EE573: Digital Communications II:

Fading & Diversity

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V3.3

Contents

Digital Communications through Fading Multipath Channels

Introduction

What is Fading Multipath Channels?

Channels having randomly time-variant impulse responses due to signal arriving at the receiver via different propagation paths at different delays.

- Signals transmitted at two widely separated delays will be received **differently** and this difference is **random**.
- Multipath components result in different **carrier-phase offset** which may add up **constructively** or **destructively**. This could result in small signal or zero signal (deep fading)

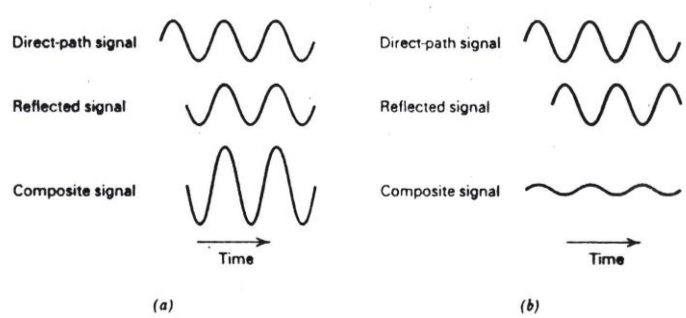


Figure 12.7 (*a*) Constructive and (*b*) destructive forms of the multipath phenomenon for sinusoidal signals.

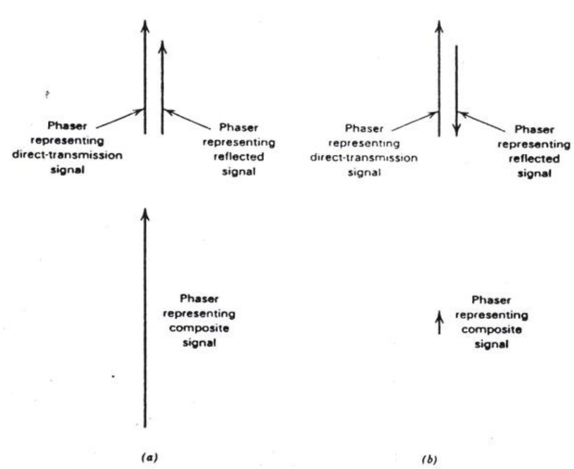


Figure 12.8 Phasor representations of (*a*) constructive and (*b*) destructive forms of multipath.

Examples of Fading Multipath channels:

1. Ionospheric Propagation with HF Band (3-30 MHz)

Sky-wave results from signals being bent or refracted by the *ionosphere* (several layers of charged particles which are always in random motion). The channel response is time-variant.

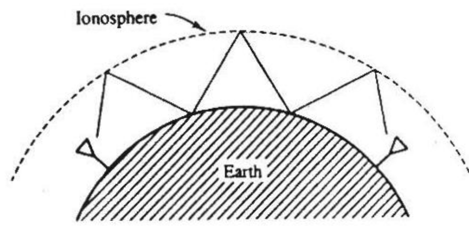


FIGURE 1.6. Illustration of sky-wave propagation.

2. LOS Microwave Transmission.

- Antennas are installed at high positions to avoid obstruction.
- Reflection from ground with changing weather condition→ change delay. (Fig).

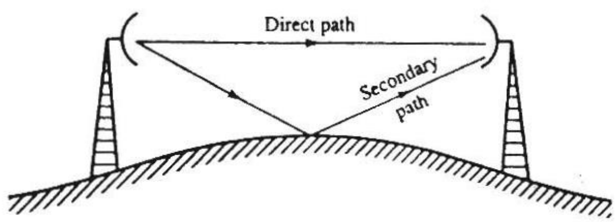


FIGURE 9.54. Illustration of multipath propagation in LOS microwave transmission.

3. Mobile Radio Channels.

- Attract most researches due to:
  - Its wide use.
  - Severity in fading.
- LOS might not exist >>> NLOS (scattering and different).
- We will refer to this channel.

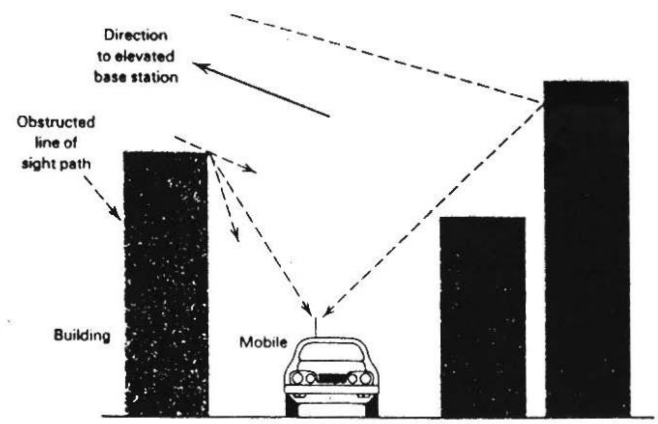
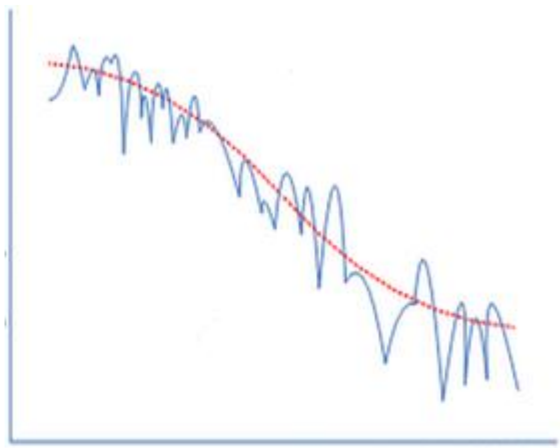
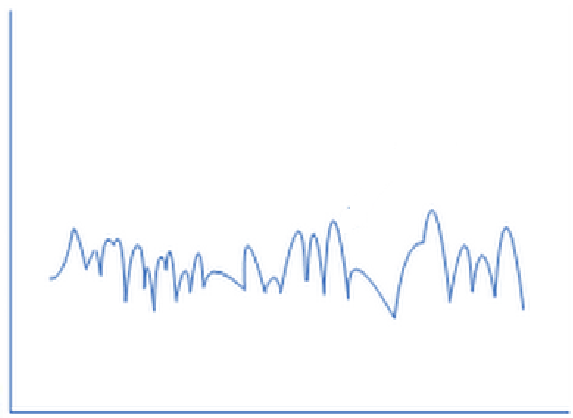


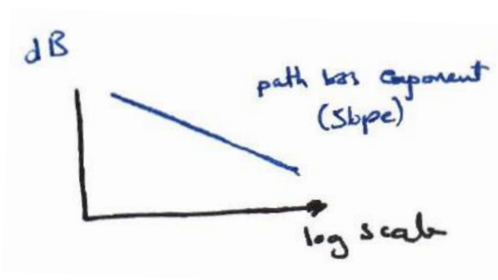
Figure 12.6 Illustrating the mechanism of radio propagation in urban areas. (From Parsons, 1992.)

Large Scale and Small Scale Fading

- The transmitted (RF) wave attenuates with distance. ($\mathcal{E}_b$ refers to the received).
- Simplest situation model is Free Space where the path loss is given by:
  - $L_s(d) = (\frac{4\pi d}{\lambda})^2$
- Due to multipath phenomenon (reflection, diffraction, & scattering) & other scenarios (NLOS) , free space model is not enough.

Example: Typical mobile channel (received signal power vs. distance) from base station.

Instead of 

Large-Scale Fading

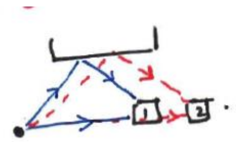
- Can be modeled as : (Fitted to)

$$L_p(d)(dB) = L_s(d_0)(dB) + 10\, n \log\left(\frac{d}{d_0}\right) + X_\sigma(dB)$$

- Large-scale fading can fit to this model by selecting the appropriated $d_0, n, \sigma\ of\ X_\sigma$.
- $d_0 = reference\ distance.$ (1 km for large cells, 100m for microcells, 1 m for indoor cells.)
- n : depends on propagation environment:
  - For free space $n = 2$.
  - Waveguide (Urban streets, corridors) $n < 2$.
  - Obstructions $n > 2$ (up to 6).

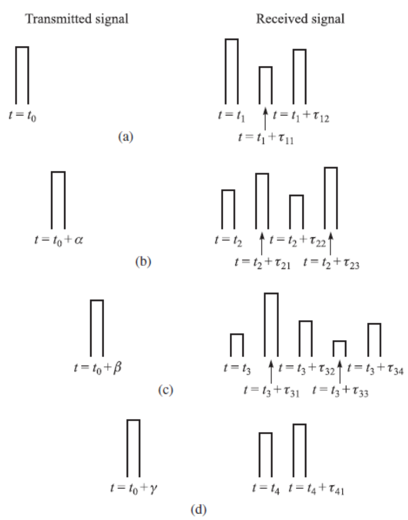
- The first two parameters give an average path loss.
- The environment of different sites may be quite different for similar Tx-Rx separation.
- Measurements have shown that for any value of d, the path loss $L_p(d)$ is a r.v having a log-normal distribution about the mean $X_\sigma(dB)$ denotes a zero-mean r.v (in dB) with s.d. σ (in dB).

