New Formulation for Evaluating Complex Permittivity of Low-Loss Materials

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Abstract

A widely used method for non-contact electromagnetic characterization of materials is based on the measurement of an 'insertion transfer function'. This function is related to the complex dielectric constant of the material through a transcendental equation which can be solved numerically using a two-dimensional root search technique. Solving this complex equation is often time consuming due to slow convergence and the existence of spurious solutions. In this article, a new formulation is presented which facilitates the evaluation of complex dielectric constant of low-loss materials by means of real equations, thus requiring only one-dimensional root search techniques. A wooden slab is characterized using the new formulation. The results for loss tangent and dielectric constant are in excellent agreement with those obtained from the complex equation.

I. Introduction

Electromagnetic characterization of building materials is essential to studying of wave propagation in indoor environments. Various techniques have been developed for the characterization of materials, each with its unique capabilities and advantages. Here, attention is focused on a characterization method based on the measurement of an *insertion transfer function* using radiated fields. This method is particularly useful for accurate characterization of construction materials needed for ultra-wideband indoor propagation studies. The complex dielectric constant of the material under test can be extracted from the measured insertion transfer function [1].

From the analysis of a plane-wave normally incident upon a one-dimensional slab material, an expression for the insertion transfer function can be derived. This expression, in addition to the insertion transfer function, includes the complex dielectric constant of the material under test, the slab thickness, and the frequency of operation. With the insertion transfer function obtained by measurements and the slab thickness known, the expression is in fact a complex transcendental equation which can be solved for the complex dielectric constant of the material at a given frequency [2].

Solving a complex equation requires a two-dimensional root search which is often time consuming. For most materials of interest losses are small, yet a two-dimensional root search technique has been used to calculate their complex dielectric constants. In this article, a new formulation is presented for the evaluation of complex dielectric constant of materials with relatively small losses. In this formulation, only real equations are solved using one-dimensional root search techniques, thus reducing the computation time significantly.

II. Measurement of Insertion Transfer Function

The insertion transfer function is obtained through two measurements as shown schematically in Figure 1. The measurements may be performed in either the time domain using short duration pulses, or in the frequency domain using sinusoidal signals. The measurement procedure is as follows: the transmitter and receiver antennas are kept at fixed locations and aligned for maximum reception. The material to be measured is placed at nearly the mid-point between the two antennas. The distance between the antennas should be sufficiently large such that the material under test is in the far field of each antenna. With this arrangement, the electromagnetic field incident on the material is essentially a plane wave. The material is assumed to be in the form of a slab with thickness d and held in position such that the plane wave is normally incident on it, as shown in Fig. 1. After the measurement system is set up, first we

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measure the time-domain signal $v_t^{fs}(t)$ with a sampling oscilloscope or the frequency-domain signal $V_t^{fs}(j\omega)$ with a network analyzer in the absence of the material. Then, we measure the time-domain signal $v_t(t)$ or the frequency-domain signal $V_t(j\omega)$ with the material layer in place. The insertion transfer function is then calculated as

$$H(j\omega) \equiv \frac{E_t(j\omega)/E_i(j\omega)}{E_t^{fs}(j\omega)/E_i(j\omega)} = \frac{E_t(j\omega)}{E_t^{fs}(j\omega)} = \frac{FFT(v_t(t))}{FFT(v_t^{fs}(t))} = \frac{V_t(j\omega)}{V_t^{fs}(j\omega)}, \tag{1}$$

where ω=$2\pi f$ is the angular frequency and fast Fourier transform (FFT) is used to convert the sampled signal to the frequency domain data. Care must be taken to ensure that the conditions for the free-space measurement are as closely identical as possible to those for the measurement through the material slab.

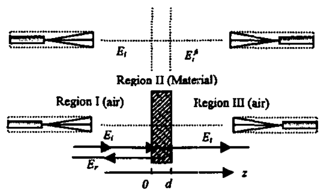


Figure 1. Propagation through a slab for the radiated measurement of insertion transfer function

