Spread Spectrum

Table of Contents

Error Rate Performance of the Spread Spectrum Decoder

For the BPSK $P_2(m) = Q\left(\frac{mean}{\sqrt{\operatorname{var}iance}}\right) = Q\left(\sqrt{\frac{2\varepsilon_b}{J_0}}\right)$

Assuming $N_0 << J_0$

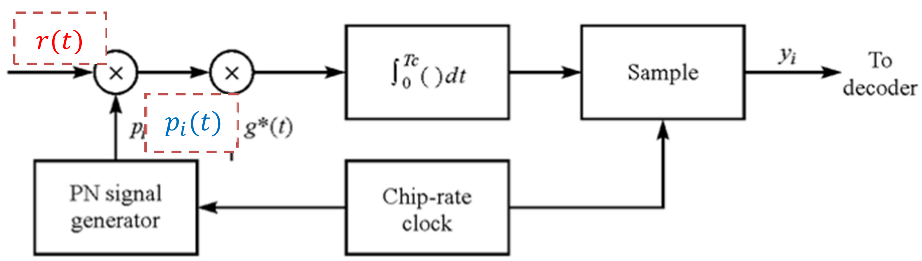
Performance of the un-coded system

Uncoded system like repetition codes results in NO coding gain but they result in processing gain

$$P_2(m) = Q\left(\sqrt{\frac{2W/R}{J_{av}/P_{av}}}\right)$$

Performance of the coded systems

One possible configuration for the receiver is shown here



Note that $p(t)$ is a square wave

$$y_j(t) = \int_0^{T_c} r(t)p_j(t)g^*(t)dt$$

$p(t)$ is a square wave representing the code $(2b_j - 1)$, i.e. $p(t)$=1 or -1 for $T_j \leq t \leq T_{j+1}$

$$= \int_0^{T_c} \left[(2b_j-1)(2c_j-1)g(t-jT_c)+z(t)\right](2b_j-1)g^*(t)dt$$

$$= \int_0^{T_c} \left[(2b_j-1)(2c_j-1)g(t-jT_c)\right](2b_j-1)g^*(t)dt + \int_0^{T_c}\left[z(t)\right](2b_j-1)g^*(t)dt$$

$$\int_0^{T_c} \left[(2c_j-1)\right](2b_j-1)^2 g(t-jT_c)g^*(t)dt + (2b_j-1)\int_0^{T_c}\left[z(t)\right]g^*(t)dt$$

The limits of the integrateion is properly synchronized over one chip (*see green color*)

Correlating the signal with itself results in the energy , since ε_c is the energy per chip for the passband signal (*cos), the energy in baseband will be double.

$$y_j(t) = 2\varepsilon_c(2c_j - 1) + (2b_j - 1)\nu_j(t)$$

where $\nu_j(t)$ is the samples of noise after the matched filter.

Employing soft-decsion , the correlation metrics

$$\mathrm{CM_i} = \sum_{j=1}^{n}(2c_{ij} - 1)y_j(t) \qquad i = 1, 2, ...2^k$$

j^{th} bit

i^{th} codeword

Assume the all zero-codeword is tranmsitted (code#1),

the correlation metric corresponding to the all zero code word.

$$\mathrm{CM_i} = \sum_{j=1}^{n}(2c_{ij} - 1)\left[2\varepsilon_c(2c_{ij} - 1) + (2b_j - 1)\nu_j(t)\right]$$

$$= \sum_{j=1}^{n}2\varepsilon_c(2c_{ij} - 1)^2 + \sum_{j=1}^{n}(2c_{ij} - 1)(2b_j - 1)\nu_j(t)$$

for the all-zero codeword $(2c_{ij} - 1)$ is allways -1

$$CM_1 = 2n\varepsilon_c - \sum_{j=1}^{n}(2b_j - 1)\nu_j(t) ....(1)$$

if we already made a hard decision on y_i then CM_i, corresponds to finding the hamming distance

$$\mathrm{CM_{im}} = \sum_{j=1}^{n}(2c_{ij} - 1)(2c_{mj} - 1) = \text{\# of terms in common} - \text{\# of terms which are different}$$

The correlation metric for a codeword of weight w_m with the all zero code word

n- $w_m - w_m = n$-$2w_m$

Let us continue for the soft case

$$CM_i = \sum_{j=1}^{n}(2c_{ij} - 1)y_j(t)$$

$$CM_m = \sum_{j=1}^{n}(2c_{mj} - 1)\left[2\varepsilon_c(2c_{ij} - 1) + (2b_j - 1)\nu_j(t)\right]$$

$$CM_m = \sum_{j=1}^{n}2\varepsilon_c(2c_{mj} - 1)(2c_{ij} - 1) + \sum_{j=1}^{n}(2c_{mj} - 1)(2b_j - 1)\nu_j(t)$$

$$CM_m = 2\varepsilon_c(n - 2w_m) + \sum_{j=1}^{n}(2c_{mj} - 1)(2b_j - 1)\nu_j(t) ....(2)$$

An error will occur if $CM_m > CM_1$

What is CM_1 substitute in (2) with w_m=0 ? or

Recall $CM_1 = 2n\varepsilon_c - \sum_{j=1}^{n}(2b_j - 1)v_j(t)$.........................(1)

$$D = CM_1 - CM_m = 4\varepsilon_c w_m - 2\sum_{j=1}^{n} c_{mj}(2b_j - 1)v_j \text{......................(3)}$$

Notes:

- D= signal + noise
- The second term has w_m non-zero terms.
- We did not assume v_i to be Gaussian, because we want the analysis to be valid for different types of Jammers.
- Assuming w_m is large enough to justify central limit theorem, the noise term is considered Gaussian. In practice this assumption is very justified, because we are requiring d_{min}=10 or any large value. In DSSS, L_e is large, and the code rate is very low which results in high d_{min}.

The mean and variance of v_i

$E(v_i) = 0$, this is because $E\left[(2b_i - 1)\right] = 0$

$$\sigma_m^2 = 4\sum_{j=1}^{n}\sum_{i=1}^{n} C_{mi}C_{mj}E\left[(2b_j - 1)(2b_i - 1)\right]E\left[v_i v_j\right]$$

If the binary sequence from the PN generator are assumed uncorrelated then

$E\left[(2b_j - 1)(2b_i - 1)\right] = \delta_{ij}$, hence

$\sigma_m^2 = 4w_m E\left[v_j^2\right]$, Note that $\sum_{j=1}^{n} c_{mj}^2 = w_m$, now we know what the receiver does to the noise.

