Problem 9.10 :

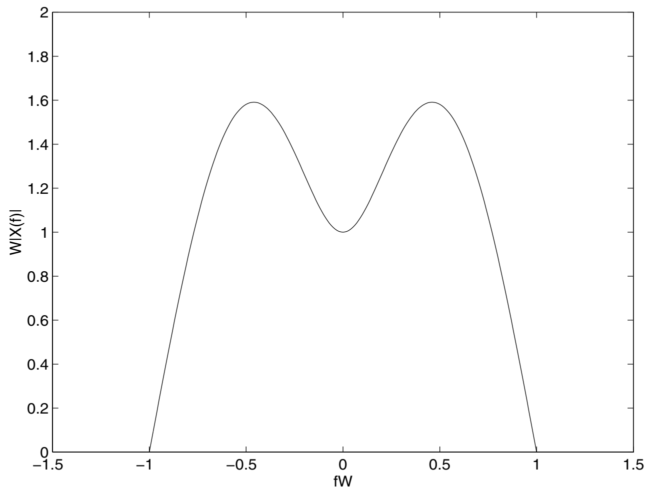
(a)
(i) $x_0 = 2,\ x_1 = 1,\ x_2 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi Wt)}{2\pi Wt} + \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)} - \frac{\sin(2\pi W(t-1/W))}{2\pi W(t-1/W)}$$

and :

$$\begin{array}{rl} X(f) = \frac{1}{2W}\left[2 + e^{-j\pi f/W} - e^{-j2\pi f/W}\right], & |f| \leq W \Rightarrow \\ |X(f)| = \frac{1}{2W}\left[6 + 2\cos\frac{\pi f}{W} - 4\cos\frac{2\pi f}{W}\right]^{1/2}, & |f| \leq W \end{array}$$

The plot of $|X(f)|$ is given in the following figure :



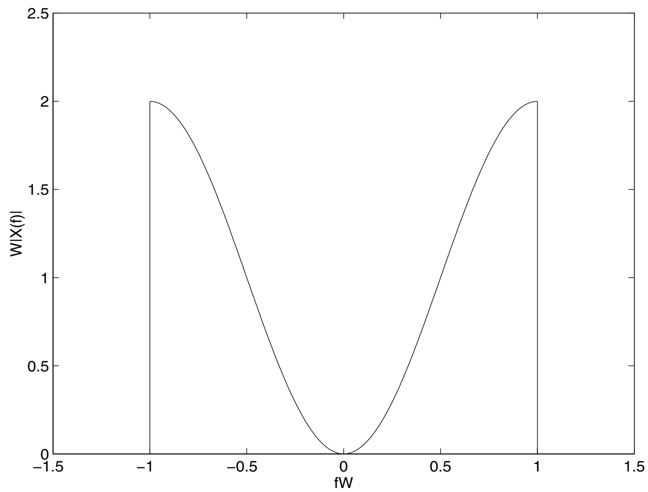
(ii) $x_{-1} = -1,\ x_0 = 2, x_1 = -1,$ otherwise $x_n = 0$. Then :

$$x(t) = 2\frac{\sin(2\pi Wt)}{2\pi Wt} - \frac{\sin(2\pi W(t+1/2W))}{2\pi W(t+1/2W)} - \frac{\sin(2\pi W(t-1/2W))}{2\pi W(t-1/2W)}$$

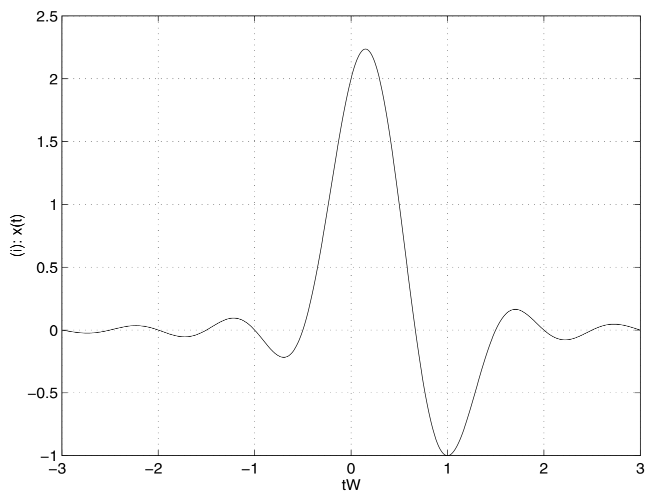
and :