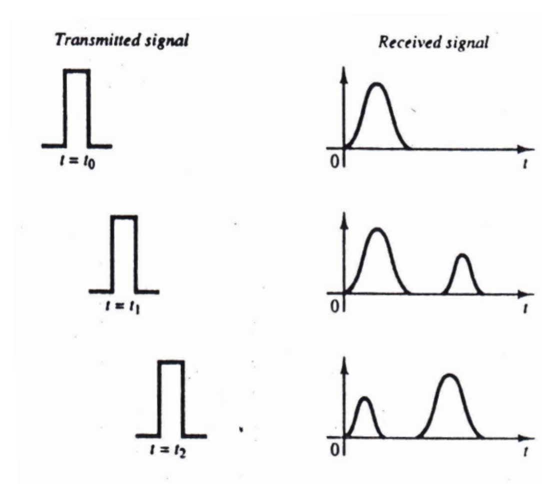
Understanding fading due to Multipath propagation



- Static example.
- Dynamic environment.
  - Assume two dominant paths; there is continuous change in the relative phase shift. (see figures below).
- Effects of fading:

1. **Time-spread (dispersion):** a transmitted pulse will spread in time.
2. **Time-variation :** if we repeat the exp. We could observe changes on the received signal. These changes include:
   a. Attenuations.
   b. Delays.
   c. Number of received paths.





These variations are random. Hence, we are in need to characterize the time-variant multipath channel statistically. We will us the following road map:

- Statistical Characterization (Modeling)
- Performance Evaluation (Penalty in SNR)
- Mitigation through efficient modulation/coding.

Bandpass Signal

- With no loss of generality and for mathematical convenience, it is desirable to reduce all signals & channels to lowpass equivalent to make the analysis general & independent of carrier freq. & channel band.

A band pass signal $s(t) = a(t)\cos[2\pi f_c t + \theta(t)]$

may be written as $s(t) = Re[S_l(t)e^{j2\pi f_c t}]$ where $s_l(t) = a(t)e^{j\theta(t)}$

$s_l(t)$: lowpass equivalent. $s(t)$: bandpass signal . (some book use $u(t)$ rather than s)

It carries all necessary information for any Tx, Rx processing mainly amplitude and phase.

Let $s(t)$ be transmitted,

$$s(t) = Re\left[S_l(t)e^{j2\pi f_c t}\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

$x(t)$ is the received multipath band pass signal (N is the # of paths)

$$x(t) = \sum_{n}^{N} \alpha_n(t)s[t-\tau_n(t)] \ldots\ldots\ldots\ldots\ldots\ldots.(2)$$

α_n & τ_n are attenuation and delay of the n^{th} path which are function of time.

- The summation should be carried over maximum number of paths, clearly there at some instants of time some α_n may be zeros. (Substituting (1) in (2))

$$x(t) = Re\left(\left\{\sum \alpha_n(t)\mathrm{e}^{-\mathrm{j}2\pi f_c\tau_n(t)}s_l[t-\tau_n(t)]\right\}e^{j2\pi f_c t}\right)$$

The equivalent lowpass received signal is

$$r_l(t) = \sum \alpha_n(t)e^{-j2\pi f_c\tau_n(t)}s_l[t-\tau_n(t)]$$

Since $r_l(t) = c(\tau;t) * s_l$, then the impulse response of the equivalent low pass channel:

$$c(\tau;t) = \sum \alpha_n(t)e^{-j2\pi f_c\alpha_n(t)}\delta[t-\tau_n(t)]$$

(Discrete channel model, if continuous remove $\delta[\ ]$ and summation, integration becomes summation)

- For some channels (e.g troposphere scatter channel), the channel is a continuum of multipath.
- From now on, let's drop the word 'lowpass' and take $s_l(t)$, $r_l(t)$ and $c(\tau;t)$ to be the transmitted signal, received and channel.

Transmission of unmodulated carrier at frequency fc

$s(t) = \cos 2\pi f_c t$, then $s_l(t) = 1$ and

$$r_l(t) = \sum_n \alpha_n(t) e^{-j2\pi f_c \tau_n(t)}$$

$$= \sum_n \alpha_n(t) e^{-j\theta_n(t)}$$

$= \sum X + jY$ when there are large number of paths using the central limit theorem X and Y are Gaussian, and α is Rayleigh distributed.

- The amplitude variation are result of time varying phases.
- Present the Matlab examples from Haykin ch.1 : $a \cos 2\pi f_c t + \sum b_n$

Channel Impulse Response

The channel impulse response is usually modeled as a ***zero mean complex-value Gaussian process.***

i.e $c(\tau; t) = c_r(\tau; t) + jc_i(\tau; t)$ where c_r *and* c_i represent real values Gaussian random processes.

Then the envelope of $(\tau; t)$, $|c(\tau; t)| = \sqrt{c_r{}^2(\tau; t) + c_i{}^2(\tau; t)}$ is ***Rayleigh distributed***.

The phase $\theta_n(t) = tan^{-1} \frac{c_i(\tau;t)}{c_r(\tau;t)}$ is ***uniformly distributed*** [0,2π).

- If there are fixed-delay paths in the medium (fixed scatters) in addition to randomly-delay paths, $c(\tau; t)$ can no longer be modeled as having zero-mean. (Example LOS channels). In this case the envelope has a ***Rice distribution*** and the channel is said to be ***Rician fading channel***.
- ***Rayleigh model*** has been widely accepted for many fading channels.
- A third widely used model for the fading envelope is ***Nakagami-m distribution***.

Check : http://www.wirelesscommunication.nl/reference/chaptr03/ricenaka/nakagami.htm

Channel Correction Function & Power Spectra.

We shall now develop a number of useful correlation & power spectral density functions that characterize multipath time-variant fading channels.

The channel impulse response is given by $c(\tau;t)$.

Auto correlation

Q. What is the significance of the auto correlation of a process (t) ?

Expectation of the product of two random variables $X(t_1)$ and $X(t_2)$, i.e obtained by observing the process $X(t)$ at times t_1 & t_2.

$$R_X(t_1,t_2) = E[X^*(t_1)X(t_2)] \quad , \quad \text{conjugate if } X(t) \text{ is complex.}$$

- For W.S.S, the autocorrelation depends on the time difference and not the specific time.

$$R_X(t_1,t_2) = R_X(t_2-t_1) = R_X(\Delta t) = E[X(t)X(t+\Delta t)]$$

Ans.:-

- $R_X(\Delta t)$ provides a mean for describing the "interdependence" of two random variables obtained by observing a random process $X(t)$ at times Δt seconds a part.
- The maximum of the autocorrelation is at $\Delta t = 0$, $R_X(0)$
- It is apparent that the more rapidly the random process $X(t)$ changes with time, the more rapidly will $R_X(\Delta t)$ decreases from its max $R_X(0)$ as Δt increase.
- We may be able to define $(\Delta t)_0$ such that for $\Delta t > (\Delta t)_0$, $R_X(\Delta t)$ is very negligible.

Effect of Multipath:

Time domain Analysis

Let $c(\tau;t)$ be the equivalent lowpass impulse response, (also W.S.S reasonable from experiment)

$$R_c(\tau_1,\tau_2;\Delta t) = \frac{1}{2}\, E\,[c^*(\tau_1;t)c(\tau_2;t+\Delta t)]$$

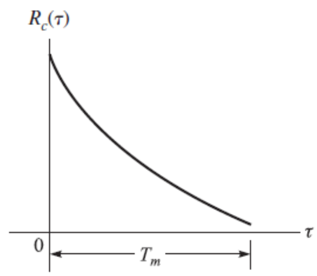
This function gives the interdependence of path propagation delay τ_1 observed at t_1, τ_2 observed at t_2.

- In most radio communication: the attenuation & phase shift associated with delay τ_1 are uncorrelated with those associated with path delay τ_2. (*Uncorrelated scattering*)

$$E[c^*(\tau_1;t)c(\tau_2;t+\Delta t)] = R_c(\tau_1;\Delta t)\,\delta(\tau_1-\tau_2)$$

- Some books will have a factor of half in the left side of the above equation.
- Let $\Delta t = 0$, results in $R_c(\tau;0) \equiv R_c(\tau)$
- $R_c(\tau)$ *multipath intensity profile* or *power delay profile*.
- The average power output of the channel as a function of the time delay τ.
- $R_c(\tau_1;\Delta t)$ gives the average power output as function of the delay and the difference Δt in observation time.
- Measurement: the function $R_c(\tau;\Delta t)$ is measured by transmitting very narrow pulses (or equivalently a wide band signal) & cross correlates the received signal with a delayed version of it.

- The channel response is correlated for $0 < \tau < T_m$, T_m is called multipath spread of the channel.



Summary of variables in time domain

$c(\tau;t)$ is the channel impulse response.

t: time of measurement.

τ : delay.

To analyze the effect of the multipath in time domain. (Time-variant)

$$R_c(\tau_1,\tau_2;\Delta t) = E\,[c^*(\tau_1;t)c(\tau_2;t+\Delta t)]$$

Assuming uncorrelated scattering (phase, attenuation, τ_1 & τ_2)

$$= R_c(\tau_1;\Delta t)\,\delta(\tau_1-\tau_2)$$

Let $\Delta t = 0$, $R_c(\tau;0) \equiv R_c(\tau)$ multipath. Intensity profile or delay power spectrum.