But what is $E\left[v_j^2\right]$?

$$E\left[v_j^2\right] = \int_{-\infty}^{\infty} |G(f)|^2 S_{zz}(f)df$$

Summary of steps

The decision will be made based on the following:

$$D = CM_1 - CM_m = 4\varepsilon_c w_m - 2\sum_{j=1}^{n} c_{mj}(2b_j - 1)v_j \text{......................(3)}$$

Error will be made if D<0

$$P_2(m) = Q\left(\frac{mean}{\sqrt{\text{var}\,iance}}\right) \qquad\qquad P_m \le \sum_{m=2}^{M} Q\left(\frac{mean}{\sqrt{\text{var}\,iance}}\right)$$

$$\text{Mean} = 4\varepsilon_c w_m$$

$$\sigma_m^2 = 4w_m E\left[v_j^2\right]$$

$$E\left[v_j^2\right] = \int_{-\infty}^{\infty} |G(f)|^2 S_{zz}(f)df$$

Broad Band Jamming

Assume that the interference is flat over certain bandpass bandwidth W,

$S_{zz}(f) = J_0$ for $|f| \le \frac{W}{2}$, note that the bandwidth of the equivalent lowpass is half of the bandpass signal. (Notes that some books define $S_{zz}(f) = 2J_0 \; for \; |f| \le \frac{W}{2} \; and \; \frac{J_{aav}}{W} = 2J$) there will be a factor of 2 in the analysis…concept remains)

Now substitute in the above equation $E\left[v_j^2\right] = \int_{-\infty}^{\infty} |G(f)|^2 S_{zz}(f)df = J_0 \int_{-\frac{W}{2}}^{\frac{W}{2}} |G(f)|^2 df = J_0 2\varepsilon_c$

$$\sigma_m^2 = 4w_m E\left[v_j^2\right] = 4w_m J_0 2\varepsilon_c = 8\varepsilon_c J_0 w_m$$

Now probability of $D < 0$ (see equation 3)

$$P_2(m) = Q\left(\frac{mean}{\sqrt{\text{var}\,iance}}\right) = Q\left(\frac{4\varepsilon_c w_m}{\sqrt{8\varepsilon_c J_0 w_m}}\right) = Q\left(\sqrt{\frac{2\varepsilon_c}{J_0} w_m}\right)$$

But $\varepsilon_b = \frac{n}{k}\varepsilon_c$, then $\varepsilon_c = R_c \varepsilon_b$

$$P_2(m) = Q\left(\sqrt{2\frac{\varepsilon_b}{J_0} R_c w_m}\right) = Q\left(\sqrt{2\gamma_b R_c w_m}\right)$$

Using the union bound (explain the Union Bound)

$$P_M \le \sum_{m=2}^{M} Q\left(\sqrt{2\frac{\varepsilon_b}{J_0} R_c w_m}\right) = \sum_{m=2}^{M} Q\left(\sqrt{2\frac{P_{av}/R}{J_{av}/W} R_c w_m}\right) = \sum_{m=2}^{M} Q\left(\sqrt{2\frac{W/R}{J_{av}/P_{av}} R_c w_m}\right)$$
$$= \sum_{m=2}^{M} Q\left(\sqrt{2\frac{G_p}{J_{av}/P_{av}} R_c w_m}\right)$$

G_p is the processing gain=W/R

$R_c w_m$ is the coding gain.

Proakis showed that a close performance is achieved for partial band jamming.

The above bound can be further upper bounded by

$$P_M \le (M-1)Q\left(\sqrt{2\frac{G_p}{J_{av}/P_{av}} R_c d_{\min}}\right),$$

For the uncoded system this reduces to

$$P_2 = Q\left(\sqrt{2\frac{G_p}{J_{av}/P_{av}}}\right) = Q\left(\sqrt{2\frac{E_b}{J_0}}\right)$$ as expected.

Note : The binary repetition code is considered as uncoded system as far as the error is concerned, because it is gain is 1

$$R_c \cdot d_{\min} = \frac{1}{n} \cdot n = 1$$

Partial Band Jamming

$$S_{zz}(f) = \begin{cases} \frac{J_{av}}{W_1} = \frac{J_0 W}{W_1} & |f| \le \frac{W_1}{2} \\ 0 & otherwise \end{cases}$$

We reach at

$$E\left[v_j^2\right] = \int_{-\infty}^{\infty} |G(f)|^2 S_{zz}(f) df$$

The value of $E[v_j^2]$ depends on $G(f)$.

We will consider two examples with two different pulse shapes.

Example 1: Rectangular Pulse Shape

Narrowband interference centered at the carrier (DC for the equivalent low-pass signal)

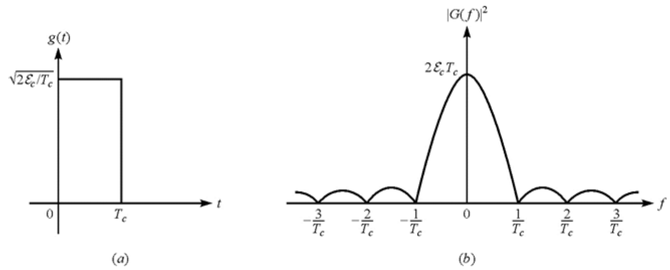
$$J_{av} = J_0 W$$

$$S_z(f) = \begin{cases} \frac{J_{av}}{W_1} = \frac{J_0 W}{W_1} & |f| \le \frac{1}{2} W_1 \\ 0 & |f| > \frac{1}{2} W_1 \end{cases}$$

$W \gg W_1$

$$E[v^2] = \frac{J_{av}}{W_1} \int_{-\frac{W_1}{2}}^{\frac{W_1}{2}} |G(f)|^2 df$$

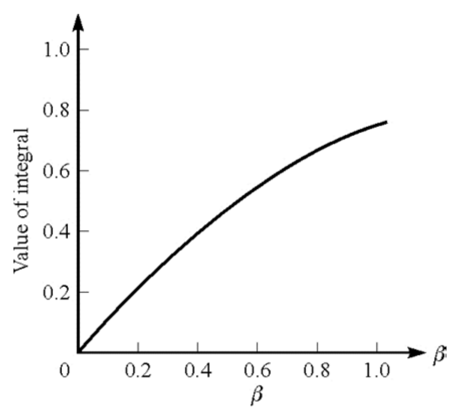
Let $g(t)$ be a rectangular pulse with its power spectral density shown in the figure



$$\sigma_m^2 = 4w_m E[v^2] = \frac{8\varepsilon_c w_m T_c J_{av}}{W_1} \int_{-\frac{W_1}{2}}^{\frac{W_1}{2}} \left(\frac{\sin \pi f T_c}{\pi f T_c}\right)^2 df = \frac{8\varepsilon_c w_m J_{av}}{W_1} \int_{-\frac{\beta}{2}}^{\frac{\beta_1}{2}} \left(\frac{\sin \pi x}{\pi x}\right)^2 dx$$

By change of variable f to $= fT_c$. Note that $\beta = W_1 T_c$

$0 \leq \beta \leq 1$ (why?)



The integral is upper bounded by $W_1 T_c$

$$\sigma_m^2 \leq 8\varepsilon_c w_m T_c J_{av}$$

Special Case (continuous Single Tone Jammer)

In the limit as W_1 approaches 0, we get the single tone scenario (continuous signal)

$$S_{zz}(f) = J_{av}\delta(f)$$

$$D = CM_1 - CM_m$$

$$\sigma_m^2 = 4w_m J_{av}|G(0)|^2 = 8w_m \varepsilon_c T_c J_{av}$$

$$P_M \le \sum_{m=2}^{M} Q\left(\frac{4\varepsilon_c w_m}{\sqrt{8 w_m \varepsilon_c T_c J_{av}}}\right) = \sum_{m=2}^{M} Q\left(\sqrt{\frac{2\varepsilon_c}{J_{av} T_c} w_m}\right)$$

Since $\varepsilon_c = R_c \varepsilon_b,\ T_c \approx \frac{1}{W},\ \ \frac{J_{av}}{W} = J_0$

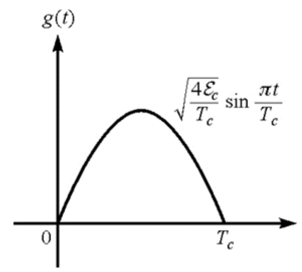
$$P_M \le \sum_{m=2}^{M} Q\left(\sqrt{\frac{2\varepsilon_c}{J_0} R_c w_m}\right)$$

Compare the last expression with the broadband jammer case. **The CW Jammer has the same effect on the performance as an equivalent broadband jammer**.

