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KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
ELECTRICAL ENGINEERING DEPARTMENT
Fall 2012
EE 242/571 Digital Communications and Coding

Home Work #1

(*due Sep. 24, 2012*)

Q1. Consider the sinusoidal signal $g(t) = 2 * \cos(2 * \Pi * 100 * t)$

a. Draw the spectrum (Fourier Transform) of this signal. What is the least sampling rate that guarantees that we can recover the signal from its samples? What is the sampling period?

b. The signal is sampled at $f_s = 400$ samples/sec. Find the values of the samples at $t = 0$, $t = 2T_s$, and $t = 100T_s$.

c. Draw the Fourier Transform of the sampled signal.

d. At the receiver, we use the filter shown in the Figure 1 to reconstruct the original signal. Let $y(t)$ be the filter output.

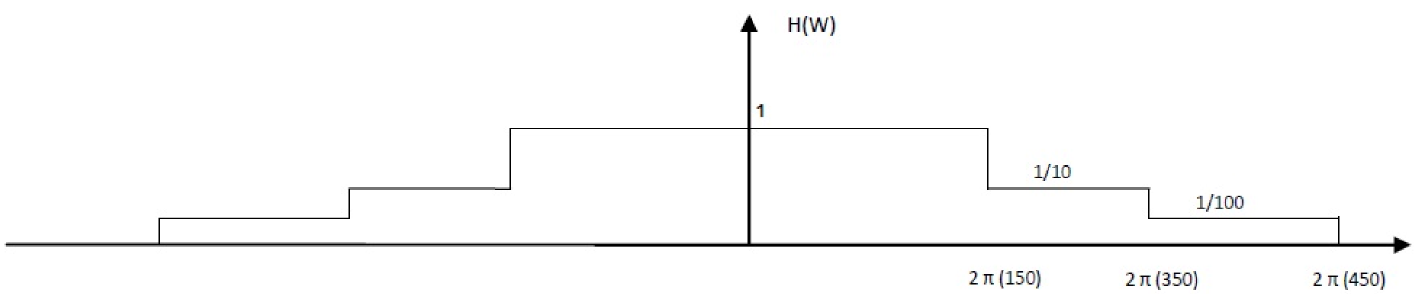

Figure 1: Filter

i. Determine the output signal $y(t)$. Why is $y(t)$ different from $g(t)$?

ii. The signal $y(t)$ consists of a desired component and an undesired one. Determine the powers in the desired and undesired parts (P_d and P_u).

iii. Calculate the signal to noise ratio in dB.

iv. The transmitter decides to increase the sampling rate to 500 samples/sec to improve the performance of the system. What is the new signal to noise ratio in dB. Is there another way to improve the SNR.

Q2. Consider the sinusoidal $m(t) = 8 * \cos(2 * \Pi * 3000 * t)$. This signal is sampled and quantized according to the following:

Sampling rate = 10 times the Nyquist rate.

Quantized using a quantizer with 500 levels.

Quantizer has a quantization range of -10 to 10 V.Find the SNR of the PCM signal in dBs.

a. Find the SNR of the PCM signal in dBs.

b. How many bits are required to represent 900 sec of this signal.

Q3. A discrete memoryless source has a symbol alphabet with $|A| = 10$. Find the upper and lower bounds of the source entropy.

Q4. Consider a source with source alphabet $A = \{a_0, a_1, a_2, a_3\}$ and alphabet probabilities

$$p(a_0) = 1/2, \quad p(a_1) = 1/4, \quad p(a_2) = 1/4, \quad p(a_3) = 1/8.$$

a. Find the entropy of this source.

b. Suppose that the source emits its symbols at a rate of 100 symbols every second. Calculate the information rate of the source.

c. Design an encoder that achieves an efficiency of 100%, provided that 000 is one codeword of the encoder.

d. Now consider the following source encoder output (21 bits)

$$101011001000000000001 \tag{1}$$

i. Decode this output.

ii. What would be the source encoder output in bits if we use instead the following mapping

$$a_0 \to 00, \quad a_1 \to 01, \quad a_2 \to 10, \quad a_3 \to 11. \tag{2}$$

What is the total number of bits needed and how does it compare to (1) which is based on the code you designed.

iii. Do we conclude that code (2) is better than your code? Explain.

Q5. Consider the random variable X with the alphabet $\{1, 2, \ldots, 7\}$ and the corresponding probabilities $\{0.49, 0.26, 0.12, 0.04, 0.04, 0.03, 0.02\}$

a. Find a Huffman code.

b. Find the expected length of the code and its efficiency.

Q6. Which of the following codes can not be Huffman codes

a. $\{0, 10, 11\}$

b. $\{00, 01, 10, 110\}$

b. $\{01, 10\}$

Q7. Let A and B be two alphabets with probabilities $P_A = \{p_0, p_1, ..., p_{M-1}\}$ and $P_B = \{q_0, q_1, ..., q_{N-1}\}$ We create a new alphabet C by concatenating the alphabets of A and B. Prove that

$H(C) = H(A) + H(B).$