Problem 10.4 :

(a) Each segment of the wire-line can be considered as a bandpass filter with bandwidth $W = 1200$ Hz. Thus, the highest bit rate that can be transmitted without ISI by means of binary PAM is :

$$R = 2W = 2400 \text{ bps}$$

(b) The probability of error for binary PAM trasmission is :

$$P_2 = Q\left[\sqrt{\frac{2\mathcal{E}_b}{N_0}}\right]$$

Hence, using mathematical tables for the function $Q[\cdot]$, we find that $P_2 = 10^{-7}$ is obtained for :

$$\sqrt{\frac{2\mathcal{E}_b}{N_0}} = 5.2 \Longrightarrow \frac{\mathcal{E}_b}{N_0} = 13.52 = 11.30 \text{ dB}$$

(c) The received power P_R is related to the desired SNR per bit through the relation :

$$\frac{P_R}{N_0} = \frac{1}{T}\frac{\mathcal{E}_b}{N_0} = R\frac{\mathcal{E}_b}{N_0}$$

Hence, with $N_0 = 4.1 \times 10^{-21}$ we obtain :

$$P_R = 4.1 \times 10^{-21} \times 1200 \times 13.52 = 6.6518 \times 10^{-17} = -161.77 \text{ dBW}$$

Since the power loss of each segment is :

$$L_s = 50 \text{ Km } \times 1 \text{ dB/Km } = 50 \text{ dB}$$

the transmitted power at each repeater should be :

$$P_T = P_R + L_s = -161.77 + 50 = -111.77 \text{ dBW}$$

Problem 10.10 :

(a) The equivalent discrete-time impulse response of the channel is :

$$h(t) = \sum_{n=-1}^{1} h_n \delta(t - nT) = 0.3\delta(t + T) + 0.9\delta(t) + 0.3\delta(t - T)$$

If by $\{c_n\}$ we denote the coefficients of the FIR equalizer, then the equalized signal is :

$$q_m = \sum_{n=-1}^{1} c_n h_{m-n}$$

which in matrix notation is written as :

$$\begin{pmatrix} 0.9 & 0.3 & 0. \\ 0.3 & 0.9 & 0.3 \\ 0. & 0.3 & 0.9 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

The coefficients of the zero-force equalizer can be found by solving the previous matrix equation. Thus,

$$\begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} -0.4762 \\ 1.4286 \\ -0.4762 \end{pmatrix}$$

(b) The values of q_m for $m = \pm 2, \pm 3$ are given by

$$\begin{aligned} q_2 &= \sum_{n=-1}^{1} c_n h_{2-n} = c_1 h_1 = -0.1429 \\ q_{-2} &= \sum_{n=-1}^{1} c_n h_{-2-n} = c_{-1} h_{-1} = -0.1429 \\ q_3 &= \sum_{n=-1}^{1} c_n h_{3-n} = 0 \\ q_{-3} &= \sum_{n=-1}^{1} c_n h_{-3-n} = 0 \end{aligned}$$

Problem 10.15 :

A discrete time transversal filter equivalent to the cascade of the trasmitting filter $g_T(t)$, the channel $c(t)$, the matched filter at the receicer $g_R(t)$ and the sampler, has tap gain coefficients $\{x_m\}$, where :

$$x_m = \begin{cases} 0.9 & m = 0 \\ 0.3 & m = \pm 1 \\ 0 & \text{otherwise} \end{cases}$$

The noise ν_k, at the output of the sampler, is a zero-mean Gaussian sequence with autocorrelation function :

$$E[\nu_k \nu_l] = \sigma^2 x_{k-l}, \qquad |k-l| \leq 1$$

If the $\mathcal{Z}$-transform of the sequence $\{x_m\}$, $X(z)$, assumes the factorization :

$$X(z) = F(z)F^*(z^{-1})$$

then the filter $1/F^*(z^{-1})$ can follow the sampler to white the noise sequence ν_k. In this case the output of the whitening filter, and input to the MSE equalizer, is the sequence :