Example: Half Cycle of Sinusoid

For a DSSS, CW Jammer of average power, J_{av}, The pulse shape, $g(t)$, is one half cycle of a sinusoid .

$$g(t) = \sqrt{\frac{4\varepsilon_c}{T_c}} \sin\frac{\pi t}{T_c}\,, \qquad 0 \le t \le T_c$$



$$\sigma_m^2 = 4 w_m J_{av} |G(0)|^2 = \frac{64}{\pi^2} \varepsilon_c T_c J_{av} w_m$$

Notice because we have a CW jammer all matter is at f=0.

$$P_M \le \sum_{m=2}^{M} Q\left(\sqrt{\frac{\pi^2 \varepsilon_c}{4 J_{av} T_C} R_c w_m}\right)$$

- This pulse shape resulted in 0.9dB better performance than rectangular pulse.
- It this pulse is used with QPSK we get MSK
- MSK is frequency used with DSSS

Pulse Jamming (Partial Time Jamming)

Let α be the duty cycle of the jammer.

For any average power, J_{av}, the jammer transmits pulses at a power of $\frac{P_{av}}{\alpha}$ for α% of the time.

This means whenever it jams the signal, it can jam it by J_0/α

Remember $J_{av} = J_0 W$ for broadband, there $J_{av} = (0)(1-\alpha) + \alpha\left(\frac{J_0}{\alpha}\right) W = J_0 W$

$$P_2(\alpha) = (1-\alpha) Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \alpha Q\left(\sqrt{\frac{2E_b}{N_0 + \frac{J_0}{\alpha}}}\right)$$

Again for $N_0 \ll J_0,\ P_2(\alpha) = \alpha Q\left(\sqrt{\frac{2\alpha E_b}{J_0}}\right)$

By differentiating with respect to α, we get

$$\alpha^* = \begin{cases} \dfrac{0.71}{E_b/J_0} & for\dfrac{E_b}{J_0} > 0.71 \\ 1 & for\dfrac{E_b}{J_0} \le 0.71 \end{cases}$$

Note that $\alpha \le 1$

Note that of the performance is already bad, jammer cannot make it worse by selecting a specific .

The corresponding error probabilities:

$$P_2 = \begin{cases} \dfrac{0.083}{E_b/J_0} & for\dfrac{E_b}{J_0} > 0.71 \\ Q\left(\sqrt{\dfrac{2E_b}{J_0}}\right) & for\dfrac{E_b}{J_0} \le 0.71 \end{cases}$$

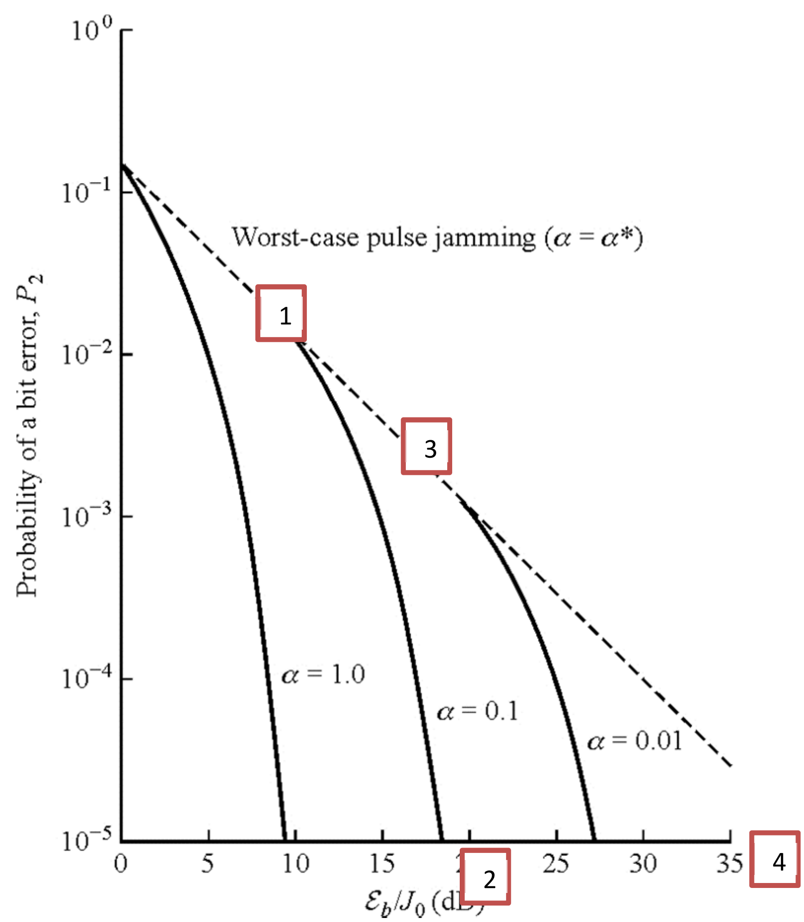


Figure: Performance of DS binary PSK with pulse jamming.

- If the Jammer can measure $\frac{E_b}{J_0}$ "at the receiver end" then the worst case α^* (best case for the jammer) can be selected. (given some examples on the figure)

1. Let's say that the system was working at $\frac{E_b}{J_0} = 10dB$ the worst case $\alpha^* = 0.1$ will yield $P_2 = 10^{-2}$.
2. If the transmitter increases his energy by 10 dB, then $\frac{E_b}{J_0} = 20dB$ and $P_2 = 10^{-6}$,
3. but the jammer by decreasing his duty cycle to 0.01 brings back the error to 10^{-3}. In general the jammer can make the performance linearly proportional to $\frac{E_b}{J_0}$ rather than exponentially proportional!

   The Jammer may also consider increasing his average power.

4. Remember that $\alpha = 1$ corresponds to continues jamming. At 10^{-5} the jammer can give up to 30dB by such smart jamming (40dB at 10^{-6})

It is worth mentioning that as αbecomes very small the pulse duration may be less than the chip duration, and very high power has to be transmitted.

Both conditions may put a limit on how small α can be. Nevertheless, the above analysis serves as a worst case analysis.

Coding Gain

The above results are for uncoded DS system. Coding gain can be applied to improve the performance. However, for coding gain to be utilized in better way, the transmitted bits must be interleaved (to randomize burst errors)

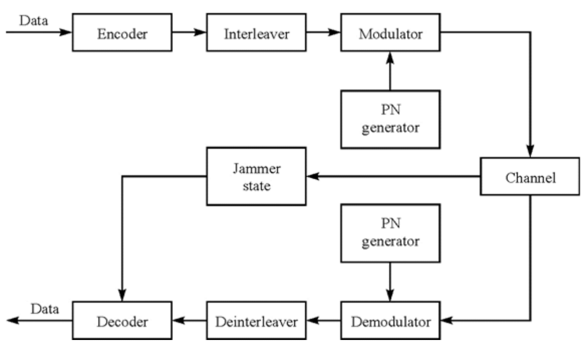


Figure: Block diagram of AJ communication system.