III. Evaluation of Complex Dielectric Constant

In order to determine the complex dielectric constant from the measured insertion transfer function, an expression for $H(j\omega)$ is needed. To calculate $H(j\omega)$ let us assume that an x-polarized uniform plane-wave, representing the local far-field of the transmitter antenna is normally incedent on a slab of material having a thickness d. The material has an unknown complex dielectric constant $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$. The incident plane-wave, as depicted in Figure 1, establishes a reflected wave in region I (air), a set of forward and backward traveling waves in region II (material), and a transmitted wave in region III (air). Imposing the boundary conditions for the electric and the magnetic fields at the slab-air interfaces, the transmission coefficient can be calculated in a straightforward manner. The result is

$$T = \frac{E_t\, e^{-j\beta_0 d}}{E_i} = 4\left(e^{\gamma d}\left(2 + \frac{\eta_1}{\eta_2} + \frac{\eta_2}{\eta_1}\right) + e^{-\gamma d}\left(2 - \frac{\eta_1}{\eta_2} - \frac{\eta_2}{\eta_1}\right)\right)^{-1} \tag{2}$$

where $\beta_0 = \omega\sqrt{\mu_0\varepsilon_0} = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$, $\gamma = \alpha + j\beta = j\omega\sqrt{\mu_0\varepsilon_0(\varepsilon_r' - j\varepsilon_r'')}$, $\eta_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 120\pi\ \Omega$, and $\eta_2 = \sqrt{\frac{\mu}{\varepsilon_0(\varepsilon_r' - j\varepsilon_r'')}}$.

The insertion transfer function is related to the transmission coefficient through $Te^{j\beta_0 d} = H(j\omega)$. Thus,

$$H(j\omega) = \frac{4e^{j\beta_0 d}}{e^{\gamma d}(2 + \frac{\eta_1}{\eta_2} + \frac{\eta_2}{\eta_1}) + e^{-\gamma d}(2 - \frac{\eta_1}{\eta_2} - \frac{\eta_2}{\eta_1})}. \tag{3}$$

It should be noted that the transmission coefficient T is equivalent to S_{21} in the scattering parameters terminology. Once the complex insertion transfer function $H(j\omega)$ is determined by measurements, equation (3) can be solved for the complex dielectric constant $\varepsilon_r = \varepsilon_r' - j\varepsilon_r''$. In terms of the scattering parameter S_{21}, the parameter that is directly measured, (3) can be easily cast into the following form [2,3],

$$\left(x+\frac{1}{x}\right)\sinh(xP)+2\cosh(xP)-\frac{2}{S_{21}}=0, \tag{4}$$

where $x=\sqrt{\varepsilon_r}$, $S_{21}(j\omega)=H(j\omega)e^{-j\omega\tau_0}$, and $P=j\beta_0 d$. This equation can be solved numerically using two-dimensional search algorithms. The convergence of this algorithm is not always guaranteed because of possible multiple solutions and noise in the measurements. In the next section, using reasonable assumptions, equation (4) is reduced to a one-dimensional problem involving real equations only.

IV A New Formulation for Characterization of Low-Loss Materials

When the material occupying region II is low loss, $\varepsilon_r''/\varepsilon_r' << 1$ and the following approximations can be used,

$$\begin{aligned}\gamma &= \alpha + j\beta = j\omega\sqrt{\mu_0\varepsilon_0(\varepsilon_r' - j\varepsilon_r'')} \cong j\omega\sqrt{\mu_0\varepsilon_0}\sqrt{\varepsilon_r}(1 - j\frac{1}{2}\frac{\varepsilon_r''}{\varepsilon_r'}) \\ &= j\beta_0\sqrt{\varepsilon_r'}(1 - j\frac{1}{2}\frac{\varepsilon_r''}{\varepsilon_r'})\end{aligned} \tag{5}$$

and

$$\eta_2=\sqrt{\frac{\mu}{\varepsilon_0(\varepsilon_r'-j\varepsilon_r'')}}\cong\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r'}}=\frac{\eta_1}{\sqrt{\varepsilon_r'}}. \tag{6}$$

Then, $\frac{\eta_1}{\eta_2}+\frac{\eta_2}{\eta_1}\cong\sqrt{\varepsilon_r'}+\frac{1}{\sqrt{\varepsilon_r'}}=\frac{\varepsilon_r'+1}{\sqrt{\varepsilon_r'}}$ and (3) reduces to

$$H(j\omega)=\frac{4e^{j\beta_0 d}}{e^{(\alpha+j\beta)d}(2+\frac{\varepsilon_r'+1}{\sqrt{\varepsilon_r'}})+e^{-(\alpha+j\beta)d}(2-\frac{\varepsilon_r'+1}{\sqrt{\varepsilon_r'}})}. \tag{7}$$