Frequency Domain Analysis:

- Time variant transfer function. $C(f;t) = \int_{-\infty}^{\infty} c(\tau;t)\ e^{-j2\pi f\tau}\ d\tau$
- Since $c(\tau;t)$ is modeled as complex-valued zero-mean Gaussian random process in t variable → $C(f;t)$ has the same statistics.
- Assuming W.S.S the autocorrelation function

$$R_C(f_1, f_2; \Delta t) = E\left[C^*(f_1;t)C(f_2;t+\Delta t)\right]$$

- We can show that $R_C(f_1, f_2; \Delta t) \leftrightarrow R_c(\tau; \Delta t)$, because $C(f;t) \leftrightarrow c(\tau;t)$
- **Proof:**

$$R_C(f_1, f_2; \Delta t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[c^*(\tau_1;t)c(\tau_2;t+\Delta t)]\, e^{j2\pi(f_1\tau_1 - f_2\tau_2)}\, d\tau_1 d\tau_2$$

Using uncorrelated scattering assumption

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_c(\tau_1;\Delta t)\,\delta(\tau_1-\tau_2)\, e^{j2\pi(f_1\tau_1 - f_2\tau_2)}\, d\tau_1 d\tau_2$$

$$= \int_{-\infty}^{\infty} R_c(\tau_1;\Delta t)\, e^{j2\pi(f_1-f_2)\tau_1}\, d\tau_1$$

$$= \int_{-\infty}^{\infty} R_c(\tau_1;\Delta t)\, e^{j2\pi(\Delta f)\tau_1}\, d\tau_1 = R_C(\Delta f;\Delta t)$$

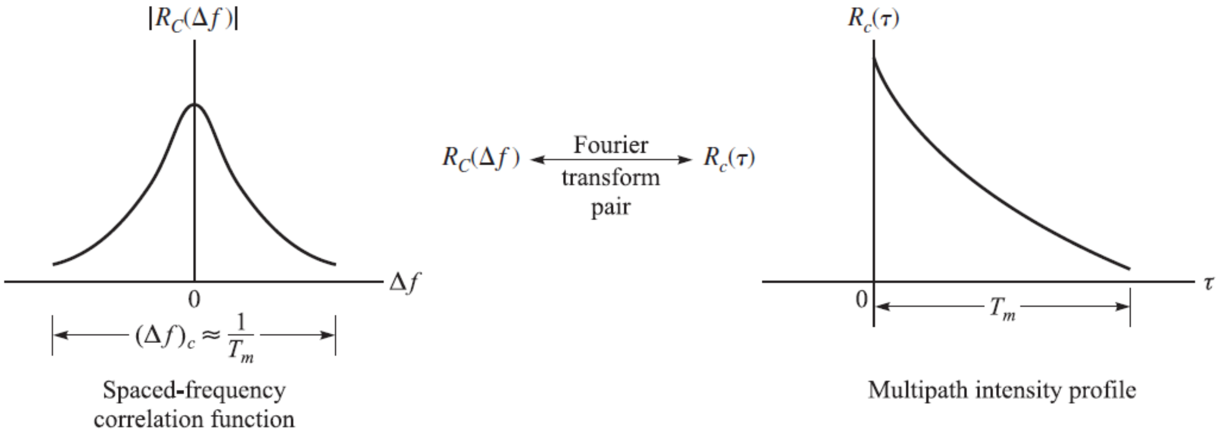
Note: the assumption of uncorrelated scattering implies that the auto correlation of $C(f;t)$ in frequency is function of only the frequency difference $\Delta f = f_2 - f_1$. (w.s.s. in frequency)

- $R_C(\Delta f;\Delta t)$ *spaced-frequency, spaced time correlation* function of the channel.
- It can be measured by transmitting a pair of sinusoids separated by Δf and cross-correlate the two received signals with relative delay Δt.
- If $\Delta t = 0$, $R_C(\Delta f;0) \equiv R_C(\Delta f)$ & $R_c(\tau;0) \equiv R_c(\tau)$, $R_C(\Delta f) \leftrightarrow R_c(\tau)$

$$R_C(\Delta f) = \int_{-\infty}^{\infty} R_c(\tau)\, e^{-j2\pi(\Delta f)\tau}\, d\tau$$

- Correlation with frequency variable provides a measure of frequency coherence of the channel.
- *Coherence Bandwidth* of the channel $(\Delta f)_c \approx \frac{1}{T_m}$, range over which $R_C(\Delta f)$ is not 0.

- Two sinusoids with freq. separation > $(\Delta f)_c$ are affected differently by the channel.



Frequency Selective channel

When an information signal is transmitted through the channel with $(\Delta f)_c <$ BW of the signal, the signal will be severely distorted.

Frequency non-selective

$$(\Delta f)_c > BW \text{ of the signal}$$

Example Consider a time-invariant channel of two paths at some instant of time with $T_m = 0.1\ ms$. Consider a transmitted signal of two frequency 1 kHz and 10 kHz.

$$s(t) = \cos[2\pi \times 10^3 t] + \cos[2\pi \times 10^4 t]$$

Worst case delay , $r(t) = \alpha_1 s(t - \tau_0) + \alpha_2 s(t - \tau_0 - T_m)$

α could be function of frequency

$$r(t) = \alpha_1 \cos[2\pi \times 10^3 (t - \tau_0)] + \grave{\alpha}_1 \cos[2\pi \times 10^4 (t - \tau_0)]$$

$$+\alpha_2 \cos[2\pi \times 10^3 (t - \tau_0 - 10^4)] + \grave{\alpha}_2 \cos[2\pi \times 10^4 (t - \tau_0 - 10^4)]$$

Consider the component for $1kHz$, $\alpha_1 \cos[2\pi \times 10^3 (t - \tau_0)] + \alpha_2 \cos[2\pi \times 10^3 (t - \tau_0 - 10^4)]$, the phase difference between the two arriving paths , $2\pi \times 10^3 \times 10^{-4} = \frac{\pi}{5}$

For $f_2 = 10\ KHz$, the phase difference $= 2\pi \times 10^4 \times 10^{-4} = 2\pi$

The two frequencies are treated differently. The channel is frequency selective, and the received signal is surely distorted.

Note that in this case, $(\Delta f)_c = 10\ kHz$ which is comparable to $f_2 - f_1$

However, if $(\Delta f)_c$ is 1 MHz → $T_m = 10^{-6} sec$

For f_1 → phase difference $= 2\pi * 10^3 10^{-6} = 0.36$

For f_2 → phase difference $= 2\pi * 10^4 10^{-6} = 3.6$

The two frequencies are treated essentially the same.

- Typical value of T_m encountered in mobile radio environment $T_m = 20\ \mu s$
- $(\Delta f)_c = \frac{1}{20*10^{-6}} = 50\ kHz.$
- $BW_c < 50\ KHz$ to get non-selective.
- $BW_c < 50\ KHz$ selective counter measures needed.

Effect of time variations:

If we send a single tone, the received signal will be

$$r(t) = \sum_n \alpha_n(t) e^{j\theta_n(t)}$$

If we assume no multipath now, at two different times, the received signal will be

$r(t_1) = \alpha(t_1)e^{j\theta(t_1)}$ and $r(t_2) = \alpha(t_2)e^{j\theta(t_2)}$, because of $\Delta t = t_2 - t_1$, the phase changed by $\Delta\theta = \theta(t_2) - \theta(t_1)$.

- Frequency change by $\frac{1}{2\pi}\frac{\Delta\theta}{\Delta t}$. The change in frequency is called *Doppler Shift*.
- If the channel was time-invariant (unit not moving), $\frac{d\theta}{dt} = 0$.
- Due to time variations, the same signal frequency will be subject to Doppler shift at different time.
- Now if we consider the *multipath* time-variant,

$$r(t) = \sum_n \alpha_n(t) e^{-j\theta_n(t)}$$

- The output of the channel for a single frequency will consist of a signal that contains many different frequency components.
- -A measure of time frequency broadening is called the *Doppler spread of the channel.* (B_d)
- Doppler shift can be used for speed measurements. $B_d = \frac{V}{\lambda}$.
- The function that characterizes B_d "Doppler spread phenomenon" is called "*Doppler Power spectrum*".
- In order to relate the Doppler effects to the time variations of the channel, we define the FT of $R_C(\Delta f; \Delta t)$,

$$S_C(\Delta f;\lambda) = \int_{-\infty}^{\infty} R_C(\Delta f;\Delta t)\ e^{-j2\pi\lambda\Delta t}\ d\Delta t$$

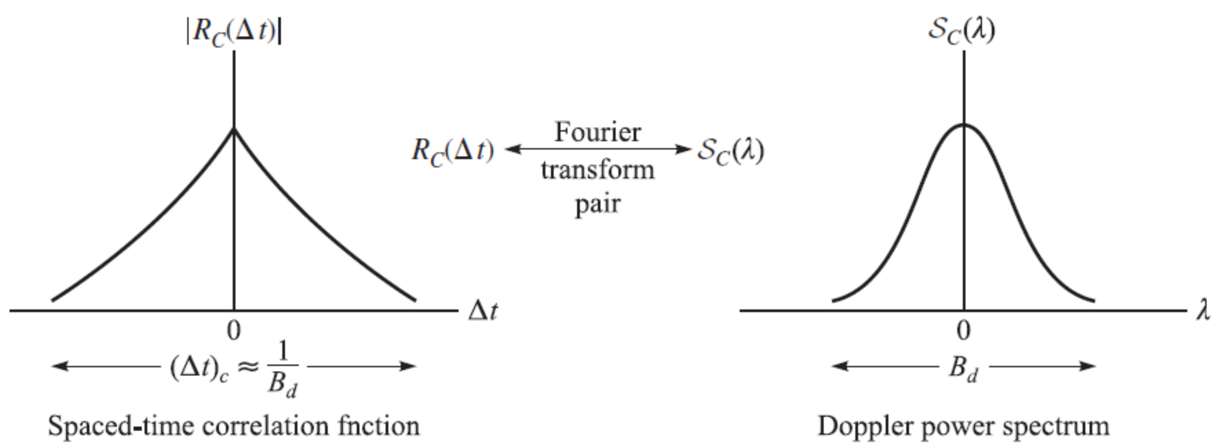
Let us describe the *frequency spread* λ due to single tone → set $\Delta f = 0$.

