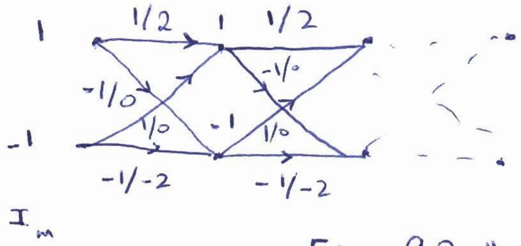
Maximum-likelihood sequence detection:

- memory can be represented with a trellis. for binary duobinary signaling.



input/output.

Fig. 9.2-11

- duobinary has a memory of length $L=1$
  $\Rightarrow$ Trellis has $S_t = 2^1$ states
- For M-ary modulation, number of Trellis states M^L
- Opimum ML: select the most probable path through the trellis upon observing $\{y_m\}$ at $t = mT$, $m=1,2 \ldots$
- Every node will have M incoming nodes with corresponding metrics.
- out of the M choose one & discard $M-1$
- surviving path will be extended to M pathes ...
- This is Viterbi algorithm.

- For a class of partial response signals $\{y_m, \; 1 \le m \le N\}$ the received sequence is described by the multivariate Gaussian pdf.

$$\underline{Y_N} = [Y_1 \; Y_2 \; Y_3 \cdots Y_N]' \qquad N > L$$

$$\underline{I_N} = [I_1 \; I_2 \; \ldots \; I_N]'$$

with additive noise which is zero mean & Gaussian.

$$f(\underline{Y_N} \mid \underline{I_N}) = \frac{1}{(2\pi \det \underline{C})^{N/2}} \exp\left[-\frac{1}{2}(\underline{Y_N} - \underline{B_N})' \, \underline{C}^{-1} (\underline{Y_N} - \underline{B_N})\right]$$

where $\underline{B_N} = [B_1 \cdots B_N]'$ mean of the vector Y_N

$\underline{C}$ is $N \times N$ covariance matrix of Y_N

→ The ML selects the sequence that maximizes the pdf.

or max. the natural logarithm of $p(\underline{Y}_N | \underline{I}_N)$

monotonic function.

$$\ln p(\underline{Y}_N | \underline{I}_N) = -\frac{1}{2} N \ln(2\pi \det \underline{C}) - \frac{1}{2}(\underline{Y}_N - \underline{B}_N)' \underline{C}^{-1} (\underline{Y}_N - \underline{B}_N)$$

Given $\{Y_m\}$ the data seq I_m that max $\ln(p(\underline{Y}_N | \underline{I}_N))$

is identical to the seq. I_N that minimizes

$$(\underline{Y}_N - \underline{B}_N)' \underline{C}^{-1} (\underline{Y}_N - \underline{B}_N)$$

$$\boxed{\hat{I}_N = \arg\min_{I_N} \left[(\underline{Y}_N - \underline{B}_N)' \underline{C}^{-1} (\underline{Y}_N - \underline{B}_N) \right]}$$

The computation is complicated by the calculation of the correlation of the noise samples v_m (at the output of the matched filter).

$$E[v_m v_{m+k}] = 0 \quad \text{for } k > L \qquad L: \text{memory.} \quad (\text{see ch \#10})$$

simplification:

let $E[v_m v_{m+k}] = 0$ for $k > 0$ $\Rightarrow$ $\underline{C} = \sigma_v^2 \mathbf{1}_N$

where $\sigma_v^2 = E[v_m^2]$

$\underline{\mathbf{1}_N} = N \times N$ identity matrix.

avoid confusion with $\underline{I_N}$

$$\hat{I}_N = \arg\min_{\underline{I_N}} \left[(\underline{Y}_N - \underline{B}_N)' (\underline{Y}_N - \underline{B}_N) \right] \quad \text{where}$$

$$= \arg\min_{\underline{I_N}} \left[\sum_{m=1}^{N} \left(y_m - \sum_{k=0}^{L} x_k I_{m-k} \right)^2 \right]$$

$$B_m = \sum_{k=0}^{L} x_k I_{m-k}$$

$x_k = x(kT)$ sample values of the partial response wave form.

Metric Computation:

$$DM_m(\underline{I_m}) = DM_{m-1}(\underline{I_{m-1}}) + \left(y_m - \sum_{k=0}^{L} x_k I_{m-k} \right)^2$$

new metric based on the new recived y_m

←

$DM_m(\underline{I}_m)$ is the distance metric @ $t = mT$

$DM_{m-1}(\underline{I}_{m-1})$ " " " " $t = (m-1)T$

⊖ • ML introduces variable delay in detecting each transmitted information symbol.

<u>solution</u> truncate to N_t most recent symbols.

$\boxed{N_t \gg 5L}$ if still disagree choose the most probable path.

• loss in performance for chosing $N_t > 5L$ is negligible.

<u>Example</u> (<u>not</u> from the book)

lets say we transmitted $\overset{I_0}{1}, \overset{I_1}{1}, \overset{I_2}{1}, \overset{I_3}{1}, \dots$

The received sequence noise free $\overset{b_1}{2}, \overset{b_2}{2}, \overset{b_3}{2}, \overset{b_4}{2}, \dots$ symbol-by symbol.

due to noise, the received seq $\overset{y_1}{2.1}, \overset{y_2}{0.9}, \overset{y_3}{1.8}, 2.2$

$I_m = y_m - I_{m-1}$ assuming I_0 was detected correctly.

• $I_1 = 1.1 \to 1$

$I_2 = 0.9 - 1 = -0.1 \to -1$ error

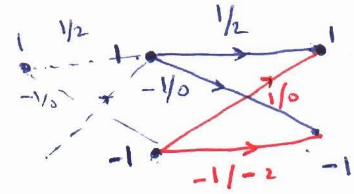
$I_3 = 2.8 \to 1$

→ assume that the first transmitted symbol was detected correctly. (from Y_0 & Y_1)

i.e. $I_0 = 1$

$\mu_1(1) = (y_1 - 2)^2 = 0.01$, $\mu_1(-1) = 4.41 = (2.1 - 0)^2 = 4.41$ ($Y_1 = 2.1$ is already recived).

→ up on reciving $Y_2 = 0.9$



survivor → $\mu_2(1,1) = (y_1 - 2)^2 + (y_2 - 2)^2 = 0.01 + 1.21 = 1.22$

$\mu_2(-1,1) = y_1^2 + y_2^2 = 4.41 + 0.81 = 5.22$

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survivor → $\mu_2(1,-1) = (y_1 - 2)^2 + y_2^2 = 0.01 + 0.81 = 0.82$

$\mu_2(-1,-1) = y_1^2 + (y_2 + 2)^2 = 4.41 + (2.9)^2 =$

$\mu_3(1,1,1) = \mu_2(1,1) + (y_3 - 2)^2 = 1.22 + 0.04 = 1.26$

$\mu_3(1\ -1, +1) = \mu_2(1, -1) + y_3^2 = 0.82 + (1.8)^2 = 4.06$

→

$\mu_3(1,1,1)$ is the survivor.

$\mu_3(1,+1,-1) = \mu_2(1,1) + y_3^2 = 1.22 + 3.24 = 4.46$

$\mu_3(1,-1,-1) = \mu_2(1,-1) + (y_3+2)^2 = 0.82 + (3.8)^2 =$

$\mu_3(1,1,-1)$ is the survivor.

$I_1 = 1$ & $I_2 = 1$

$\vdots$

$I_3 = 1$

ML is optimum.