Big|_{t=T} = \int_0^T x_i(\tau)\varphi_n(\tau)\,d\tau = x_n^i$$

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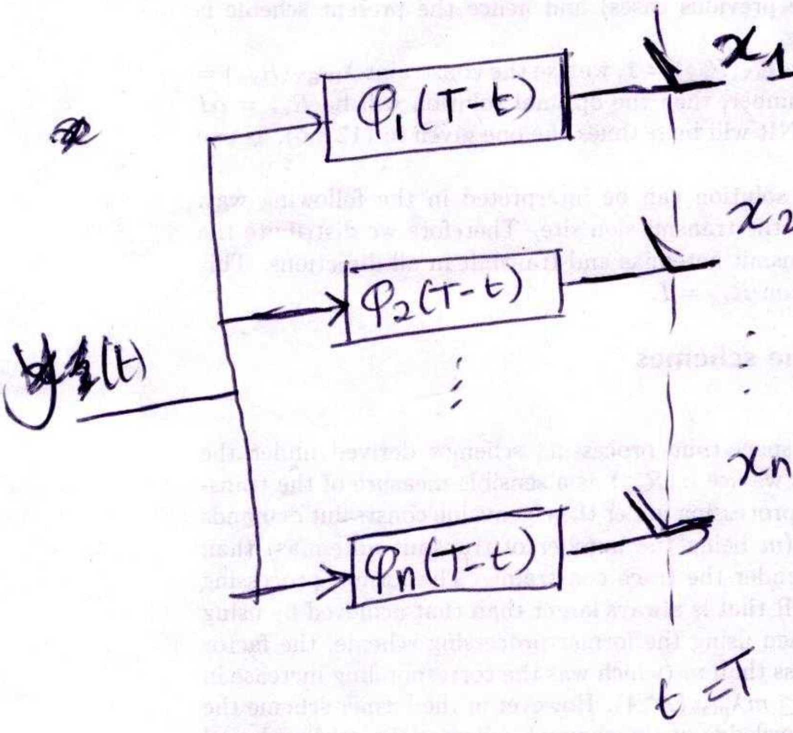
So, the demodulator looks like



In absence of noise

$$y(t) = x_i(t)$$

and we can detect the received $x_i(t)$ by concentrating on the vector $\begin{bmatrix} x_1^i \\ \vdots \\ x_N^i \end{bmatrix}$