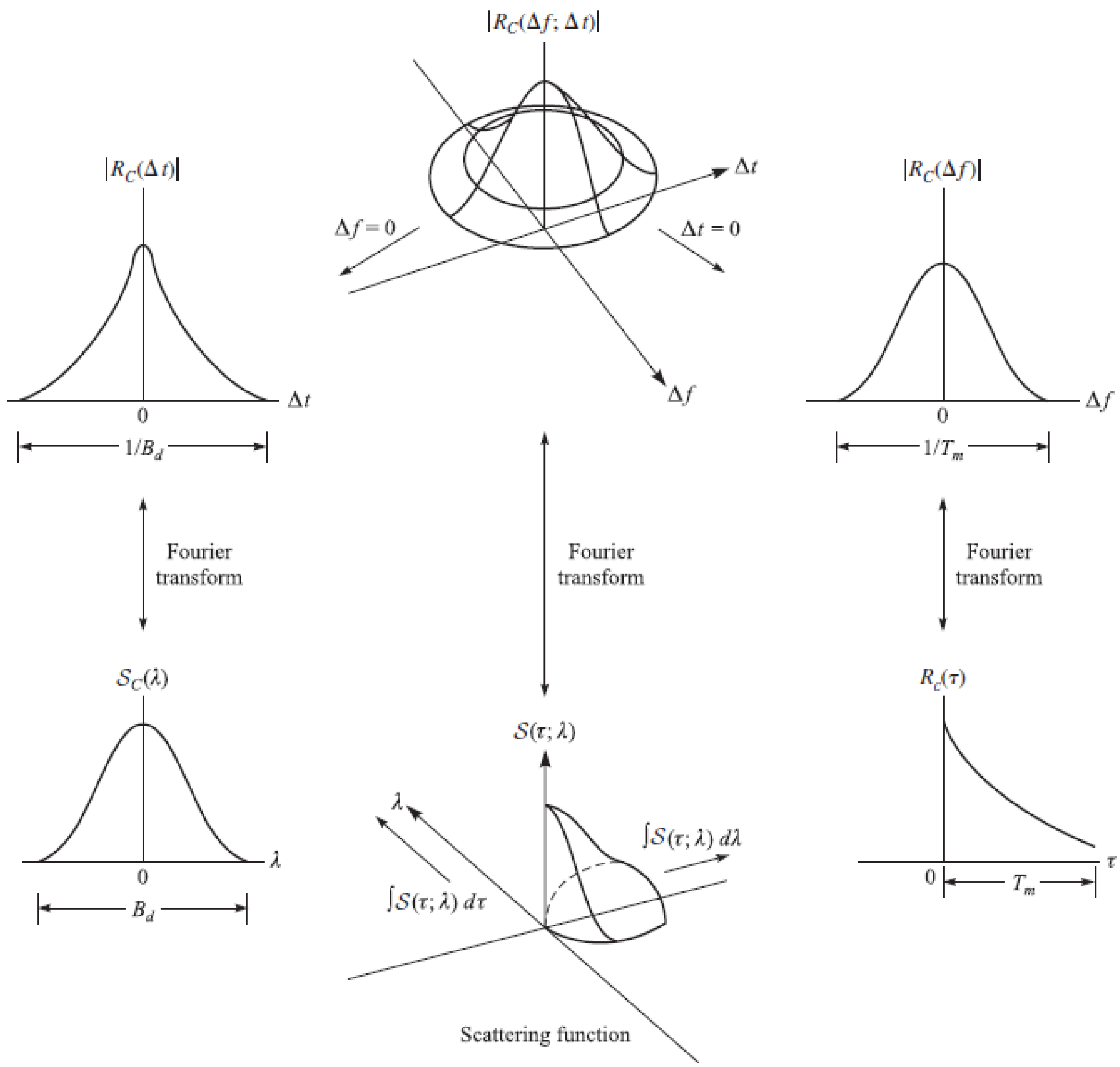
$S_c(0;\lambda) = S_c(\lambda) = Doppler\ power\ spectrum\ of\ the\ channel$ (it gives the signal intensity as a function of the Doppler freq. λ)

where

$$S_C(\lambda) = \int_{-\infty}^{\infty} R_C(0;\Delta t)\ e^{-j2\pi\lambda\Delta t}\ d\Delta t$$

$$S_C(\lambda) \overset{\Delta t,\lambda}{\Longleftrightarrow} R_C(\Delta t)$$





- If the channel is time invariant $R_c(\Delta t) = 1$, then $S_C(\lambda) = \delta(\lambda)$.
- When there are no time variations → There is no spectral broadening.
- B_d (Doppler spread of the channel): range of values over which $S_c(\lambda)$ is essentially not zero.
- Since $S_C(\lambda)$ is related to $R_c(\Delta t)$, $(\Delta t)_c \approx \frac{1}{B_d}$, $(\Delta t)_c$ is *coherence time* of the channel.
- **Relations between** $R_C(\Delta f;\Delta t)$, $R_c(\tau;\Delta t)$, $S_C(\Delta f;\lambda)$, *and* $S(\tau;\lambda)$ is illustrated in the figure above

$$R_C(\Delta f;\Delta t) \overset{\tau,\Delta f}{\Longleftrightarrow} R_c(\tau;\Delta t)$$

$$S_C(\Delta f;\lambda) \overset{\tau,\Delta f}{\Longleftrightarrow} S(\tau;\lambda)$$

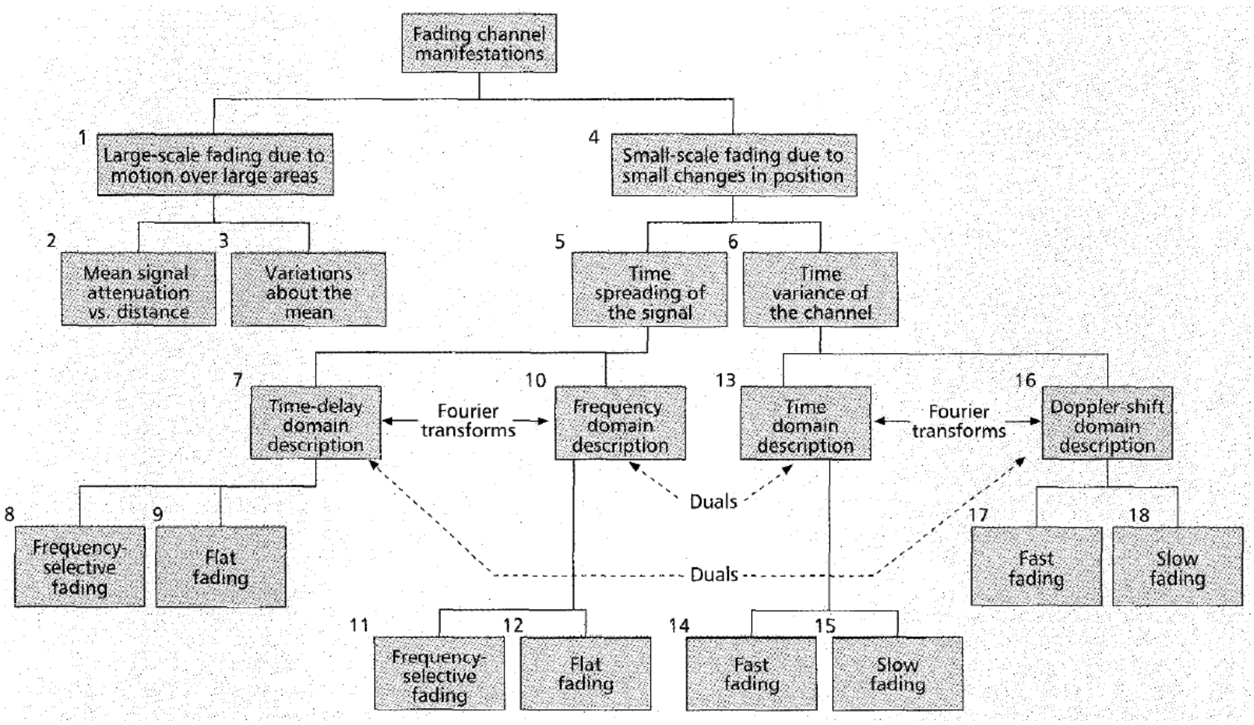
Scattering function of the channel $S(\tau;\lambda)$:

Provides a measure of the average power output of the channel as a function of the time delay τ and the Doppler frequency.