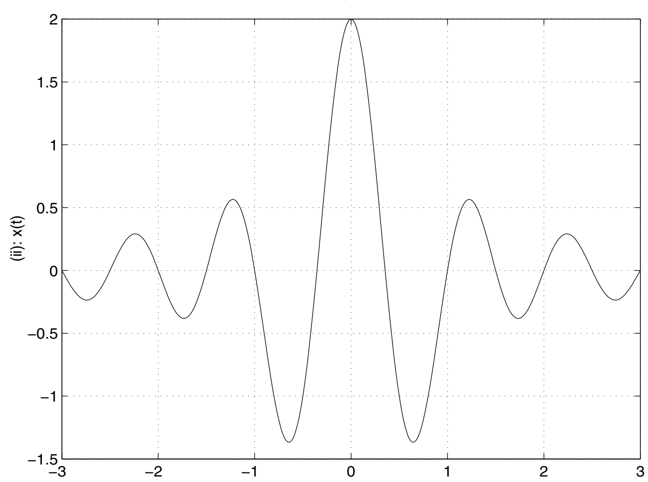
$$X(f) = \frac{1}{2W}\left[2 - e^{-j\pi f/W} - e^{+j\pi f/W}\right] = \frac{1}{2W}\left[2 - 2\cos\frac{\pi f}{W}\right] = \frac{1}{W}\left[1 - \cos\frac{\pi f}{W}\right],\ |f| \leq W$$

The plot of $|X(f)|$ is given in the following figure :



(b) Based on the results obtained in part (a) :





(c) The possible received levels at the receiver are given by :
(i)

$$B_n = 2I_n + I_{n-1} - I_{n-2}$$

where $I_m = \pm 1$. Hence :

$$\begin{array}{c} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{array}$$

(ii)

$$B_n = 2I_n - I_{n-1} - I_{n+1}$$

where $I_m = \pm 1$. Hence :

$$\begin{array}{c} P(B_n = 0) = 1/4 \\ P(B_n = -2) = 1/4 \\ P(B_n = 2) = 1/4 \\ P(B_n = -4) = 1/8 \\ P(B_n = 4) = 1/8 \end{array}$$

Problem 9.11 :

The bandwidth of the bandpass channel is $W = 4$ KHz. Hence, the rate of transmission should be less or equal to 4000 symbols/sec. If a 8-QAM constellation is employed, then the required symbol rate is $R = 9600/3 = 3200$. If a signal pulse with raised cosine spectrum is used for shaping, the maximum allowable roll-off factor is determined by :

$$1600(1 + \beta) = 2000$$

which yields $\beta = 0.25$. Since β is less than 50%, we consider a larger constellation. With a 16-QAM constellation we obtain :

$$R = \frac{9600}{4} = 2400$$

and :

$$1200(1 + \beta) = 2000$$

or $\beta = 2/3$, which satisfies the required conditions. The probability of error for an M-QAM constellation is given by :

$$P_M = 1 - (1 - P_{\sqrt{M}})^2$$

where :

$$P_{\sqrt{M}} = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left[\sqrt{\frac{3\mathcal{E}_{av}}{(M-1)N_0}}\right]$$

With $P_M = 10^{-6}$ we obtain $P_{\sqrt{M}} = 5 \times 10^{-7}$ and therefore using the last equation and the table of values for the $Q(\cdot)$ function, we find that the average transmitted energy is :

$$\mathcal{E}_{av} = 24.70 \times 10^{-9}$$

Note that if the desired spectral characteristic $X_{rc}(f)$ is split evenly between the transmitting and receiving filter, then the energy of the transmitting pulse is :

$$\int_{-\infty}^{\infty} g_T^2(t)dt = \int_{-\infty}^{\infty} |G_T(f)|^2 df = \int_{-\infty}^{\infty} X_{rc}(f)df = 1$$

Hence, the energy $\mathcal{E}_{av} = P_{av}T$ depends only on the amplitude of the transmitted points and the symbol interval T. Since $T = \frac{1}{2400}$, the average transmitted power is :

$$P_{av} = \frac{\mathcal{E}_{av}}{T} = 24.70 \times 10^{-9} \times 2400 = 592.8 \times 10^{-7}$$

If the points of the 16-QAM constellation are evenly spaced with minimum distance between them equal to d, then there are four points with coordinates $(\pm\frac{d}{2}, \pm\frac{d}{2})$, four points with coordinates $(\pm\frac{3d}{2}, \pm\frac{3d}{2})$, and eight points with coordinates $(\pm\frac{3d}{2}, \pm\frac{d}{2})$, or $(\pm\frac{d}{2}, \pm\frac{3d}{2})$. Thus, the average transmitted power is :

$$P_{av} = \frac{1}{2 \times 16} \sum_{i=1}^{16} (A_{mc}^2 + A_{ms}^2) = \frac{1}{32}\left[4 \times \frac{d^2}{2} + 4 \times \frac{9d^2}{2} + 8 \times \frac{10d^2}{4}\right] = \frac{5}{4}d^2$$

