Continuous-Phase Frequency-Shift Keying (CPFSK) & Continuous-Phase Modulation (CPM)

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Continuous Phase Frequency Shift Keying (CPFSK)

- CPM: the phase of the signal is constrained to be continuous \( \Rightarrow \) Memory.

- For FSK, there are two options

\[
f_1, f_2, f_M
\]

\[
m \Delta f, 1 \leq m \leq M, \quad M=2^k
\]

Large spectral side lobes outside of the main spectral band of the signal (bandwidth requirements)

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Representation of CPFSK

• \{a_n\} information sequence 001010111101
• \{l_n\} sequence of amplitudes obtained by mapping k-bit blocks of binary digits from the information sequence \{a_n\} into the amplitude levels \pm 1, \pm 3, \ldots, \pm(M - 1).
• \(g(t)\) pulse shape. Example: rectangular pulse of amplitude \(1/2T\) and duration \(T\) seconds.
• \(d(t)\) is used to frequency-modulate the carrier
• \(\nu(t)\) the equivalent lowpass waveform
• \(f_d\) is the peak frequency deviation and
• \(\phi_0\) is the initial phase of the carrier
• The carrier-modulated signal corresponding
• \(\phi(t; I)\) represents the time-varying phase of the carrier

\[
d(t) = \sum_n I_n g(t - nT)
\]

\[
v(t) = \sqrt{\frac{2\xi}{T}} e^{j \left[4\pi f_d \int_{-\infty}^{t} d(\tau) d\tau + \phi_0\right]}
\]

\[
s(t) = \sqrt{\frac{2\xi}{T}} \cos \left[2\pi f_c t + \phi(t; I) + \phi_0\right]
\]

\[
\phi(t; I) = 4\pi f_d \int_{-\infty}^{t} d(\tau) d\tau
\]

\[
= 4\pi f_d \int_{-\infty}^{t} \left[\sum_n I_n g(\tau - nT)\right] d\tau
\]
Although $d(t)$ contains discontinuities, the integral of $d(t)$ is continuous.

The phase of the carrier in the interval $nT \leq t \leq (n + 1)T$

$$\phi(t; I) = 4\pi f_d T \int_{-\infty}^{t} d(\tau) d\tau = 4\pi f_d T \int_{-\infty}^{t} \left[ \sum_{n} I_n g(\tau - nT) \right] d\tau$$

$$\phi(t; I) = 4\pi f_d T \int_{-\infty}^{t} \left[ \sum_{k=-\infty}^{n-1} I_k g(\tau - kT) \right] d\tau + 4\pi f_d T \int_{-\infty}^{t} I_n g(\tau - nT) d\tau$$

Note that the $g(t)$ is assumed to be rectangular pulse of amplitude $1/2T$ and duration $T$ seconds.

$$\phi(t; I) = 2\pi f_d T \int_{-\infty}^{t} \left[ \sum_{k=-\infty}^{n-1} I_k \right] d\tau + 2\pi f_d T q(\tau - nT) I_n = \theta_n + 2\pi h I_n q(\tau - nT)$$

$h = 2f_d T$, the parameter $h$ is called the modulation index.

$$\theta_n = \pi h \sum_{k=-\infty}^{n-1} I_k$$

We observe that $\theta_n$ represents the accumulation (memory) of all symbols up to time $(n - 1)T$.
Continuous-Phase Modulation (CPM)

• CPFSK is a special case of CPM

$$\phi(t; I) = 2\pi \sum_{k=-\infty}^{n} I_k h_k q(t - kT), \quad nT \leq t \leq (n + 1)T$$

When $h_k = h$ for all $k$, the modulation index is fixed for all symbols. When the modulation index varies from one symbol to another, the signal is called multi-$h$ CPM.

If $g(t) = 0$ for $t > T$, the signal is called full-response CPM. If $g(t) \neq 0$ for $t > T$, the modulated signal is called partial-response CPM.
Pulse Shapes for CPM

Pulse shapes for full response CPM

Pulse shapes for partial response CPM.

REC=Rectangle
RC=Raised Cosine

For \( L > 1 \), additional memory is introduced in the CPM signal by the pulse \( g(t) \).