Spread factor= $T_m B_d$

Multipath Spread, Doppler Spread, and Spread Factor for Several Time-Variant Multipath Channels

Type of channel	Multipath duration, s	Doppler spread, Hz	Spread factor
Shortwave ionospheric propagation (HF)	10^{-3}–10^{-2}	10^{-1}–1	10^{-4}–10^{-2}
Ionospheric propagation under distributed auroral conditions (HF)	10^{-3}–10^{-2}	10–100	10^{-2}–1
Ionospheric forward scatter (VHF)	10^{-4}	10	10^{-3}
Tropospheric scatter (SHF)	10^{-6}	10	10^{-5}
Orbital scatter (X band)	10^{-4}	10^{3}	10^{-1}
Moon at max. libration ($f_0 = 0.4$ kmc)	10^{-2}	10	10^{-1}



EXAMPLE 1: SCATTERING FUNCTION OF A TROPOSPHERIC SCATTER CHANNEL.

The scattering function $S(\tau\,;\,\lambda)$ measured on a $150mi$ tropospheric scatter link is shown in the Figure below. The signal used to probe the channel had a time resolution of $0.1\ \mu s$. Hence, the time-delay axis is quantized in increments of $0.1\ \mu s$. From the graph, we observe that the multipath spread $T_m = 0.7\mu s$. On the other hand, the Doppler spread, which may be defined as the $3dB$ bandwidth of the power spectrum for each signal path, appears to vary with each signal path. For example, in one path it is less than $1\ Hz$, while in some other paths it is several hertz. For our purposes, we shall take the largest of these $3dB$ bandwidths of the various paths and call that the *Doppler spread*.

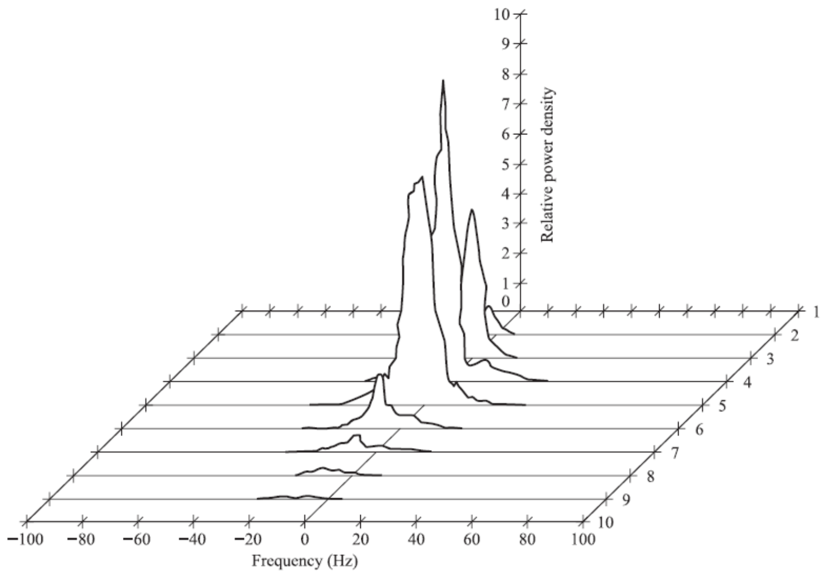


Figure: Scattering function of a medium-range tropospheric scatter channel. The taps delay increment is $0.1\ \mu s$.

Example 2: MULTIPATH INTENSITY PROFILE OF MOBILE RADIO CHANNELS.

The multipath intensity profile of a mobile radio channel depends critically on the type of terrain. Numerous measurements have been made under various conditions in many parts of the world. In **urban and suburban areas**, typical values of multipath spreads range from **1 to 10 μs**. In **rural** mountainous areas, the multipath spreads are much greater, with typical values in the range of 10 to 30 μs. Two models for the multipath intensity profile that are widely used in evaluating system performance for these two types of terrain are illustrated in the figure below.

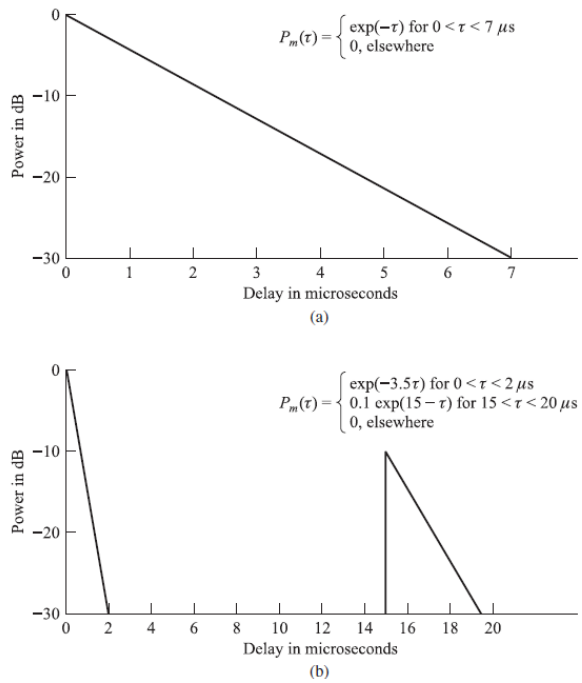


Figure: Cost 207 average power delay profiles: (a) typical delay profile for suburban and urban areas; (b) typical "bad"-case delay profile for hilly terrain. [*From Cost 207 Document 207 TD (86)51 rev 3.*]

Example 3: DOPPLER POWER SPECTRUM OF MOBILE RADIO CHANNELS.

A widely used model for the Doppler power spectrum of a **mobile radio channel** is the so called Jakes' model (Jakes, 1974). In this model, the autocorrelation of the time-variant transfer function $C(f;t)$ is given as

$$R_C(\Delta t) = E[C^*(f;t)C(f;t+\Delta t)] = J_0(2\pi f_m \Delta t)$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind and $f_m = v f_0/c$ is the maximum Doppler frequency, where v is the vehicle speed in meters per second (m/s), f_0 is the carrier frequency, and c is the speed of light $(3 \times 10^8\ m/s)$. The Fourier transform of this autocorrelation function yields the Doppler power spectrum. That is

$$S_C(\lambda) = \int_{-\infty}^{+\infty} R_C(\Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t$$

$$= \int_{-\infty}^{+\infty} J_0(2\pi f_m \Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t$$

$$= \begin{cases} \dfrac{1}{\pi f_m} \dfrac{1}{\sqrt{1-\left(\dfrac{f}{f_m}\right)^2}} & |f| \le f_m \\ 0 & |f| > f_m \end{cases}$$

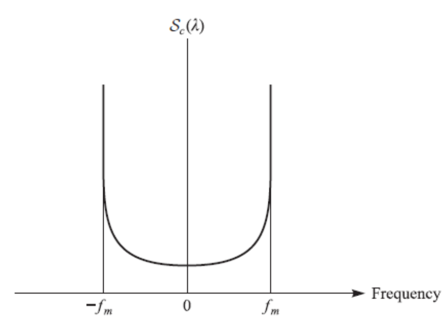
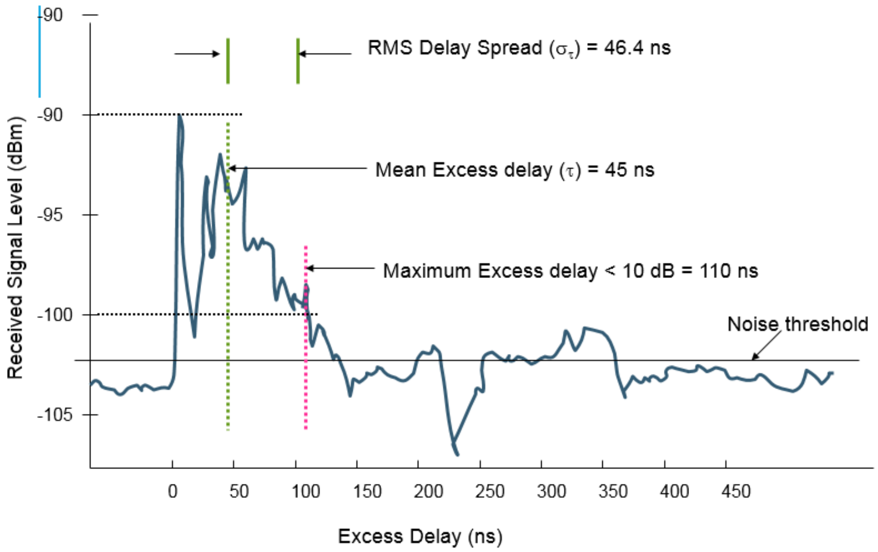


Figure: Model of Doppler spectrum for a mobile radio channel

The graph of $S_C(\lambda)$ is shown in the figure

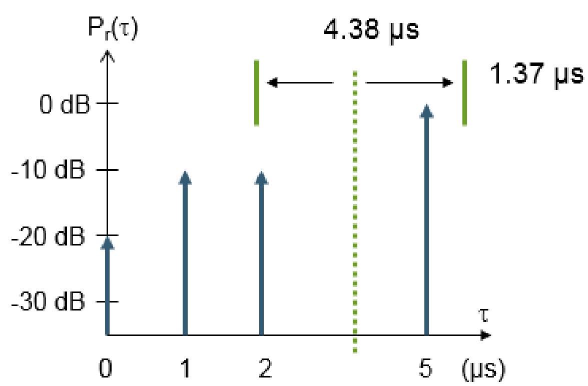
Additional Examples:

Power Delay Profile



Discrete Example (Power delay profile)

Calculate the mean excess delay, $\bar{\tau}$, mean squared excess delay, $\overline{\tau^2}$, and rms delay spread, σ_τ