Since $P_{av} = 592.8 \times 10^{-7}$, we obtain

$$d = \sqrt{4\frac{P_{av}}{5}} = 0.0069$$

Problem 9.22 :

(a) The power spectral density of $X(t)$ is given by (see (4-4-12))

$$\Phi_x(f) = \frac{1}{T}\Phi_a(f)|G_T(f)|^2$$

The Fourier transform of $g(t)$ is

$$G_T(f) = \mathcal{F}[g(t)] = AT\frac{\sin \pi f T}{\pi f T}e^{-j\pi f T}$$

Hence,

$$|G_T(f)|^2 = (AT)^2 \text{sinc}^2(fT)$$

and therefore,

$$\Phi_x(f) = A^2 T \Phi_a(f)\text{sinc}^2(fT) = A^2 T \text{sinc}^2(fT)$$

(b) If $g_1(t)$ is used instead of $g(t)$ and the symbol interval is T, then

$$\begin{aligned} \Phi_x(f) &= \frac{1}{T}\Phi_a(f)|G_{2T}(f)|^2 \\ &= \frac{1}{T}(A2T)^2 \text{sinc}^2(f2T) = 4A^2 T \text{sinc}^2(f2T) \end{aligned}$$

(c) If we precode the input sequence as $b_n = a_n + \alpha a_{n-3}$, then

$$\phi_b(m) = \begin{cases} 1+\alpha^2 & m = 0 \\ \alpha & m = \pm 3 \\ 0 & \text{otherwise} \end{cases}$$

and therefore, the power spectral density $\Phi_b(f)$ is

$$\Phi_b(f) = 1 + \alpha^2 + 2\alpha\cos(2\pi f 3T)$$

To obtain a null at $f = \frac{1}{3T}$, the parameter α should be such that

$$1 + \alpha^2 + 2\alpha\cos(2\pi f 3T)_{|_{f=\frac{1}{3}}} = 0 \Longrightarrow \alpha = -1$$

(c) The answer to this question is no. This is because $\Phi_b(f)$ is an analytic function and unless it is identical to zero it can have at most a countable number of zeros. This property of the analytic functions is also referred as the theorem of isolated zeros.

Problem 9.26 :

Let $X(z)$ denote the $\mathcal{Z}$-transform of the sequence x_n, that is

$$X(z) = \sum_n x_n z^{-n}$$

Then the precoding operation can be described as

$$P(z) = \frac{D(z)}{X(z)} \quad \text{mod} - M$$

where $D(z)$ and $P(z)$ are the $\mathcal{Z}$-transforms of the data and precoded dequences respectively. For example, if $M = 2$ and $X(z) = 1 + z^{-1}$ (duobinary signaling), then

$$P(z) = \frac{D(z)}{1 + z^{-1}} \Longrightarrow P(z) = D(z) - z^{-1}P(z)$$

which in the time domain is written as

$$p_n = d_n - p_{n-1}$$

and the subtraction is mod-2.
However, the inverse filter $\frac{1}{X(z)}$ exists only if x_0, the first coefficient of $X(z)$ is relatively prime with M. If this is not the case, then the precoded symbols p_n cannot be determined uniquely from the data sequence d_n.
In the example given in the book, where $x_0 = 2$ we note that whatever the value of d_n (0 or 1), the value of ($2d_n$ mod 2) will be zero, hence this precoding scheme cannot work.

Problem 9.29 :

A 4-PAM modulation can accomodate $k = 2$ bits per transmitted symbol. Thus, the symbol interval duration is :

$$T = \frac{k}{9600} = \frac{1}{4800} \text{ sec}$$

Since, the channel's bandwidth is $W = 2400 = \frac{1}{2T}$, in order to achieve the maximum rate of transmission, $R_{\max} = \frac{1}{2T}$, the spectrum of the signal pulse should be :

$$X(f) = T\Pi\left(\frac{f}{2W}\right)$$

Then, the magnitude frequency response of the optimum transmitting and receiving filter is (see (9-2-81))

$$|G_T(f)| = |G_R(f)| = \left[1 + \left(\frac{f}{2400}\right)^2\right]^{\frac{1}{4}} \Pi\left(\frac{f}{2W}\right) = \begin{cases} \left[1 + \left(\frac{f}{2400}\right)^2\right]^{\frac{1}{4}}, & |f| < 2400 \\ 0, & \text{otherwise} \end{cases}$$