$$u_n = \sum_k I_k f_{n-k} + n_k$$

where n_k is zero mean Gaussian with variance σ^2. The optimum coefficients of the MSE equalizer, c_k, satisfy :

$$\sum_{n=-1}^{1} c_n \Gamma_{kn} = \xi_k, \qquad k = 0, \pm 1$$

where :

$$\begin{aligned} \Gamma(n-k) &= \begin{cases} x_{n-k} + \sigma^2 \delta_{n,k}, & |n-k| \leq 1 \\ 0 & \text{otherwise} \end{cases} \\ \xi_(k) &= \begin{cases} f_{-k}, & -1 \leq k \leq 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

With

$$X(z) = 0.3z + 0.9 + 0.3z^{-1} = (f_0 + f_1 z^{-1})(f_0 + f_1 z)$$

we obtain the parameters f_0 and f_1 as :

$$f_0 = \begin{cases} \pm\sqrt{0.7854} \\ \pm\sqrt{0.1146} \end{cases}, \qquad f_1 = \begin{cases} \pm\sqrt{0.1146} \\ \pm\sqrt{0.7854} \end{cases}$$

The parameters f_0 and f_1 should have the same sign since $f_0 f_1 = 0.3$. However, the sign itself does not play any role if the data are differentially encoded. To have a stable inverse system $1/F^*(z^{-1})$, we select f_0 and f_1 in such a way that the zero of the system $F^*(z^{-1}) = f_0 + f_1 z$ is inside the unit circle. Thus, we choose $f_0 = \sqrt{0.1146}$ and $f_1 = \sqrt{0.7854}$ and therefore, the desired system for the equalizer's coefficients is

$$\begin{pmatrix} 0.9+0.1 & 0.3 & 0.0 \\ 0.3 & 0.9+0.1 & 0.3 \\ 0.0 & 0.3 & 0.9+0.1 \end{pmatrix} \begin{pmatrix} c_{-1} \\ c_0 \\ c_1 \end{pmatrix} = \begin{pmatrix} \sqrt{0.7854} \\ \sqrt{0.1146} \\ 0 \end{pmatrix}$$

Solving this system, we obtain

$$c_{-1} = 0.8596, \quad c_0 = 0.0886, \quad c_1 = -0.0266$$

Problem 10.22 :

(a) We have that :

$$\frac{1}{2T} = 900, \ \frac{1+\beta}{2T} = 1200 \Rightarrow$$
$$1 + \beta = 1200/900 = 4/3 \Rightarrow \beta = 1/3$$