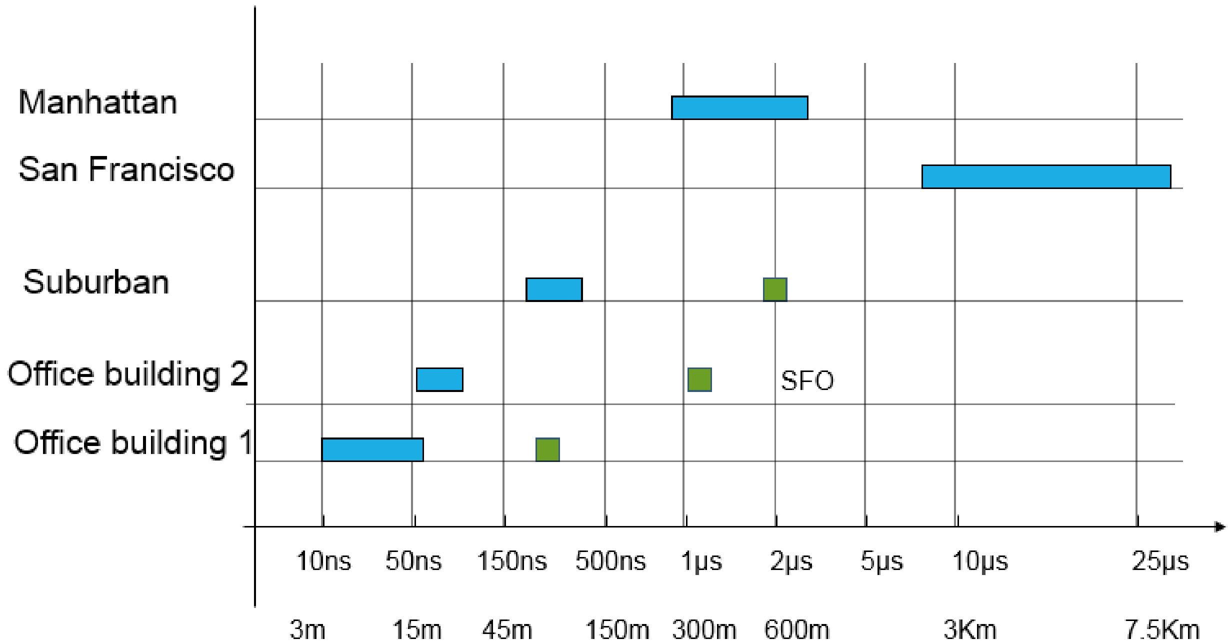
$$\bar{\tau} = \frac{(1)(5)+(0.1)(1)+(0.1)(2)+(0.01)(0)}{[0.01+0.1+0.1+1]} = 4.38\mu s$$

$$\overline{\tau^2} = \frac{(1)(5)^2+(0.1)(1)^2+(0.1)(2)^2+(0.01)(0)^2}{[0.01+0.1+0.1+1]} = 21.07\mu s^2$$

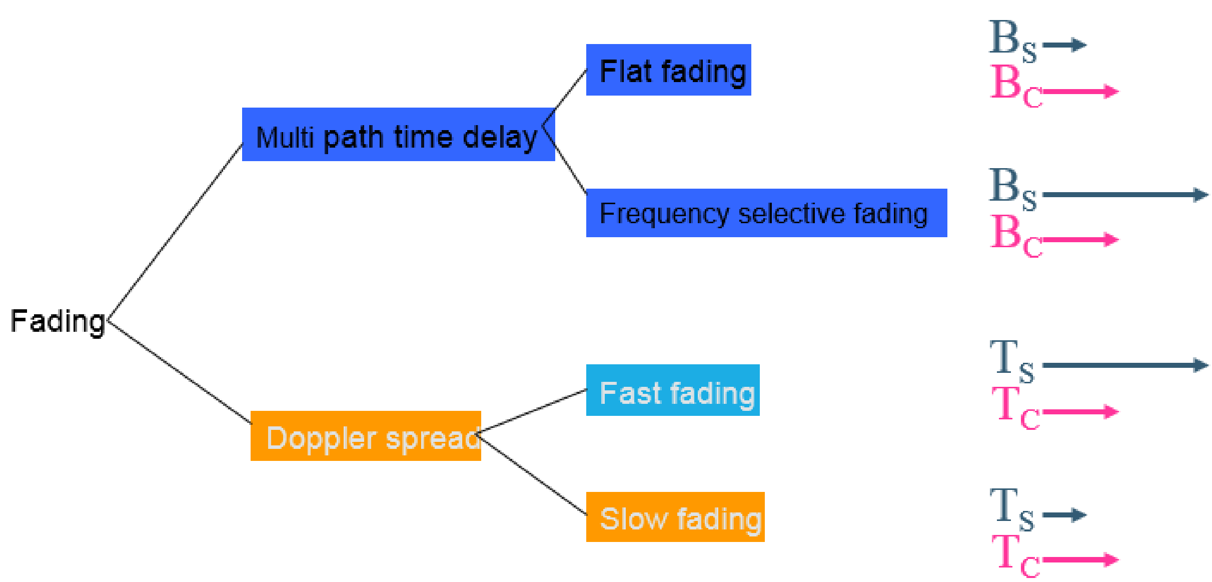
$$\sigma_\tau = \sqrt{21.07 - (4.38)^2} = 1.37\mu s$$

RMS Delay Spread: Typical values

Delay spread is a good measure of Multipath (Note the equivalent distance in the x-axis)



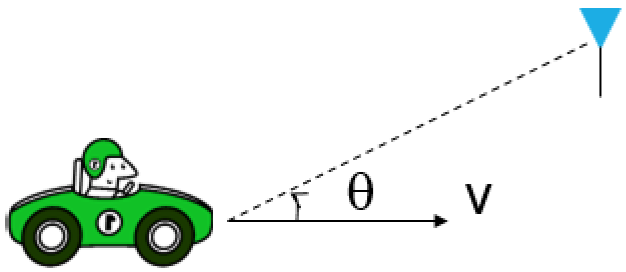
Small Scale Fading



Signal bandwidth for Analog Cellular = 30 KHz

Signal bandwidth for GSM = 200 KHz

Example: Doppler Shift



$$\Delta f = \frac{v \cos\theta}{\lambda}$$

The carrier frequency f_c = 1850 MHz (i.e. λ = 16.2 cm) and vehicle speed v = 60 mph = 26.82 m/s.

- If the vehicle is moving directly towards the transmitter, $\Delta f = \frac{26.82}{0.162} = 165 Hz$
- If the vehicle is moving perpendicular to the angle of arrival of the transmitted signal $\Delta f = 0$

Binary Signaling Over Frequency Nonselective (FNS) Slowly Fading (SF) Channels

In general for $s_l(t)$ transmitted over $c(\tau;t)$

$$r_l(t) = \int_{-\infty}^{\infty} C(f;t)\, S_l(f) e^{j2\pi f t} df + z(t)$$

If the channel is frequency non-selective then

$$C(f;t) = K = C(0;t) \qquad -W \le f \le W$$

When $c(0;t)$ is modeled as a zero mean complex–valued Gaussian Random Process

$$c(0;t) = c_r(t) + jc_i(t) = \alpha(t)e^{-j\emptyset(t)}$$

where $c_r(t)$ *and* $c_i(t)$ are real valued Gaussian random process, then the envelope

$$\alpha(t) = |c(0;t)| = \sqrt{{c_r}^2 + {c_i}^2}$$

$\alpha(t)$ is **Rayleigh distributed**,

$$p(\alpha) = \frac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2} \qquad \alpha > 0$$

The phase

$$\emptyset(t) = \tan^{-1}\frac{c_i(t)}{c_r(t)}$$

The phase , $\emptyset(t)$, is **uniformly distributed over [0,2π)**

The first equation now becomes

$$r(t) = \alpha(t)e^{-j\emptyset(t)} s_l(t) + z(t) \qquad 0 \le t \le T$$

Frequency non-selective channels results in *multiplicative* distortion of the transmitted signal $s_l(t)$

Note that α and Ø are constant over one period, which is the implication of slowly fading.

In the first course in digital communication, we have expressed BER due to AWGN for different Modulation Schemes. For example for Binary PSK $P_2 = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right)$

For a channel that introduces attenuation of α, E_b has to be modified to $\alpha^2 E_b$

Define γ_b to be the received signal-to-noise ratio SNR per information bit. For the case of single channel with fading, $\gamma_b = \frac{\alpha^2 E_b}{N_0}$ $(no\ diversity\ yet, L = 1)$

Binary PSK: $P_b(\gamma_b) = \frac{1}{2} erfc(\sqrt{\gamma_b})$

Note : we must assume that the channel fading is sufficiently slow such that the phase Ø can be estimated from the received signal without error , as required by coherent modulation

$P_b(\gamma_b)$ **gives the bit error probability given that γ_b is fixed**. The average bit error probability could be obtained by averaging $P_b(\gamma_b)$ over all possible value of γ_b. i.e.

$$P_b(\bar{\gamma}_b) = \int P_b(\gamma_b) p(\gamma_b) d\gamma_b$$

but what is $p(\gamma_b)$?

$\gamma_b = \frac{\alpha^2 E_b}{N_0}$, where α is Rayleigh distributed.

