KING ABDULLAH UNIVERSITY OF SCIENCE AND TECHNOLOGY ELECTRICAL ENGINEERING DEPARTMENT Fall 2012

EE 242 Digital Communications and Coding

Home Work #4 (*due Nov. 18, 2012*)

Q1. In this problem, analyze the degradation in performance of a binary demodulator due to the use of a filter different from the optimum matched filter. The system is shown in Fig. 1. Assume,

$$s_1(t) = 0, \quad s_2(t) = \sqrt{\frac{\epsilon}{T}}, \qquad 0 \le t < T$$

and n(t) a Gaussian noise process with power density $\frac{N_0}{2}$. The optimum demodulator would require a matched filter, i.e., one with impulse response

$$h_{opt}(t) = \sqrt{\frac{1}{T}}, \qquad 0 \le t < T$$

In other words, $h_{opt}(t)$ is the basis function. Assume instead an approximation of a simple *RC* filter with impulse response $h(t) = e^{-At}$, $0 \le t < T$.

- a. Compute the error probability of this nonoptimum demodulator.
- **b.** Find the value of A that minimizes the error probability found in (\mathbf{a}) .
- **c.** Evaluate the increase of transmitted energy required to get the same error probability as with the optimum demodulator.



Figure 1: System for Q1.

Q2. Consider the 16-QAM constellation with minimum distance $d_{min} = 2$ which is observed in additive noise with variance $N_0/2 = 0.01$. Answer the following questions.

- **a.** Obtain an upper bound for the probability of error in terms of the minimum distance of the constellation.
- **b.** Consider the 16-QAM with Gray coding. Use the union bound procedure to calculate a bound on the bit error rate. Express your answer in terms of the minimum distance.

Q3. In our class, we focused on the design and analysis of communication systems that deal with AWGN channels. In this problem, we show how our design and analysis can be extended to channels that introduce amplitude and phase distortion. Thus, consider the following channel.



Figure 2: Channel for Q3.

Assume that $\mathcal{X}_i = +1$ or -1 with equal probability. Also assume that the memory element is initially empty (i.e., it contains the zero element).

- **a.** Find the possible values of Y_1 and Y_2 .
- **b.** Draw the constellation points on the two dimensional plane and determine the decision regions.
- c. What is the optimal decision if the received signal is $Y_1 = 0.2$ and $Y_2 = 0.3$?
- **d.** What challenge does the receiver face when the transmitter sends L symbols, $\mathcal{X}_1, \mathcal{X}_2, \cdots, \mathcal{X}_L$?