Coding with interleaving (*it is a way to rearrange data in a non-contiguous manner to increase robustness to burst errors, find more about interleaving and de-interleaving*) can recover a significant portion of this loss. We summarize the results of a research paper [Martin &McAdam]. They used a rate ½ convolutional code

- The optimum duty cycle in the coded case is also proportional to 1/SNR but slightly different from the uncoded system.
- The 40dB gain at 10^{-6} by the jammer due to the pulse-jamming is reduced to less than 5dB. That is coding with interleaving recovers more than 35dB.

- Knowing the jammer state help in designing anti-jamming systems.
- For the uncoded system: to achieve 10^{-5} you need 40 dB. For the coded system: to achieve 10^{-5} you need 5dB (k=9, soft-decision)

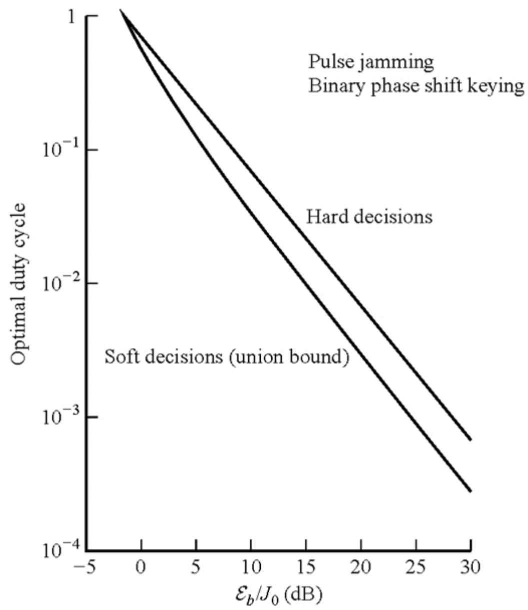


Figure: Optimal duty cycle for pulse jammer. [*From Martin and McAdam (1980). © 1980 IEEE.*

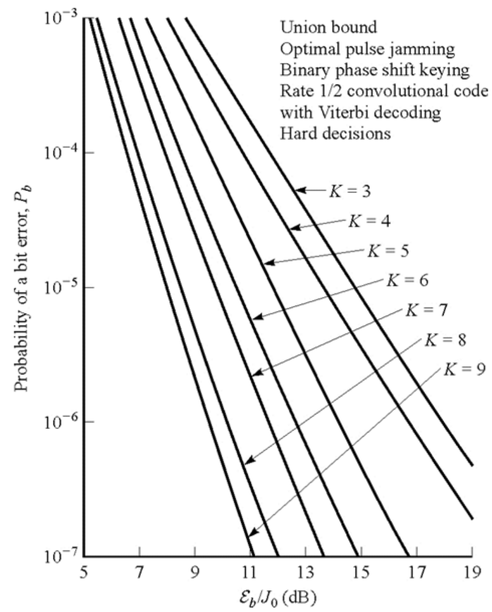


Figure: Performance of rate 1/2 convolutional codes with hard-decision Viterbi decoding binary PSK with optimal pulse jamming. [*From Martin and McAdam (1980). © 1980 IEEE.*]

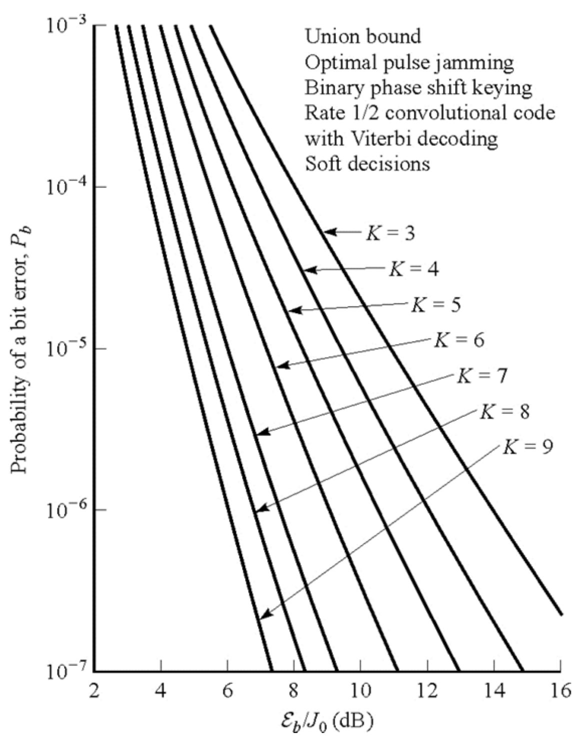


Figure: Performance of rate ½ convolutional codes with soft-decision Viterbi decoding binary PSK with optimal pulse jamming. [*From Martin and McAdam (1980). © IEEE.*]

Generation of PN Sequence

- This subject received lots of attention in research.
- A long and pseudorandom sequence is needed to make it difficult for the jammer to recover the sequence. Sometimes nonlinear operations are used to increase the difficulty of recovering the sequence.
- Ideally PN sequences used in SS must have two properties:

1) Auto correlation function is an impulse (to suppress multipath=to suppress self interference)

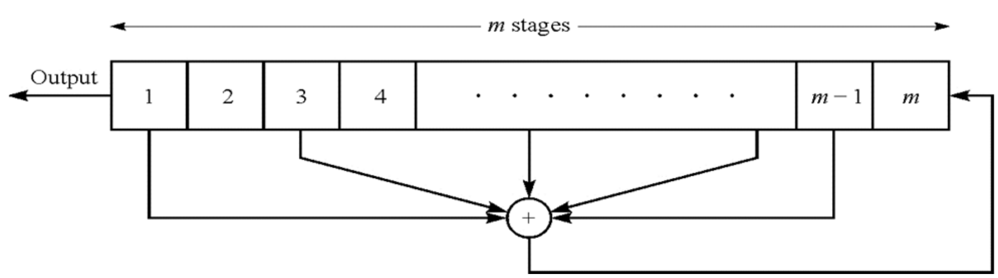
$$R(j)=\sum_{i=1}^{n}(2b_i-1)\big(2b_{i+j}-1\big)\qquad ,0\le j\le n-1$$

$$R(j)=\begin{cases} n & j=0 \\ 0 & 1\le j\le n-1 \end{cases}$$

2) Cross-correlation is also an impulse (to support many users for CDMA applications)

$$R(k,l)=\sum_{i=1}^{n}(2b_{ik}-1)(2b_{il}-1)\qquad ,0\le j\le n-1$$

Maximum-Length Shift-Register Sequence (*m*-sequence), MLS



- By far the most widely known PN sequence is the maximum-length shift register. A max-length shift register sequence has length $n=2^m-1$ generated by m-stage shift register with linear feedback.
- There are tables to make the proper feedback connections (Table 8-1-5)
- As MLS are periodic and shift registers cycle through every possible binary value (with the exception of the zero vector), registers can be initialized to any state, with the exception of the zero vector.
- For more information about maximum-length shift register sequences (see your text)

Table 8-1-5: Shift-Register Connections for Generating Maximum-Length Sequence

m	Stages connected to modulo-2 adder	m	Stages connected to modulo-2 adder	m	Stages connected to modulo-2 adder
2	1, 2	13	1, 10, 11, 13	24	1, 18, 23, 24
3	1, 3	14	1, 5, 9, 14	25	1, 23
4	1, 4	15	1, 15	26	1, 21, 25, 26
5	1, 4	16	1, 5, 14, 16	27	1, 23, 26, 27
6	1, 6	17	1, 15	28	1, 26
7	1, 7	18	1, 12	29	1, 28
8	1, 5, 6, 7	19	1, [illegible]	30	1, 8, 29, 30
9	1, 6	20	1, 18	31	1, 29
10	1, 8	21	1, 20	32	1, 11, 31, 32
11	1, 10	22	1, 22	33	1, 21
12	1, 7, 9, 12	23	1, 19	34	1, 8, 33, 34

Source: Forney (1970).