By straight transformation , we can show that γ_b is Chi Square distributed (exponential)

$$p(\gamma_b) = \frac{1}{\bar{\gamma}_b} e^{-\frac{\gamma_b}{\bar{\gamma}_b}} \qquad \gamma_b \geq 0 \qquad \text{where} \qquad \bar{\gamma}_b = \frac{\mathcal{E}_b}{N_0} E(\alpha^2)$$

Carrying out the integration, which fortunately happens to have a closed form answer, it is found that

$$P_b(\bar{\gamma}_b) = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_b}{1+\bar{\gamma}_b}}\right] \qquad \text{PSK, Rayleigh fading channel}$$

For Coherent FSK

$$P_b(\gamma_b) = \frac{1}{2} erfc\left(\sqrt{\frac{\gamma_b}{2}}\right)$$

Averaging over γ_b:

$$P_b(\bar{\gamma}_b) = \frac{1}{2}\left[1 - \sqrt{\frac{\bar{\gamma}_b}{2+\bar{\gamma}_b}}\right] \qquad \text{FSK Coherent}$$

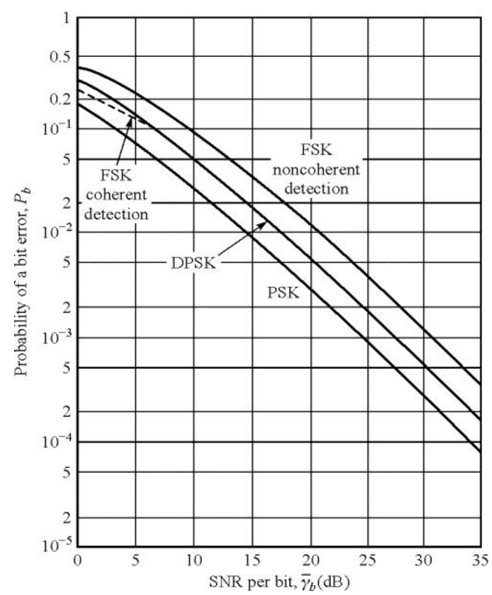
DPSK: If fading is relatively fast that, DPSK could be used, which requires phase stability over only two consecutive signaling intervals, then

$$P_b(\gamma_b) = \frac{1}{2} e^{-\gamma_b} \quad \text{and} \quad P_b(\bar{\gamma}_b) = \frac{1}{2(1+\bar{\gamma}_b)}$$

Non coherent FSK is used , then

$$P_2(\gamma_b) = \frac{1}{2} e^{-\frac{\gamma_b}{2}} \quad \text{and} \quad P_b(\bar{\gamma}_b) = \frac{1}{2+\bar{\gamma}_b}$$

Figure Performance of binary signaling on a Rayleigh fading channel.



Approximate Performance at Large $\bar{\gamma}_b$

For large $\bar{\gamma}_b \gg 1$, the above bit error probability could be approximated by straight line in the log scale. See the performance curves.

$\frac{1}{4\,\bar{\gamma}_b}$ Coherent PSK, $\frac{1}{2\,\bar{\gamma}_b}$ Coherent FSK and DPSK, $\frac{1}{\bar{\gamma}_b}$ non-coherent FSK

Diversity and Combining Techniques

Diversity techniques are useful assuming errors occur in reception when the attenuation is large. i.e. the channel is in deep fade.

If P is the probability of a channel to be in deep fade, then if L channels are used to transmit the same information, the probability that all independent channels are in deep fade simultaneously is P^L.

Types of Diversity

1) Frequency Diversity (When separation between carrier frequencies $> (\Delta f)_c$)
2) Time diversity (When separation between transmission intervals $> (\Delta t)_c$)
    - Similarity to repetition code. Time diversity (and frequency diversity) could be looked at as applying repetition coding & interleaving.
    - We could do much better by:
        i. Using powerful codes, repletion codes are very primitive.
        ii. Using burst error correcting codes, instead of spreading the burst error.
3) Space Diversity (Multiple antennas spaced by 10λ, or a reasonable fraction of λ)

Analysis of Different Combining Techniques

- Assume that we are using L channels that are (Frequency non selective, Slowly fading, independent)

$$r_k(t) = \alpha_k e^{-j\phi_k} s_{km}(t) + z_k(t) \qquad k = 1, \dots\dots L \qquad m = 1,2, \dots . M$$

- For binary we have one of two signals $s_{k1}(t)$ or $s_{k2}(t)$
- The optimum modulator for the single received from the k^{th} channel consists of two matched filters: $b_{k1}(t)= s^*_{k1}(T-t)$ and $b_{k2}(t)= s^*_{k2}(T-t)$
- For PSK, since $s_{k1}(t) = - s_{k2}(t)$, one matched filter could be used. In the case of a single channel transmission, the matched filters are compared and a decision is made.
- Here, we have to have some rule for decision based on the outputs of the L channels.
- Basically there are three combining techniques [Brennan 1959]
    i. Selective Diversity
    ii. Equal gain diversity
    iii. Maximum ratio combining diversity (MRC) : optimal.
- MRC will be analyzed next. To make the analysis tractable, we will make logical extension of previous single channel transmission analysis.

Combining Techniques

The three combining methods mentioned above can be described with $r(t) = \sum_{k=1}^{L} w_k r_k$

Maximum Ratio Combining

- Here, each matched filter output is multiplied by the conjugate of the channel gain , i.e. by $\alpha_k e^{j\emptyset_k}$.
- This assumes that α_k and $\emptyset_k$can be estimated correctly.
- The effect of this multiplication is to:
  - Compensate for the phase shift in the channel. This way all output signals are in phase.
  - Weight the signal by a factor that is proportional to the signal strength. Thus a strong signal carries a large weight than a weak signal.
- The effect of this multiplication, as we will see soon, is to have output energy per bit as the sum of the bit energy over each channel, thus maximizing the SNR of the system of L channels.

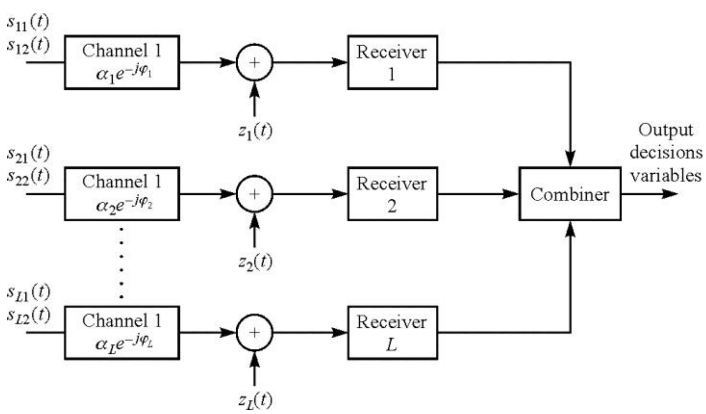


Figure 13.4-1 Model of binary digital communication system with diversity. **[Proakis and Salehi, 5th Edition]**

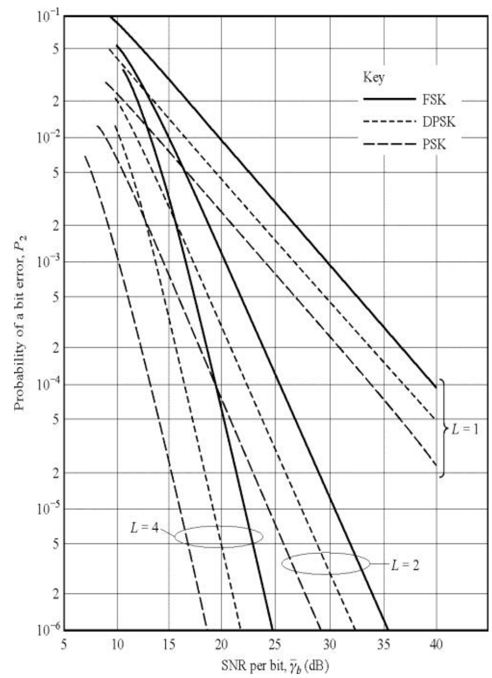


Figure 13.4-2 Performance of binary signals with diversity. **[Proakis and Salehi, 5th Edition]**

For PSK s_{km} is transmitted, and the received signal is

$$r_k(t) = \alpha_k e^{-j\emptyset_k} s_{km}(t) + z_k(t) \qquad k = 1, \dots\dots L \qquad m = 1,2, \dots . M$$

For post detection the output of the matched filter, matched to $s_{km}(t)$ over the k^{th} channel is

$$y_k = \alpha_k e^{-j\emptyset_k}\ 2\mathcal{E} + \int_0^T z_k(t) s_k^*(t) dt = 2\varepsilon \alpha_k e^{-j\emptyset_k} + N_k$$

Forming the decision variable U by weighting each channel by $\alpha_k e^{j\phi_k}$, then summing the results over L channels.

$$U = 2\mathcal{E}\sum_{k=1}^{L} {\alpha_k}^2 + \sum_{k=1}^{L} \alpha_k N_k e^{j\phi_k}$$

For a fixed set of $\{\alpha_k\}$, U is Gaussian with

$E[U] = 2\varepsilon \sum_{k=1}^{L} {\alpha_k}^2$ and $\sigma_U^2 = 2\varepsilon N_0 \sum_{k=1}^{L} {\alpha_k}^2$

The probability of error conditioned on a fixed set of attenuation factors is calculated first.

