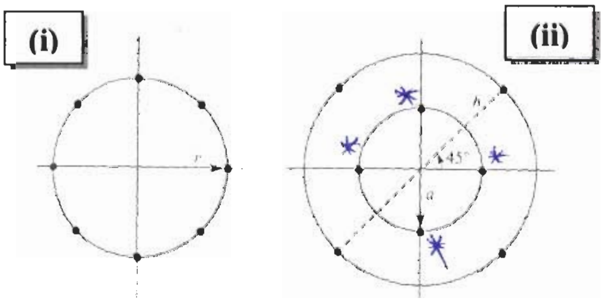
Problem 4: Signal Representation and Passband Communication (Ch5&Ch6) (8 points)

Consider the octal signal point constellations in the figure
Let the basis functions be as follows:



$$\phi_1 = \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \quad ,0 \le t \le T$$

where $f_c = \frac{2}{T}$

$$\phi_2 = \sqrt{\frac{2}{T}}\sin(2\pi f_c t) \quad ,0 \le t \le T$$

a) If one of these constellations is used, determine the symbol rate if the desired rate is 90 Mbit/s.

from the constellation 8 points ⇒ $\log_2 8 = 3$ bits/symbol.

Symbol rate = $90 \frac{Mbit}{s} \cdot \frac{symbol}{3\ bits} = 30$ MSymbol/s

(1 point)

b) What types of modulation are being used for constellation **(i)** and **(ii)** (ASK, PSK, QAM or FSK)? *Justify your answer.*

constellation **(i)**: PSK, all points have same amplitude & differ in the phase (circle) } Same freq. ϕ_1 & ϕ_2

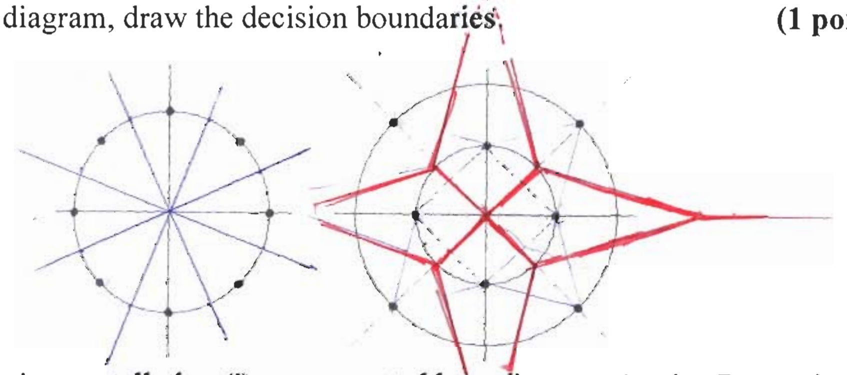
constellation **(ii)**: QAM, points differ in Amp & phase

(0.5 point)

c) Consider constellation **(ii)**, is it possible to assign three data bits to each point of the signal constellation such that nearest (adjacent) points differ in only one bit position. **(0.5 point)**
If possible show your assignment, if not explain why.

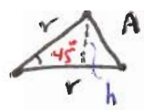
No it is not possible because the points indicated with * have four different constellations which are close & Equidistanse.

d) On the new constellation diagram, draw the decision boundaries. **(1 point)**

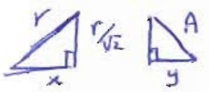


e) The adjacent signal points in constellation **(i)** are separated by a distance A units. Determine the radius r of the circle as function of A. *Hint : you may use trigonometry* **(1 points)**

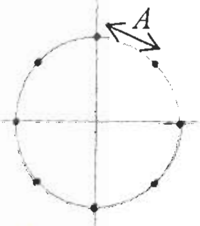
$h = r\cos 45 = \frac{r}{\sqrt{2}}$



We have two right angle triangle.