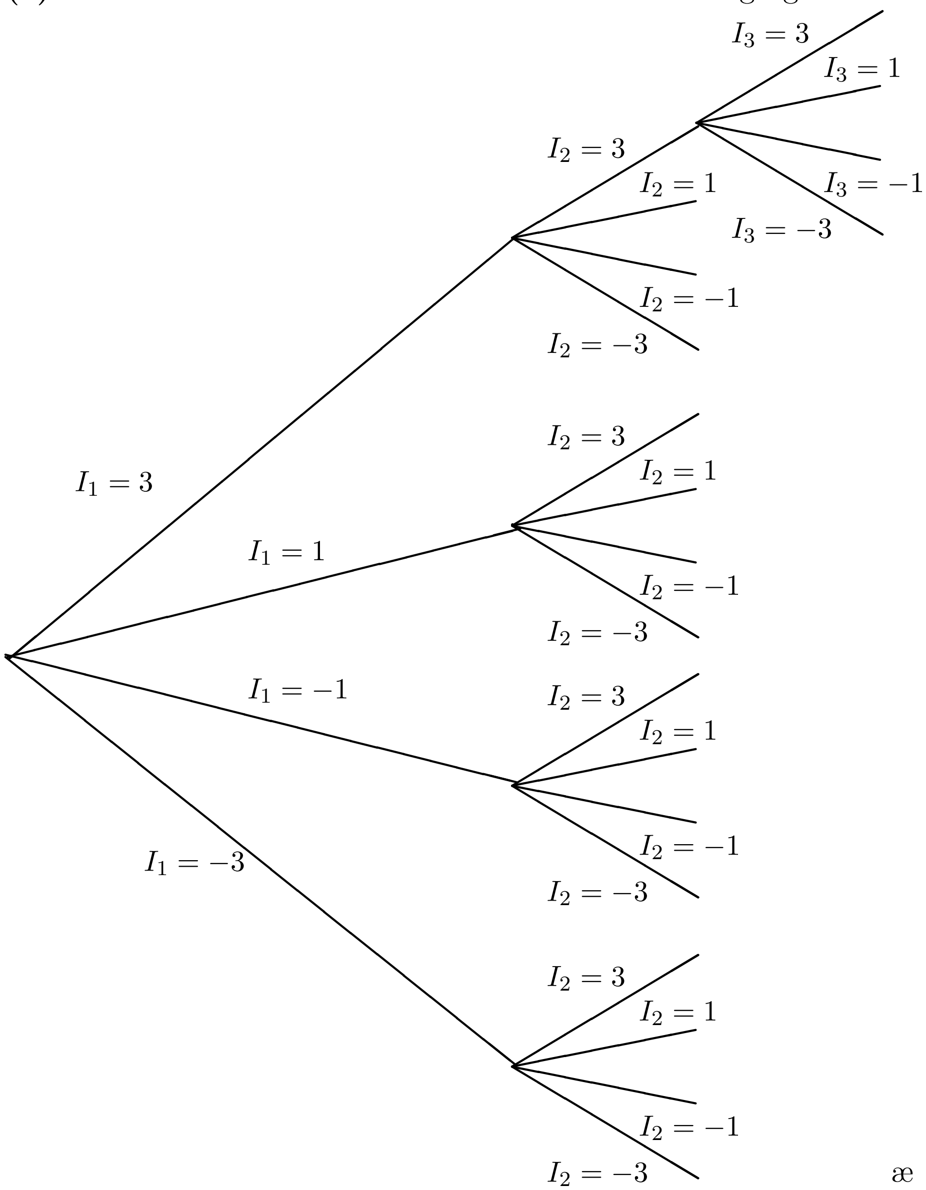
(b) Since $1/2T = 900$, the pulse rate $1/T$ is 1800 pulses/sec.

(c) The largest interference is caused by the sequence : $\{1, -1, s, 1, -1, 1\}$ or its opposite in sign. This interference is constructive or destructive depending on the sign of the information symbol s. The peak distortion is $\sum_{k=-2,k\neq 0}^{3} f_k = 1.6$

(d) The probability of the worst-case interference given above is $\left(\frac{1}{2}\right)^5 = 1/32$, and the same is the probability of the sequence that causes the opposite-sign interference.

Problem 10.24 :

(a) Part of the tree structure is shown in the following figure :



(b) There are four states in the trellis (corresponding to the four possible values of the symbol I_{k-1}), and for each one there are four paths starting from it (corresponding to the four possible values of the symbol I_k). Hence, 16 probabilities must be computed at each stage of the Viterbi algorithm.

(c) Since, there are four states, the number of surviving sequences is also four.

(d) The metrics are

$$(y_1 - 0.8I_1)^2\,,\ i = 1 \ \text{ and } \sum_i (y_i - 0.8I_i + 0.6I_{i-1})^2\,,\ i \geq 2$$

$$\begin{aligned}
\mu_1(I_1 = 3) &= [0.5 - 3 * 0.8]^2 = 3.61 \\
\mu_1(I_1 = 1) &= [0.5 - 1 * 0.8]^2 = 0.09 \\
\mu_1(I_1 = -1) &= [0.5 + 1 * 0.8]^2 = 1.69 \\
\mu_1(I_1 = -3) &= [0.5 + 3 * 0.8]^2 = 8.41
\end{aligned}$$

