14.1 The scattering function $S(\tau; \lambda)$ for a fading multipath channel is nonzero for the range of values $0 \leqslant \tau \leqslant 1$ ms and -0.1 Hz $\leqslant \lambda \leqslant 0.1$ Hz. Assume that the scattering function is approximately uniform in the two variables.

a) Give numerical values for the following parameters:
  (i) The multipath spread of the channel.
  (ii) The Doppler spread of the channel.
  (iii) The coherence time of the channel.
  (iv) The coherence bandwidth of the channel.
  (v) The spread factor of the channel.

b) Explain the meaning of the following, taking into consideration the answers given in (a):
  (i) The channel is frequency-nonselective.
  (ii) The channel is slowly fading.
  (iii) The channel is frequency-selective.

c) Suppose that we have a frequency allocation (bandwidth) of 10 kHz and we wish to transmit at a rate of 100 bits/s over this channel. Design a binary communication system with frequency diversity. In particular, specify
  (i) The type of modulation.
  (ii) The number of subchannels.
  (iii) The frequency separation between adjacent carriers.
  (iv) The signaling interval used in your design.
  Justify your choice of parameters.

14.2 Consider a binary communication system for transmitting a binary sequence over a fading channel. The modulation is orthogonal FSK with third-order frequency diversity ($L = 3$). The demodulator consists of matched filters followed by square-law detectors. Assume that the FSK carriers fade independently and identically according to a Rayleigh envelope distribution. The additive noises on the diversity signals are zero-mean Gaussian with autocorrelation functions $\frac{1}{2}E[z_k^*(t)z_k(t+\tau)] = N_0\delta(\tau)$. The noise processes are mutually statistically independent.

a) The transmitted signal may be viewed as binary FSK with square-law detection, generated by a repetition code of the form

$$1 \to \mathbf{C}_1 = [1 \quad 1 \quad 1], \qquad 0 \to \mathbf{C}_0 = [0 \quad 0 \quad 0]$$

Determine the error rate performance P_{2h} for a hard-decision decoder following the square-law-detected signals.

b) Evaluate P_{2h} for $\bar{\gamma}_c = 100$ and 1000.

c) Evaluate the error rate P_{2s} for $\bar{\gamma}_c = 100$ and 1000 if the decoder employs soft-decision decoding.

d) Consider the generalization of the result in (a). If a repetition code of block length L (L odd) is used, determine the error probability P_{2h} of the hard-decision decoder and compare that with P_{2s}, the error rate of the soft-decision decoder. Assume $\bar{\gamma} \gg 1$.

14.3 Suppose that the binary signal $\pm s_l(t)$ is transmitted over a fading channel and the received signal is

$$r_l(t) = \pm a s_l(t) + z(t), \qquad 0 \leqslant t \leqslant T$$

where $z(t)$ is zero-mean white Gaussian noise with autocorrelation function

$$\phi_{zz}(\tau) = N_0\delta(\tau)$$

The energy in the transmitted signal is $\mathcal{E} = \frac{1}{2}\int_0^T |s_l(t)|^2\, dt$. The channel gain a is specified by the probability density function

$$p(a) = 0.1\delta(a) + 0.9\delta(a-2)$$

a) Determine the average probability of error P_2 for the demodulator that employs a filter matched to $s_l(t)$.

b) What value does P_2 approach as $\mathcal{E}/N_0$ approaches infinity?

c) Suppose that the same signal is transmitted on two statistically *independently fading* channels with gains a_1 and a_2, where

$$p(a_k) = 0.1\delta(a_k) + 0.9\delta(a_k - 2), \qquad k = 1, 2$$

The noises on the two channels are statistically independent and identically distributed. The demodulator employs a matched filter for each channel and simply adds the two filter outputs to form the decision variable. Determine the average P_2.

d) For the case in (c) what value does P_2 approach as $\mathcal{E}/N_0$ approaches infinity?

14.4 A multipath fading channel has a multipath spread of $T_m = 1$ s and a Doppler spread $B_d = 0.01$ Hz. The total channel bandwidth at band-pass available for signal transmission is $W = 5$ Hz. To reduce the effects of intersymbol interference, the signal designer selects a pulse duration $T = 10$ s.

a) Determine the coherence bandwidth and the coherence time.

b) Is the channel frequency selective? Explain.

c) Is the channel fading slowly or rapidly? Explain.

d) Suppose that the channel is used to transmit binary data via (antipodal) coherently detected PSK in a frequency diversity mode. Explain how you would use the available channel bandwidth to obtain frequency diversity and determine how much diversity is available.

e) For the case in (d), what is the *approximate* SNR required per diversity to achieve an error probability of 10^{-6}?

f) Suppose that a wideband signal is used for transmission and a RAKE-type receiver is used for demodulation. How many taps would you use in the RAKE receiver?

g) Explain whether or not the RAKE receiver can be implemented as a coherent receiver with maximal ratio combining.

h) If binary orthogonal signals are used for the wideband signal with square-law postdetection combining in the RAKE receiver, what is the *approximate* SNR required to achieve an error probability of 10^{-6}? (Assume that all taps have the same SNR.)

14.11 A DS spread spectrum system is used to resolve the multipath signal components in a two-path radio signal propagation scenario. If the path length of the secondary path is 300 m longer than that of the direct path, determine the minimum chip rate necessary to resolve the multipath components.