Rewriting the insertion transfer function in terms of magnitude and phase, we obtain

$$|H(j\omega)|=4\left(e^{2\alpha d}\left(2+\frac{\varepsilon_r'+1}{\sqrt{\varepsilon_r'}}\right)^2+e^{-2\alpha d}\left(2-\frac{\varepsilon_r'+1}{\sqrt{\varepsilon_r'}}\right)^2+2\cos(2\beta d)\left(4-\left(\frac{\varepsilon_r'+1}{\sqrt{\varepsilon_r'}}\right)^2\right)\right)^{-1/2} \tag{8}$$

and

$$\angle H(j\omega)=\beta_0 d-\phi \tag{9}$$

where

$$\phi=\tan^{-1}\left\{\left[\frac{1-e^{-2\alpha d}Q}{1+e^{-2\alpha d}Q}\right]\cdot\tan(\beta d)\right\} \quad \text{and} \quad Q=-\left(\frac{\sqrt{\varepsilon_r'}-1}{\sqrt{\varepsilon_r'}+1}\right)^2 \tag{10}$$

Letting $e^{-2\alpha d}=X$ *in (8) and rearranging the terms, yields*

$$X^2\left(\sqrt{\varepsilon_r'}-1\right)^4-2\left[\cos(2\beta d)(\varepsilon_r'-1)^2+8\frac{\varepsilon_r'}{|H(j\omega)|^2}\right]X+\left(\sqrt{\varepsilon_r'}+1\right)^4=0, \tag{11}$$

which is a quadratic equation in terms of X. Solving this equation for X, we have

$$X=e^{-2\alpha d}=\frac{\left[\cos(2\beta d)(\varepsilon_r'-1)^2+8\frac{\varepsilon_r'}{|H(j\omega)|^2}\right]\pm\sqrt{\left[\cos(2\beta d)(\varepsilon_r'-1)^2+8\frac{\varepsilon_r'}{|H(j\omega)|^2}\right]^2-(\varepsilon_r'-1)^4}}{\left(\sqrt{\varepsilon_r'}-1\right)^4} \tag{12}$$

Only the solution with negative sign in (12) is valid. The solution with positive sign results in non-realistic behavior of amplification rather than attenuation. Substituting for X from (12) in the phase expression (10), we obtain the following equation which is only in terms of ε_r'.

$$\tan[\beta_0 d - \angle H(j\omega)] + \frac{1-QX}{1+QX}\tan(\beta d) = 0 \tag{13}$$

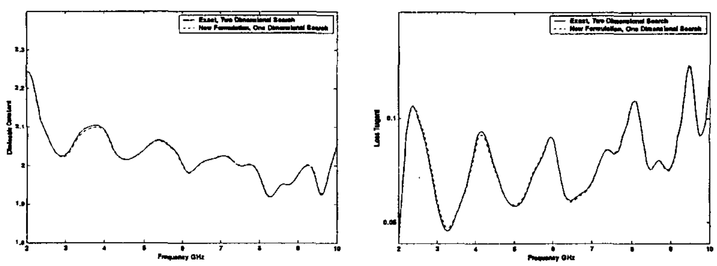
Solving this equation numerically, ε_r' is readily determined. Then, X and subsequently α are found from (12). Finally, ε_r'' is calculated using

$$\varepsilon_r'' = \frac{2c\alpha\sqrt{\varepsilon_r'}}{\omega}. \tag{14}$$

V Measurement Results

Measurements were carried out for a sample wooden door representing the slab. Two wideband TEM horn antennas were used. For the sample door, the time-domain and frequency-domain results for the insertion transfer function were essentially the same.

The results for the real part of relative permittivity and the loss tangent obtained from the complex equation (4) and the real equations (13) and (14) are compared in Figures 2a and 2b. It is noted that the results from the two solutions are nearly identical, primarily because the door material is low loss.



(a) (b)

Figure 2. Comparison of (a) dielectric constant and (b) loss tangent for a wooden slab obtained from the exact two-dimensional search and from the new one-dimensional formulation.

VI Conclusion

A new formulation for the characterization of low-loss materials has been presented which requires solving real equations only. This formulation converges more rapidly and thus requires much less computation time than that based on solving the complex equation relating the insertion transfer function to the dielectric constant of the material under test. The new formulation can be used to accurately characterize many materials of practical applications.

References

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