$$\begin{aligned}
&\mu_2(I_2 = 3, I_1 = 3) = \mu_1(3) + [2 - 2.4 + 3 * 0.6]^2 = 5.57 \\
&\mu_2(3, 1) = \mu_1(1) + [2 - 2.4 + 1 * 0.6]^2 = 0.13 \\
&\mu_2(3, -1) = \mu_1(-1) + [2 - 2.4 - 1 * 0.6]^2 = 6.53 \\
&\mu_2(3, -3) = \mu_1(-3) + [2 - 2.4 - 3 * 0.6]^2 = 13.25
\end{aligned}$$

$$\begin{aligned}
\mu_2(1, 3) &= \mu_1(3) + [2 - 0.8 + 3 * 0.6]^2 = 12.61 \\
\mu_2(1, 1) &= \mu_1(1) + [2 - 0.8 + 1 * 0.6]^2 = 3.33 \\
\mu_2(1, -1) &= \mu_1(-1) + [2 - 0.8 - 1 * 0.6]^2 = 2.05 \\
\mu_2(1, -3) &= \mu_1(-3) + [2 - 0.8 - 3 * 0.6]^2 = 8.77
\end{aligned}$$

$$\begin{aligned}
\mu_2(-1, 3) &= \mu_1(3) + [2 + 0.8 + 3 * 0.6]^2 = 24.77 \\
\mu_2(-1, 1) &= \mu_1(1) + [2 + 0.8 + 1 * 0.6]^2 = 11.65 \\
\mu_2(-1, -1) &= \mu_1(-1) + [2 + 0.8 - 1 * 0.6]^2 = 6.53 \\
\mu_2(-1, -3) &= \mu_1(-3) + [2 + 0.8 - 3 * 0.6]^2 = 9.41
\end{aligned}$$

$$\begin{aligned}
\mu_2(-3, 3) &= \mu_1(3) + [2 + 2.4 + 3 * 0.6]^2 = 42.05 \\
\mu_2(-3, 1) &= \mu_1(1) + [2 + 2.4 + 1 * 0.6]^2 = 25.09 \\
\mu_2(-3, -1) &= \mu_1(-1) + [2 + 2.4 - 1 * 0.6]^2 = 16.13 \\
\mu_2(-3, -3) &= \mu_1(-3) + [2 + 2.4 - 3 * 0.6]^2 = 15.17
\end{aligned}$$

The four surviving paths at this stage are $\min_{I_1}[\mu_2(x, I_1)]\,,\ x = 3, 1, -1, -3$ or :

$$\begin{aligned}
&I_2 = 3, I_1 = 1 \text{ with metric } \mu_2(3, 1) = 0.13 \\
&I_2 = 1, I_1 = -1 \text{ with metric } \mu_2(1, -1) = 2.05 \\
&I_2 = -1, I_1 = -1 \text{ with metric } \mu_2(-1, -1) = 6.53 \\
&I_2 = -3, I_1 = -3 \text{ with metric } \mu_2(-3, -3) = 15.17
\end{aligned}$$

Now we compute the metrics for the next stage :

$$\begin{aligned}
&\mu_3(I_3 = 3, I_2 = 3, I_1 = 1) = \mu_2(3, 1) + [-1 - 2.4 + 1.8]^2 = 2.69 \\
&\mu_3(3, 1, -1) = \mu_2(1, -1) + [-1 - 2.4 + 0.6]^2 = 9.89 \\
&\mu_3(3, -1, -1) = \mu_2(-1, -1) + [-1 - 2.4 - 0.6]^2 = 22.53 \\
&\mu_3(3, -3, -3) = \mu_2(-3, -3) + [-1 - 2.4 - 1.8]^2 = 42.21
\end{aligned}$$

$$\begin{aligned}
\mu_3(1,3,1) &= \mu_2(3,1) + [-1-0.8+1.8]^2 = 0.13 \\
\mu_3(1,1,-1) &= \mu_2(1,-1) + [-1-0.8+0.6]^2 = 7.81 \\
\mu_3(1,-1,-1) &= \mu_2(-1,-1) + [-1-0.8-0.6]^2 = 12.29 \\
\mu_3(1,-3,-3) &= \mu_2(-3,-3) + [-1-0.8-1.8]^2 = 28.13
\end{aligned}$$