$x^2 = r^2 - \frac{r^2}{2} = \frac{r^2}{2} \Rightarrow x = \frac{r}{\sqrt{2}}$



$y^2 = A^2 - \frac{r^2}{2}$

$x + y = r \Rightarrow y = r - x = r - \frac{r}{\sqrt{2}}$

$\left(r - \frac{r}{\sqrt{2}}\right)^2 = A^2 - \frac{r^2}{2} \Rightarrow r^2 - \sqrt{2}r^2 + \frac{r^2}{2} = A^2 - \frac{r^2}{2} \Rightarrow r^2(2-\sqrt{2}) = A^2$

$\Rightarrow r = \frac{A}{\sqrt{2-\sqrt{2}}} = 1.306\,A$

f) What is the average energy per bit for constellation **(i)** as function of A? **(1 point)**

$E_{per\ symbol} = r^2$ Energy per bit $= \frac{r^2}{3} = \frac{A^2}{3(2-\sqrt{2})}$

g) The nearest-neighbor signal points in constellation **(ii)** are separated by A units. **Show** that the radii a and b of the inner and outer circles are given by **(1.5 points)**

$$a = \frac{A}{\sqrt{2}}, \qquad b = A\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)$$

for a

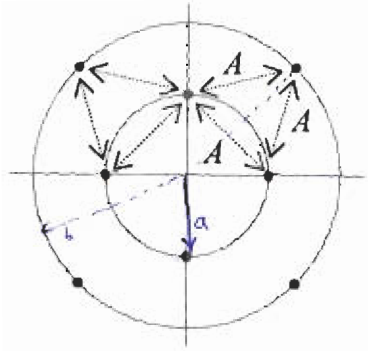
$$a^2 + a^2 = A^2 \Rightarrow a = \frac{A}{\sqrt{2}}$$

for b

$$y^2 = A^2 - \frac{A^2}{4} = \frac{3}{4}A^2$$

$$x^2 = a^2 - \frac{A^2}{4} = \frac{A^2}{2} - \frac{A^2}{4} = \frac{A^2}{4}$$

$$x + y = A\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = A\left(\frac{1+\sqrt{3}}{2}\right)$$



h) Determine the average transmitter powers for the two signal constellations and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume that all points are equally probable) **(1.5 points)**

(ii) $E_{\text{average symbol}} = \frac{4a^2 + 4b^2}{8} = \frac{1}{2}(a^2+b^2) = \frac{1}{2}\left(\frac{A^2}{2} + A^2\, 1.866\right) = 1.18\, A^2$

(i) $E_{\text{symbol}} = 1.707\, A^2$ $\qquad \frac{(ii)}{(i)} = \frac{1.18}{1.707} = 70\,\%$ $\qquad -1.6$ dB lower.

note the difference in Error probability

ii more energy efficient

<u>Problem 5:</u> Baseband Pulse Transmission (Ch4) (3 points)

An analog signal is sampled, quantized, and encoded into a binary PCM wave. The number of quantization levels used is 128. A synchronizing pulse is added at the end of each code word representing a sample of the analog signal. The resulting PCM wave is transmitted over a channel of bandwidth ~~#~~ 12 kHz using a 16-ary PAM system with raised-cosine spectrum. The rolloff factor is 0.5.

(a) Find the rate (bits/sec) at which information is transmitted through the channel. **(1.5 points)**

$BW = (1+r)\frac{R}{2} \Rightarrow 12K = \left(\frac{1+0.5}{2}\right)R \Rightarrow R = 16\ K$ Symbol/sec

16-ary $\Rightarrow \log_2 16 = 4$ bits/symbol

rate $\left(\frac{\text{bits}}{\text{sec}}\right) = 16K$ Symbol/sec $\cdot\ 4\ \frac{\text{bits}}{\text{symbol}} = 64\ \frac{\text{K bit}}{\text{sec}}$.

(b) <u>Find</u> the rate at which the analog signal is sampled. <u>What</u> is the maximum possible value for the highest frequency component of the analog signal? **(1.5 points)**

# of bits per symbol $= \log_2 128 = 7$ bits with synchronization 8 bits

8 bits/sample $\Rightarrow$ Sampling rate $= \frac{64\ \text{K bit/sec}}{8\ \text{bit/sample}} = 8\ K\ \frac{\text{samples}}{\text{sec}}$.

maximum possible value for the highest freq. Component $= \frac{8K}{2} = 4$ KHz (Nyquist rate).