The following link provide more feedback connections

http://www.newwaveinstruments.com/resources/articles/m_sequence_linear_feedback_shift_register_lfsr.htm

Table 13-2-1 Peak Cross Correlation of m Sequences and Gold Sequences

m	$n = 2^m - 1$	Number of m-sequences	θ_c	$\theta_c/\theta_u(0)$	$t(m)$	$t(m)/\theta_u(0)$
3	7	2	5	0.71	5	0.71
4	15	2	9	0.60	9	0.60
5	31	6	11	0.35	9	0.29
6	63	6	23	0.36	17	0.27
7	127	18	41	0.32	17	0.13
8	255	16	95	0.37	33	0.13
9	511	48	113	0.22	33	0.06
10	1023	60	383	0.37	65	0.06
11	2047	176	287	0.14	65	0.03
12	4095	144	1407	0.34	129	0.03

Description & Properties

- The length of the memory elements is *m* *(m-stage register)*
- The sequence is periodic with period $n=2^m-1$.
- Each period contains 2^{m-1} ones & $2^{m-1}-1$ zeros (# of ones= # of zeros +1).
- It has the **auto correlation** function

$$R(j) = \begin{cases} n & j = 0 \\ -1 & 1 \le j \le n-1 \end{cases}$$

For large values of *n*, $R_{max}/\ R(0)\ = -1/n$ which is small => The auto correlation function is almost ideal.

- **Cross Correlation** (See Table 13.2-1) has relatively some large peaks
  - Solution: select a small subset that has relatively small cross-correlation peak values.
    - Drawback: this solution results in a small set of sequences for CDMA systems.

Gold sequences

It was proved that certain pairs of *m*-sequences of length *n* exhibits a three values cross-correlation function with values {-1, -*t*(*m*),*t*(*m*)-2}, where

$$t(m) = \begin{cases} 2^{(m+1)/2} + 1 & odd\ m \\ 2^{(m+2)/2} + 1 & even\ m \end{cases}$$

These two sequences are called *preferred sequences* (how to find them?!).

From the pair for preferred sequences, say *a* & *b*, we can generate *n* other sequences by shift and add.

$$\underline{a} + \underline{b}^{(i)} \qquad \text{for } i=0,\ n-1 \text{ (shifts)}$$

Including *a* and *b* , we have now *n*+2 sequences. Note that *n* of them are not *m*-sequences.

Gold showed that the resultant *n*+2 sequences have the following correlation function.

(1) Cross-correlation between any two sequences is one of three values {-1, -*t*(*m*),*t*(*m*)-2}
(2) Off-peak autocorrelation takes on value from the set {-1, -*t*(*m*),*t*(*m*)-2}

See Figure below

Example: *m*=10

For *m*=10,

Using maximum length shift registers:

The length of the maximum shift *m*-sequence is $2^m-1=2^{10}-1=1023$.

There are 60 *m*-sequences corresponding to different feedback connections. The maximum cross correlation for these sequences is $R_{max} = 383$ which corresponds to $R_{max}/\ R(0)\ = 383/1023 = 0.37$

Using Gold Codes:

There is a certain pair that results in $t(10)=2^6+1=65$, and the three possible values of the periodic cross correlation are {-1,-**65**,+63} which corresponds to $R_{max}/\ R(0)\ = 65/1023 = 0.06$ which is much better than before.

Example: *m*=5

Generation of Gold sequences of length 31 (*m*=5). From table 13-2-1 there are 6 different feedback connections to generate *m* sequences. The peak cross correlation between these sequences is R_{max} =11. Two sequences have less cross correlation *t*(5)=9 (not a big difference but it is meant for illustration). These are preferred codes and are generated by the following feedback polynomials

$$g_1(p)=p^5+p^2+1,\ \text{and}\ g_2(p)=p^5+p^4+p^2+p+1$$

The required system is shown below. The circuit was also tested by Matlab (*See posted Matlab Code*)

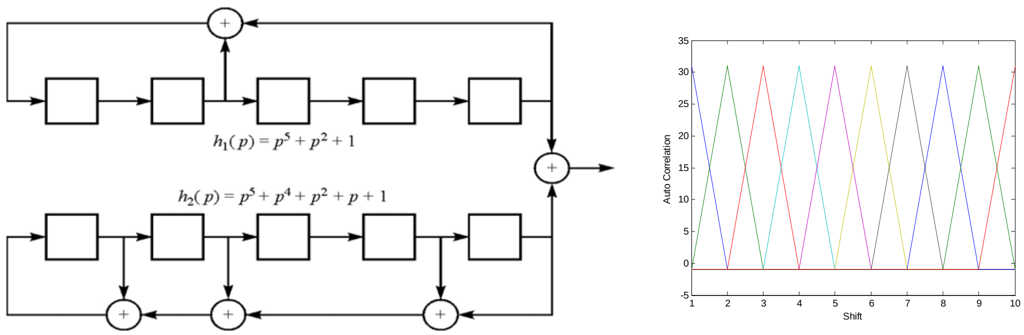


Figure: Generation of Gold sequences of length 31.

Welch Bound:

Welch bound is a lower bound on the minimum possible cross correlation

M: # of sequences

n: length of the sequence

m: # of stages in the shift register

$$R_{max} \geq n\sqrt{\frac{M-1}{Mn-1}} = n\sqrt{\frac{M-1}{n(M-\frac{1}{n})}} \approx \sqrt{n}\ \ for\ large\ values\ of\ M\ \&\ n$$

Example: Check Gold Codes against Welch bound.

For Gold codes $n=2^m-1$, and $M=n+2=2^m-1+2=2^m+1$ now substitute for *M* and *n* in the bound above

$$R_{max} \geq n\sqrt{\frac{M-1}{n(M-\frac{1}{n})}} = n\sqrt{\frac{2^m+1-1}{n(2^m+1-\frac{1}{n})}}$$

For *n*>>1

$$R_{max} \geq \frac{n}{\sqrt{n}}\sqrt{\frac{2^m}{(2^m+1)}} = \frac{\sqrt{n}}{\sqrt{(2^m+1)}}\sqrt{2^m}$$

For large m , 2^m+1 can be approximated to n=2^m-1

$$R_{max} \geq \frac{n}{\sqrt{n}}\sqrt{\frac{2^m}{(2^m+1)}} = \frac{\sqrt{n}}{\sqrt{n}}2^{m/2} \approx 2^{m/2}$$

While the true value for the maximum cross correlation of Gold codes is $t(m)$, which is

$$t(m) = \begin{cases} 2^{(m+1)/2}+1 & odd\ m \\ 2^{(m+2)/2}+1 & even\ m \end{cases}$$

Again for large values of m,

$$t(m) \cong \begin{cases} 2^{(m+1)/2} & odd\ m \\ 2^{(m+2)/2} & even\ m \end{cases} = \begin{matrix} 2^{1/2}\ 2^{m/2} \\ 2 \quad 2^{m/2} \end{matrix} \begin{matrix} odd\ m \\ even\ m \end{matrix} = \begin{matrix} \sqrt{2}\ 2^{m/2} \\ 2 \quad 2^{m/2} \end{matrix} \begin{matrix} odd\ m \\ even\ m \end{matrix}$$