Some Commonly Used CPM Pulse Shapes

<table>
<thead>
<tr>
<th>Pulse Shape</th>
<th>Pulse Function</th>
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<tbody>
<tr>
<td>LREC</td>
<td>( g(t) = \begin{cases} \frac{1}{2LT} &amp; 0 \leq t \leq LT \ 0 &amp; \text{otherwise} \end{cases} )</td>
</tr>
<tr>
<td>LRC</td>
<td>( g(t) = \begin{cases} \frac{1}{2LT} \left(1 - \cos \frac{2\pi t}{LT}\right) &amp; 0 \leq t \leq LT \ 0 &amp; \text{otherwise} \end{cases} )</td>
</tr>
<tr>
<td>GMSK</td>
<td>( g(t) = \frac{g\left(2\pi B(t-\frac{L}{2})\right) - g\left(2\pi B(t+\frac{L}{2})\right)}{2\pi B} )</td>
</tr>
</tbody>
</table>
Gaussian minimum-shift keying (GMSK)

- GMSK with $BT = 0.3$ is used in the European digital cellular communication system, called **GSM**. We observe that when $BT = 0.3$, the GMSK pulse may be truncated at $|t| = 1.5T$ with a relatively small error incurred for $t > 1.5T$.

$B$, which represents the $-3$-$dB$ bandwidth of the Gaussian pulse.
Phase trajectory for CPFSK.

Phase trajectory for binary CPFSK

Phase trajectory for quaternary CPFSK
Phase trajectories for binary CPFSK (dashed) and binary, partial response CPM based on raised cosine pulse of length $3T$ (solid).

[From Sundberg (1986), © 1986 IEEE.]
Phase cylinder for binary CPM with $h = \frac{1}{2}$ and a raised cosine pulse of length $3T$. [From Sundberg (1986), © 1986 IEEE.]

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The phase transitions from one state to another are not true phase trajectories. They represent phase transitions for the (terminal) states at the time instants $t = nT$. 

State trellis for binary CPFSK with $h = \frac{1}{2}$
Minimum Shift Keying (MSK)

MSK is a special form of binary CPFSK (and, therefore, CPM) in which the modulation index $h = \frac{1}{2}$ and $g(t)$ is a rectangular pulse of duration $T$

$$\phi(t; I) = \frac{1}{2} \pi \sum_{k=-\infty}^{n-1} I_k + \pi I_n q(t - nT)$$

$$= \theta_n + \frac{1}{2} \pi I_n \left( \frac{t - nT}{T} \right), \quad nT \leq t \leq (n + 1)T$$

$$s(t) = A \cos \left[ 2\pi f_c t + \theta_n + \frac{1}{2} \pi I_n \left( \frac{t - nT}{T} \right) \right]$$

$$= A \cos \left[ 2\pi \left( f_c + \frac{1}{4T} I_n \right) t - \frac{1}{2} n\pi I_n + \theta_n \right], \quad nT \leq t \leq (n + 1)T$$

$$f_1 = f_c - \frac{1}{4T} \quad s_i(t) = A \cos \left[ 2\pi f_i t + \theta_n + \frac{1}{2} n\pi (-1)^{i-1} \right], \quad i = 1, 2$$
Offset QPSK (OQPSK)

A possible mapping for QPSK

The in-phase and quadrature components for QPSK
Possible phase transitions in QPSK signaling

Possible phase transitions in OQPSK signaling

\[ s(t) = A \left[ \left( \sum_{n=-\infty}^{\infty} I_{2n}g(t - 2nT) \right) \cos 2\pi f_c t + \left( \sum_{n=-\infty}^{\infty} I_{2n+1}g(t - 2nT - T) \right) \sin 2\pi f_c t \right] \]

\[ s_f(t) = A \left[ \sum_{n=-\infty}^{\infty} I_{2n}g(t - 2nT) \right] - j \left[ \sum_{n=-\infty}^{\infty} I_{2n+1}g(t - 2nT - T) \right] \]
$s(t) = A \left[ \sum_{n=-\infty}^{\infty} I_{2n} g(t - 2nT) \right] - j \left[ \sum_{n=-\infty}^{\infty} I_{2n+1} g(t - 2nT - T) \right]$ 

MSK may also be represented as a form of OQPSK. Specifically, we may express the equivalent lowpass digitally modulated MSK signal

$$g(t) = \begin{cases} 
\sin \frac{\pi t}{2T} & 0 \leq t \leq 2T \\
0 & \text{otherwise}
\end{cases}$$
Comparing MSK, OQPSK, and QPSK

(a) MSK

(b) Offset QPSK

(c) QPSK

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