$$\begin{aligned}
\mu_3(-1,3,1) &= \mu_2(3,1) + [-1+0.8+1.8]^2 = 2.69 \\
\mu_3(-1,1,-1) &= \mu_2(1,-1) + [-1+0.8+0.6]^2 = 2.69 \\
\mu_3(-1,-1,-1) &= \mu_2(-1,-1) + [-1+0.8-0.6]^2 = 7.17 \\
\mu_3(-1,-3,-3) &= \mu_2(-3,-3) + [-1+0.8-1.8]^2 = 19.17
\end{aligned}$$

$$\begin{aligned}
\mu_3(-3,3,1) &= \mu_2(3,1) + [-1+2.4+1.8]^2 = 10.37 \\
\mu_3(-3,1,-1) &= \mu_2(1,-1) + [-1+2.4+0.6]^2 = 2.69 \\
\mu_3(-3,-1,-1) &= \mu_2(-1,-1) + [-1+2.4-0.6]^2 = 7.17 \\
\mu_3(-3,-3,-3) &= \mu_2(-3,-3) + [-1+2.4-1.8]^2 = 15.33
\end{aligned}$$

The four surviving sequences at this stage are $\min_{I_2,I_1}[\mu_3(x,I_2,I_1)]$, $x = 3,1,-1,-3$ or :

$$\begin{aligned}
&I_3 = 3, I_2 = 3, I_1 = 1 \text{ with metric } \mu_3(3,3,1) = 2.69 \\
&I_3 = 1, I_2 = 3, I_1 = 1 \text{ with metric } \mu_3(1,3,1) = 0.13 \\
&I_3 = -1, I_2 = 3, I_1 = 1 \text{ with metric } \mu_3(-1,3,1) = 2.69 \\
&I_3 = -3, I_2 = 1, I_1 = -1 \text{ with metric } \mu_3(-3,1,-1) = 2.69
\end{aligned}$$

(e) For the channel, $\delta^2_{\min} = 1$ and hence :

$$P_4 = 8Q\left(\sqrt{\frac{6}{15}\gamma_{av}}\right)$$

CHAPTER 11

Problem 11.1 :

(a)

$$F(z) = \frac{4}{5} + \frac{3}{5}z^{-1} \Rightarrow X(z) = F(z)F^*(z^{-1}) = 1 + \frac{12}{25}\left(z + z^{-1}\right)$$

Hence :

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & \frac{12}{25} & 0 \\ \frac{12}{25} & 1 & \frac{12}{25} \\ 0 & \frac{12}{25} & 1 \end{bmatrix} \quad \xi = \begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix}$$

and :

$$\mathbf{C}_{opt} = \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \mathbf{\Gamma}^{-1}\xi = \frac{1}{\beta}\begin{bmatrix} 1-a^2 & -a & a^2 \\ -a & 1 & -a \\ a^2 & -a & 1-a^2 \end{bmatrix}\begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix}$$

where $a = 0.48$ and $\beta = 1 - 2a^2 = 0.539$. Hence :

$$\mathbf{C}_{opt} = \begin{bmatrix} 0.145 \\ 0.95 \\ -0.456 \end{bmatrix}$$

(b) The eigenvalues of the matrix $\mathbf{\Gamma}$ are given by :

$$|\mathbf{\Gamma} - \lambda\mathbf{I}| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 0.48 & 0 \\ 0.48 & 1-\lambda & 0.48 \\ 0 & 0.48 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1,\ 0.3232,\ 1.6768$$

The step size Δ should range between :

$$0 \le \Delta \le 2/\lambda_{\max} = 1.19$$

(c) Following equations (10-3-3)-(10-3-4) we have :

$$\psi = \begin{bmatrix} 1 & 0.48 \\ 0.48 & 0.64 \end{bmatrix}, \quad \psi\begin{bmatrix} c_{-1} \\ c_0 \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} c_{-1} \\ c_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.25 \end{bmatrix}$$

and the feedback tap is :

$$c_1 = -c_0 f_1 = -0.75$$