Compared with the bound above, Gold codes are above the lower bound by $\sqrt{2}$ or 2. This motivated the researchers to find other codes which achieve the bound (Kasami Codes)

Kasami Codes

Kasami codes are optimal in term of cross correlation (they achieve Welch bound) but they generate a smaller set of codes)

Procedure:

- Choose an m-sequence $\underline{a}$.
- Form $\underline{b}$ by deciminating $\underline{a}$ by $2^{m/2}$+1 (take every other $2^{m/2}$+1 bit of $\underline{a}$)
- Form the set by adding $\underline{a}+\underline{b}^{(i)}$ $0 \leq i \leq 2^{m/2}-2$. This way we have generated $2^{m/2}$ sequences (We include $\underline{a}$).
- The autocorrelation and the cross correlation values of any pair take values from the set $\{-1, -(\mathbf{2^{\frac{m}{2}}+1}), 2^{\frac{m}{2}}-1\}$

Frequency Hopping (FH)

Main Ideas:

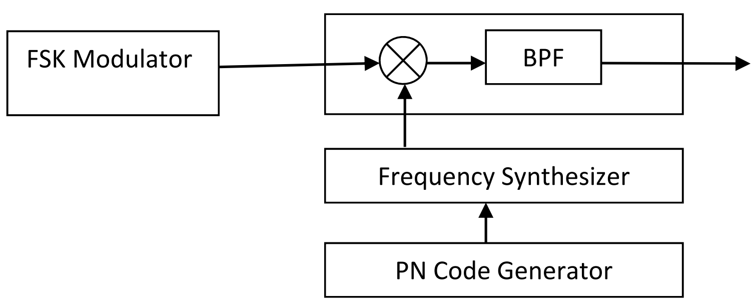
- FH definition & features
- Slow FH (SFH) vs Fast FH (FFH)
- Detection (SFH,FFH)
- Redefining processing gain for FH
- Frequency Hopping vs. Direct Sequence
- Performance of frequency hopping under jamming condition

Definition

Usually used with FSK (or MFSK). The frequency of the transmitted symbol relative to the carrier is determined. The carrier frequency is not fixed but pseudorandomly selected from a set of frequencies.

Features

- The spectrum of the transmitted signal is spread **sequentially** rather than instantaneously.
- At any hop, m-bit segment of the PN sequence is used to determine on the 2^m carrier frequency.
- The number of carrier frequencies (slots) is determined by W_{ss} and minimum frequency spacing between two adjacent carriers (based on the orthogonality condition of MFSK)



SFH vs FFH

Symbol duration T_s	Symbol Rate R_s
Hop duration T_H	Hope rate R_H

- The hop duration T_H is the duration of using the same carrier (or band)
- Chip duration is the shortest duration of the same tone.

Chip duration T_c=min (T_H,T_S)	Chip rate R_c=max (R_H,R_S)

- Minimum spacing between tone for orthogonality $\Delta f=R_c$

1) SFH if $R_H \leq R_S$

In SFH, one or several symbols are transmitted at each frequency hop.

$R_c = R_s$, and $\Delta f = R_s$

2) FFH if $R_H > R_S$

In FFH, the carrier frequency hops several times during the transmission of one symbol

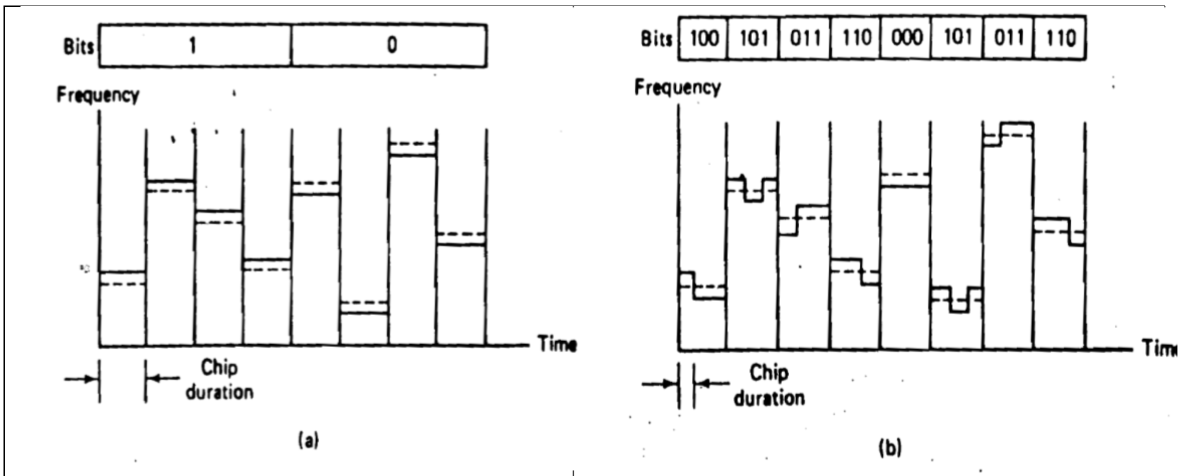
$R_c = R_H$, and $\Delta f = R_H$

Example 1 : Binary FSK

Let R_H=20 hops/sec, R_s=60 bits/sec

Then R_c=Rs which is (SFH) and Δf=60 Hz.

(*From Sklar text book see figure 10.15b below*)



(a) Fast hopping example 4 hops/bit

(b) Slow hopping example 3bits/hops

Figure 10.15: Fast hopping versus slow hoping in binary systems

Example 2: M-ary FSK (MSFK)-SFH

Let M=8 symbols, R_H=25 hops/sec, R_s=50 symbols/sec, R_b=150 bits/sec

Then R_c=R_s which is (SFH) and Δf=50 Hz.

(*From Sklar text book see figure 10.15b below*)

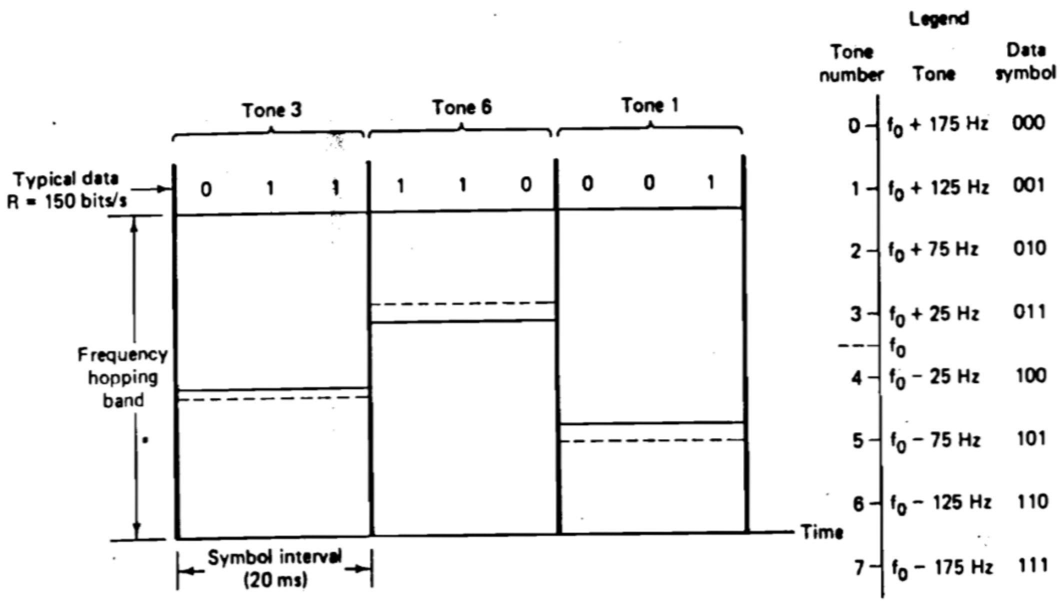


Figure 10.12 Frequency hopping example using 8-FSK Example (SFH)