An error is committed whenever $U < 0$. This leads to (where γ_b is the SNR per bit)

$$P_b(\gamma_b) = \frac{1}{2} erfc\left(\sqrt{\gamma_b}\right) = Q\left(\sqrt{2\gamma_b}\right)$$

Where for MRC $\gamma_b = \frac{\varepsilon}{N_0}\sum_{k=1}^{L} \alpha_k^2 = \sum_{k=1}^{L} \gamma_k$

(note that, assuming all the *channels are mutually independent*, in MRC the energy per bit, γ_b, is equal to the sum of the instantaneous SNR, $\gamma_k = \mathcal{E}\alpha^2/N_0$)

The above probability of error is conditional probability on a fixed set of $\{\alpha_k\}$, or $\{\gamma_k\}$, i.e. instantaneous γ_b. The average error is obtained by averaging $P_2(\gamma_b)$ over all possible values of γ_b.

We need to find the pdf of γ_b, then carry out the integral

$$P_b = \int_0^\infty P_b(\gamma_b) p(\gamma_b) d\gamma_b$$

Under Rayleigh fading, MRC results in (without derivation)

$\gamma_b = \sum_{k=1}^{L} \gamma_k$ and $p(\gamma_b) = \frac{1}{(L-1)!\bar{\gamma}_c{}^L} \gamma_b^{L-1} e^{-\frac{\gamma_b}{\bar{\gamma}_c}}$

We can see the improvement due to diversity, the error rate decreases inversely with the L^{th} power of SNR. Similar analysis could be carried out to find P_b for other modulation schemes (Coherent FM, DPSK, non coherent FM) . See Figure 13.4-2 above.

What are we paying for that achievement?

By increasing L, first note that the horizontal axis is the average energy per bit, $\bar{\gamma}_b$, which is defined as $\bar{\gamma}_b = L\bar{\gamma}_c$

So we are comparing them finally in terms of required transmitted power (if using transmitter-based diversity). However, we are scarifying bandwidth by $\frac{1}{L}$.

Coding

One very interesting result of coding is that it provides diversity of the order of the minimum distance of the code. Binary repletion codes are very primitive, the minimum distance $= 1/R$.

For majority of classes of codes, minimum distance is $>>\ 1/R$.

This means that coding provides diversity without much penalizing the transmission rate over the given bandwidth.

For example the $(24,12)$ Golay code has a minimum distance of 8. It provide a diversity of the order of $L = 8$, but penalizing the bandwidth by ½ only

---

We can repeat the above analysis for other combining techniques. The following relation will be different:

1) γ_b in terms of γ_k
2) $p(\gamma_b)$
3) $P_b(\gamma_b)$

Equal Gain Combining

$$r(t) = \sum_{k=1}^{L} r_k$$

Following the analysis of the MRC, the output of the matched filter is

$$y_k = \alpha_k e^{-j\emptyset_k}\ 2\varepsilon + \int_0^T z_k(t)s^*dt$$

$$U = 2\varepsilon \sum_{k=1}^{L} \alpha_k + \sum N_k e^{j\emptyset_k}$$

For a fixed set of $\{\alpha_k\}$, U is Gaussian with

$E[U] = 2\varepsilon \sum_{k=1}^{L} \alpha_k$ and $\sigma_U^2 = 2\varepsilon N_0 L$

$$P_b(\gamma_b) = Q\left(\sqrt{\frac{(2\varepsilon \sum_{k=1}^{L} \alpha_k)^2}{2\varepsilon N_0 L}}\right) = Q\left(\sqrt{\frac{2\varepsilon(\sum_{k=1}^{L} \alpha_k)^2}{N_0 L}}\right) = Q(\sqrt{2\gamma_b})$$

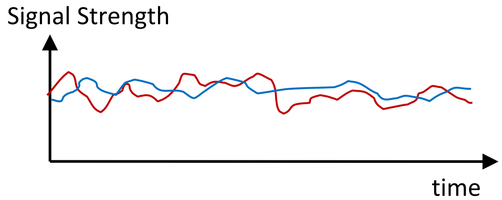
where $\gamma_b = \frac{\varepsilon(\sum_{k=1}^{L} \alpha_k)^2}{N_0 L}$

What is the pdf of γ_b? From which we can find (does not have a closed form)

$$P_b(\bar{\gamma}_b) = \int_0^\infty P(\gamma_b)p(\gamma_b)d\gamma_b$$

Selective Diversity

Selecting the best signal among all of the signals received from different branches, at the receiving end.



Let ω_m denote the index of the channel for which $\gamma_m > \gamma_k$, then $\omega_k = \begin{cases} 1 & k = m \\ 0 & k \neq m \end{cases}$

Where γ_k is the instantaneous SNR on the k^{th} channel.

Switched combining is a more realistic version of selective diversity. In switched combining, the receiver switches to another signal when current signal drops below a predefined threshold. This is a less efficient technique than selection combining. [http://en.wikipedia.org/wiki/Diversity_combining]

Since we are selecting one branch at a time, the BER for PSK:

$$P_b(\gamma_b) = Q(\sqrt{2\gamma_b})$$

Where $\gamma_b = \max \gamma_k \quad k = 1, \ldots .. L$

Given γ_k is exponentially distributed $p(\gamma_k) = \frac{1}{\bar{\gamma}_c} e^{-\frac{\gamma_k}{\bar{\gamma}_c}}$, where $\bar{\gamma}_c$ is the average SNR per channel, which is assumed to be identical for all channels

What is $p(\gamma_b)$?

The pdf of the k^{th} branch of L assuming iid is a basic result of order statistics

For $L = 1$ $\quad p(\gamma_b) = \frac{1}{\gamma_b} e^{-\frac{\gamma_b}{\gamma_c}}$

For $L = 2$ $\quad p(\gamma_b) = \frac{2}{\gamma_b} e^{-\frac{\gamma_b}{\gamma_c}} \left[1 - e^{-\frac{\gamma_b}{\gamma_c}}\right] = \frac{2}{\gamma} e^{-\frac{\gamma_b}{\gamma_c}} - \frac{2}{\gamma_b} e^{-\frac{\gamma_b}{\gamma_c}}$

In general for any L $\quad p(\gamma_b) = \frac{L}{\gamma_b} e^{-\frac{\gamma_b}{\gamma_c}} \left[1 - e^{-\frac{\gamma_b}{\gamma_c}}\right]^{L-1}$

The average of $\bar{\gamma}_b$ is given by $\bar{\gamma}_b = \bar{\gamma}_c \sum_{k=1}^{L} \frac{1}{k}$ (Diversity gain)

It is worth comparing the results with those of MRC

Correlated Branches:

Note in the presence of correlation, the diversity gain will not be the same. See the figure below

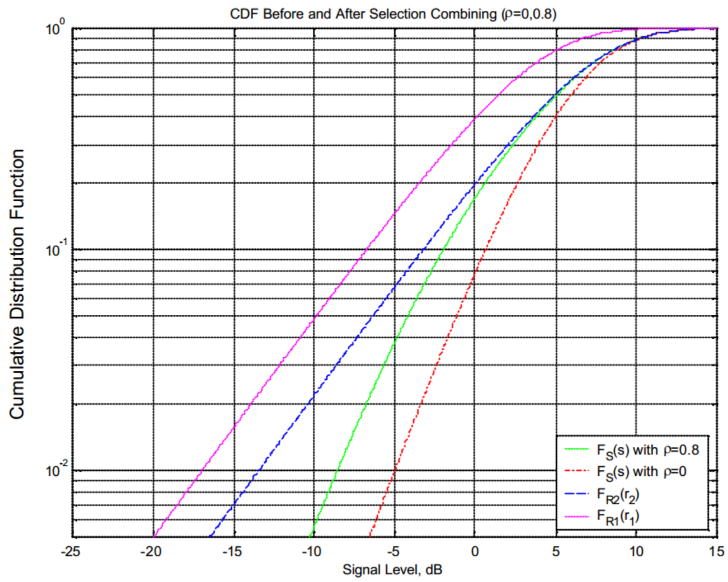


Figure: Cumulative distribution function of r1, r2, and s for an envelope correlation of 0.8 and 0. [Thesis by Kai Dietze submitted to Virginia Tech]

References

[Brennan 1959] D.G. Brennan, "Linear diversity combining techniques," Proc. IRE, vol.47, no.1, pp.1075–1102, June 1959.

[Proakis 5th Edition] John Proakis and Masoud Salehi, Digital Communications, McGraw-Hill 2001

See Matlab Demo: Check Matlab/Toolbox/Communications/Channel Modeling/Multipath Fading Channels

EE573: Digital Communications II:

Summary Fading & Diversity

Summary of Main Points
Prepared by Dr. Ali Muqaibel

- Digital Communications through Fading Multipath Channels
    - What is Fading Multipath Channels?
    - Example of fading multipath Channels
- Large Scale and Small Scale Fading
    - Statistical Characterization (Modeling)
    - Performance Evaluation (Penalty in SNR)
    - Mitigation through efficient modulation/coding.

- Channel Correlation Functions & Power Spectra
    - Significance of the autocorrelation function
    - I. **Effect of Multipath**
        - Time Domain Analysis: multipath intensity profile or delay power spectrum
        - Frequency Domain Analysis: Spaced frequency spaced time correlation function
            - Frequency selective vs. frequency nonselective channels
    - II. **Effect of Time Variations**
            - Doppler spread of the channel
- Binary Signaling Over FNS SF Channels
    - Binary PSK
    - Coherent FSK
- Diversity and Combining Techniques
- Def.
- Types:
    - Frequency Diversity
    - Time Diversity
    - Space Diversity
- Combining Techniques
    - Maximum Ratio Combining
    - Selective Combining
    - Switched Combining
    - Equal Gain Combining
- Coding
- The two papers
    - Digital Communications through Fading Multipath Channels