Example 3: M-ary FSK (MSFK)-FFH

Let M=8 symbols, symbol duration, T_s= 20 m Sec, 4 hops/symbols

Chip duration T_c=20/4= 5 m sec, R_H=50 symbols/sec*4 hops/symbols=200 hops/sec

Then R_c=R_H which is (FFH) and Δf=200 Hz. (*From Sklar text book see figure 10.13*)

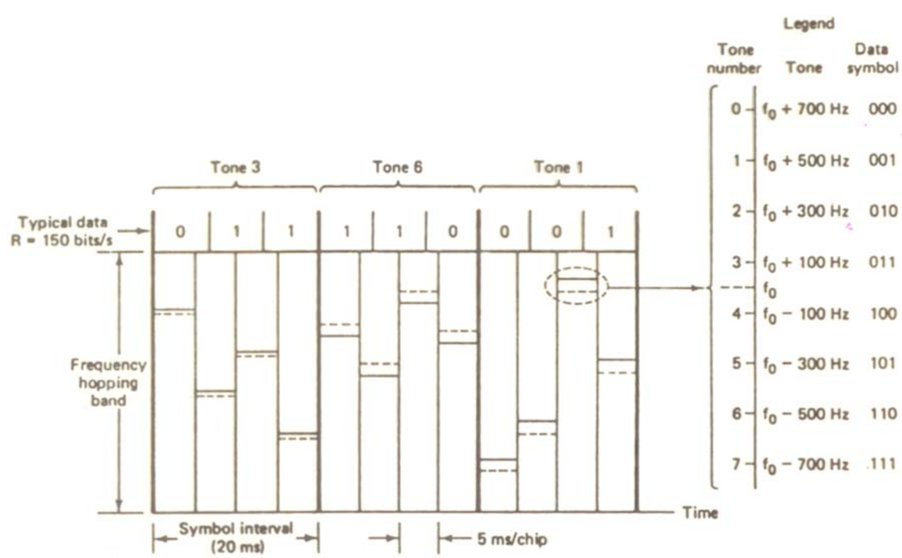


Figure 10.13 Frequency hopping example with diversity N=8

Detection Procedure

SFH:

Remove hopping and then proceed like normal FSK detection:

1. Frequency hopping is removed by down converting the received signal with a local oscillator derived by the PN sequence.
2. The output is processed by a non-coherent *M*-ary FSK detector. (A bank of M non-coherent matched filters each is matched to one of the MFSK tones).
3. An estimate of the original symbol transmitted is obtained by selecting the largest filter output.

FFH:

After de-hopping and processing by the filters, two procedures may be considered:

1) For each symbol, separate decisions are made on the *k* frequency hop chips received, and a simple rule based majority vote is used to make a decision.
2) The energy from the *k* chips are accumulated and the closest decision is selected “Maximum likelihood approach”

This shows the *diversity* effect of fast hopping

Processing Gain

l is the length of the binary sequence from which we find the current tone

$$G_p = \frac{W}{R} = \frac{2^l \ \Delta f}{{}^1/_{T_b}} = \frac{2^l \ {}^1/_{T_c}}{{}^1/_{T_b}} = 2^l \ k$$

G_p=(# of carrier frequencies=2^l)(number of hops per symbol=k)

Special case for SFH

$$G_p = \frac{W}{R} = \frac{2^l \ \Delta f}{{}^1/_{T_b}} = \frac{2^l \ {}^1/_{T_b}}{{}^1/_{T_b}} = 2^l$$

How slow the system can be? (For you to think about)

Frequency Hopping (FH) vs. Direct Sequence (DS)

1. **Processing Gain**: since technology limits the speed of the clock
    a. FH can achieve several GHz of BW larger compared with DS for given clock rate
    b. Or for the same gain, the clock rate can run at a much slower rate than DS

    When $T_c = T_b/k$ (fast hopping) => $R_H = kR_b$, the PN code generation is lkR_b (& independent of the eventual signal BW)

    Example:

    We want G_p=1000 with R_b=100K bit/sec

    <u>For DS:</u> $G_P = T_b/T_c$ => $T_c = T_b/G_p$

    $R_c = G_p R_b$=(1000)(100K)=10^8 chips /sec

    <u>For FFH:</u> $G_p = 2^l\ k$=1000

    We would like to have the proper combination

    l=5, k=30 , G_p=960, chip rate=kR_b=30x10^{5}, , clock rate =$l\ kR_b$ =150x10^5=0.15x10^8

    l=8, k=6, G_p =2^8(6)=256x6 >1000

    Clock rate = $l\ kR_b$=(8)6(1000k)=48000k

2. **Synchronization**
    a. Both DS and FH require synchronization within a fraction of a chip.
    b. DS is more demanding on synchronization.
    c. DS is a very small fraction of a bit (previous example $T_c=10^{-8}$)
    d. In FH, the chip interval is the time spent in transmitting a signal with the same frequency

    Example:

    l=8, R_b=100K bit/sec ,k=6, $T_c = T_b/6 = 10^{-5}/6$ (in general T_b/k)

3. ***Near-Far Problem***
    a. Severe in DS.
    b. It is less in FH because it is an avoidance system rather than averaging system: in FH interference occurs if two signals occupy the same slot simultaneously (power does not matter much)
4. **Modulation**

    DS uses PSK better than FSK
5. **Error Coding**
    a. Required by both DS and FH.
    b. More needed by FH because two signals occupy the same frequency slot could destroy the data.
6. **Multipath Suppression**

    FH can avoid multipath by fast change of the slot before arrival of multipath component (delay T_m) . The system will be excellent when $T_C \ll T_m$.

    DS is better in this regard as T_c is smaller.

7. **Performance under Jamming Conditions** : important criteria

Performance of SFH (R_C=R_S) in Jamming Environments

Broadband Jamming: for FSK and assuming N_0<<J_0

$$P_2 = \frac{1}{2}e^{-\gamma_b/2}, \qquad \gamma_b = {E_b}/{J_0}$$

Partial band Jamming: notice the duality with partial time jammer

$$P = \frac{1-\alpha}{2}e^{-E_b/2N_0} + \frac{\alpha}{2}e^{-E_b/2(N_0+J_0)} \approx \frac{\alpha}{2}e^{-\alpha E_b/2J_0}$$

For the optimal value of α (the ratio of the band that is jammed)

$$\frac{dP}{d\alpha} = \frac{1}{2}e^{-\alpha E_b/2J_0} - \frac{\alpha}{2}\frac{E_b}{2J_0}e^{-\alpha E_b/2J_0} = 0$$

$$\alpha^* = \begin{cases} \dfrac{2}{E_b/J_0} & E_b/J_0 > 2 \\ 1 & E_b/J_0 \le 2 \end{cases}$$

Condition $0 \le \alpha \le 1$,

The associated probability of errors

$$P_2 = \begin{cases} \dfrac{e^{-1}}{E_b/J_0} & E_b/J_0 > 2 \\ \dfrac{1}{2}e^{-E_b/2J_0} & E_b/J_0 \le 2 \end{cases}$$

We have the same effect as seen in pulse jamming (duality)

Spread Spectrum CDMA vs. TDMA/FDMA Viterbi Debate

1. Viterbi, "Spread Spectrum Communications Myths and Realities," *IEEE Communications Magazine*, 1979: *Response to some misconceptions*
2. Viterbi, "When not to Spread Spectrum- a Sequel," *IEEE Communications Magazine*, 1985: *Against CDMA !?*
3. Viterbi, "Wireless Digital Communications: A view based on Three Lessons Learned," *IEEE Communications Magazine*, May 1991 *with CDMA !?*
4. **The last two articles are summarized by Jeffery Wu**

Code Division Multiple Access (CDMA)

The received signal is $g_1(t)s_1(t) + g_2(t)s_2(t) + \cdots \ldots \ldots \ldots . +g_N(t)s_N(t)$

After de-spreading $g_1^2(t)s_1(t) + g_1(t)g_2(t)s_2(t) + \cdots \ldots \ldots \ldots . +g_1(t)g_N(t)s_N(t)$

$$g_1^2(t) = 1\,, despreaded$$

$g_1(t)g_2(t)s_2(t) + \cdots \ldots \ldots \ldots . +g_1(t)g_N(t)s_N(t)$, multiuser interference (spreaded more)

Capacity: Number of Simultaneous Users

For N users all transmitting with the same average power P_{av}.

$$\frac{P_{av}}{J_{av}} = \frac{P_{av}}{(N_u - 1)P_{av}} = \frac{1}{N-1}$$

$$P = Q\left(\sqrt{\frac{2W/R}{J_{av}/P_{av}}}\right) = Q\left(\sqrt{\frac{2W/R}{N_u - 1}}\right)$$

The number of users for a guaranteed QOS can be determined (Concurrent). T he number of subscribers is much more).

With coding

$$P_M = (M-1)Q\left(\sqrt{\frac{2W/R}{N_u - 1} R_c d_{\min}}\right)$$

The above results assume perfect orthognality between PN sequences.

What is Multiuser Detection (MUD)? *Find out*

Multipath Suppression

Received signal = $g(t)s(t) + \sum g(t-\tau)s(t-\tau_i)$

After despreading $s(t) + 0$, since $g(t)g(t-\tau_i) = 0$

Practical Example: IS-95

- IS95 is a digital cellular CDMA system based on DSSS.
- Digital cellular voice in North America (proposed and developed by Qualcomm)
- Standardized as IS-95 by TIA (Telecomm Industry Association)
- 800-1900MHz
- Froward Link: (Base to Mobile) 1.25MHz
- Reverse link 1.25MHz
- Chip rate of 1.2288×10^6

Forward link

- Code-excited linear predictive (CELP)
- Low speech activity low power lower interference
- Each base station has 64 code:

a) All zero channel measurements (pilot)
b) 1 code for synchronization
c) One or more for paging
d) 61 users

- Synchronous (No interference)
- At the receiver (RAKE) & soft Viterbi. (Find out what RAKE receivers are)

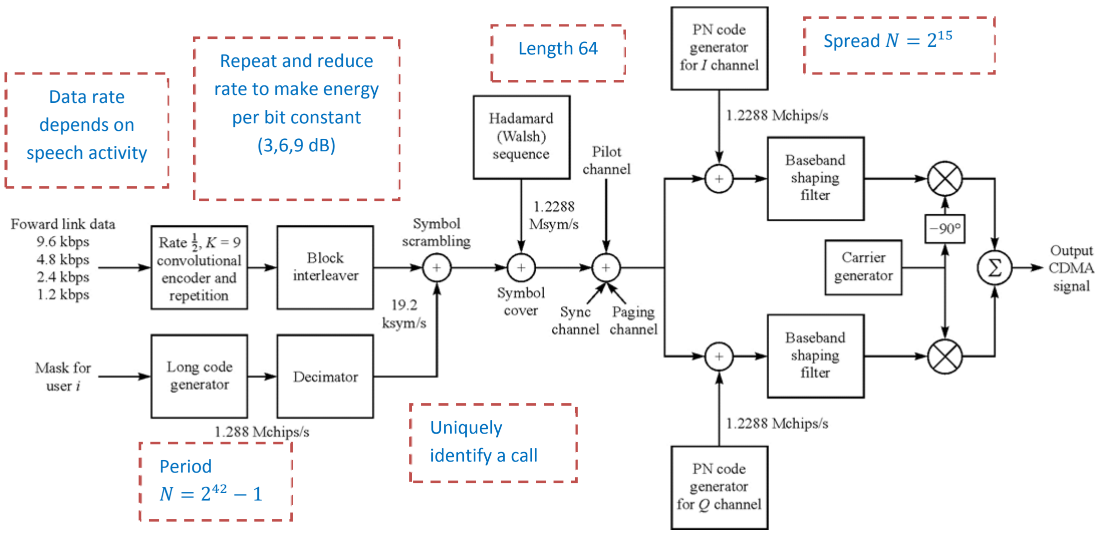


Figure: Block diagram of IS-95 forward link

Reverse link

- Asynchronous (significant more interference).
- Power limited (battery)
- Rate 1/3 (much higher gain to avoid fading) is used to compensate for the constrains
- Frame size (20ms) 576
- Read more about IS-95 as a practical example for CDMA (textbook)

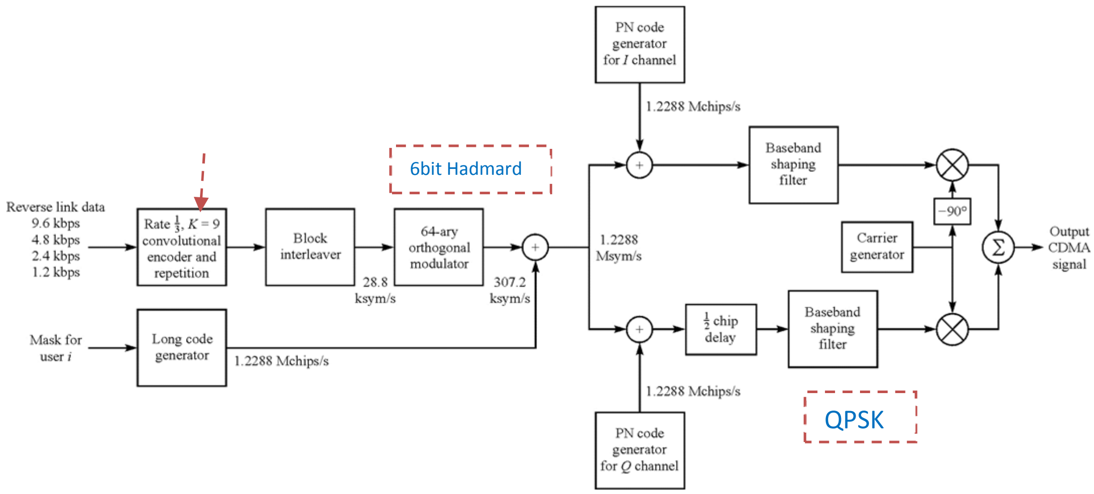


Figure: Block diagram of IS-95 reverse link