King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE571: Digital Communications I (111)

Major Exam II

December 3rd, 2011
5:00 PM-6:15 PM
Building 59-2022

Serial Number 0

Name:______Key_______

ID:_______________________________

Question	Mark
1	/10
2	/10
3	/10
Total	/30

Instructions:
1. This is a closed-books/notes exam.
2. Read the questions carefully. Plan which question to start with.
3. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH.
4. Work in your own and Show ALL IMPORTANT STEPS.
5. Strictly no mobile phones are allowed.

Useful Formulas:

$$S_{v_l}(f) = \frac{1}{T}|G(f)|^2 S_I(f)$$

$$S_I(f) = \sum_{k=-\infty}^{+\infty} R_I(k)\exp(-j2\pi fkT)$$

$$\varepsilon_g = \int_{-\infty}^{+\infty} |g(t)|^2 dt,$$

$$X(f) = \int_{-\infty}^{+\infty} x(t)\exp(-j2\pi ft)dt$$

Good luck

Dr. Ali Muqaibel

<u>Problem 1:</u> In the following statements, correct the mistake (minimum change).

a. In a set of biorthognal signals, the correlation between any waveforms can take one of two values 0 or +1.

In a set of biorthognal signals, the correlation between any waveforms can take one of two values 0 or -1.

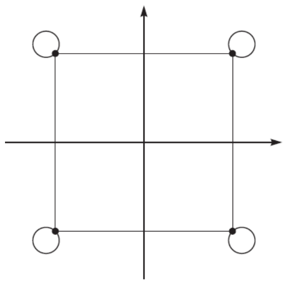
b. Simplex signals require half the energy compared with non simplex for the same error performance.

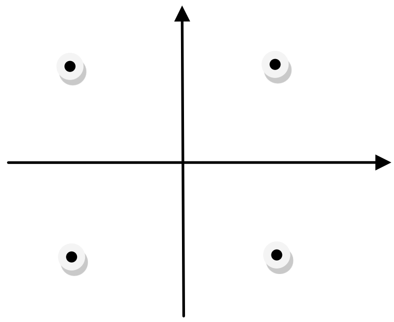
Simplex signals require <u>less</u> energy compared with non simplex for the same error performance.

c. QPSK is a special case of continuous phase frequency shift keying (CPFSK)

GMSK is a special case of continuous phase frequency shift keying (CPFSK) .some may wrote MSK instead of GMSK

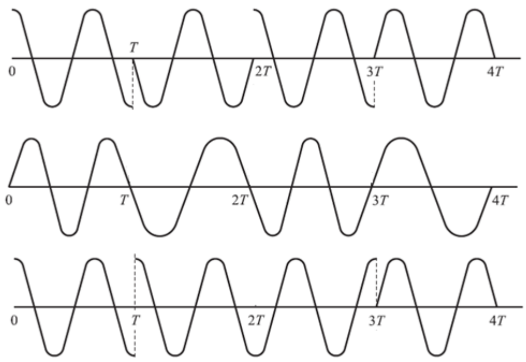
Complete the phase transition diagram for OQPSK (1 point)





For the same data sequence, identify the waveform as (FSK, QPSK, OQPSK, MSK, ASK) indicate all correct answers. **(2 points)**

Uppper (OQPSK), Middle (FSK, MSK), Lower (QPSK)



For 8-PSK, sketch the constellation diagram <u>and</u> find the minimum distance in terms of the energy per bit (E_b). **(4 points)**

$$E_s = 3E_b$$

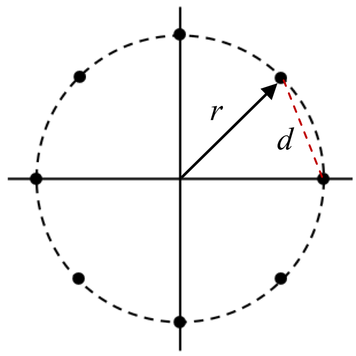
$r = \sqrt{E_s} = \sqrt{3E_b}$

With geometry (required but not shown here), we can show that d the minimum distance

$$d = \frac{r}{1.306}$$

Hence

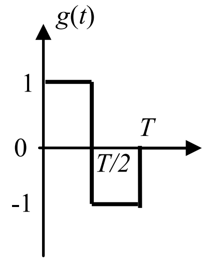
$$d = 1.326\sqrt{E_b}$$



Problem 2: Spectrum of Digitally Modulated Signals (10 points)

The information sequence $\{a_n\}_{n=-\infty}^{\infty}$ is a sequence of iid random variables, each taking values +1 and -1with equal probability. This sequence is to be transmitted at baseband as

$$s(t) = \sum_{n=-\infty}^{\infty} a_n\, g(t-nT)$$



where $g(t)$ is shown in the figure.

(a) Find the power spectral density of $s(t)$.

Since : $\mu_a = 0,\ \sigma_a^2 = 1$, we have : $\Phi_{ss}(f) = \frac{1}{T}|G(f)|^2$. But :

$$\begin{aligned}
G(f) &= \frac{T}{2}\frac{\sin \pi f T/2}{\pi f T/2}e^{-j2\pi f T/4} - \frac{T}{2}\frac{\sin \pi f T/2}{\pi f T/2}e^{-j2\pi f 3T/4} \\
&= \frac{T}{2}\frac{\sin \pi f T/2}{\pi f T/2}e^{-j\pi f T}(2j\sin \pi f T/2) \\
&= jT\frac{\sin^2 \pi f T/2}{\pi f T/2}e^{-j\pi f T} \Rightarrow \\
|G(f)|^2 &= T^2\left(\frac{\sin^2 \pi f T/2}{\pi f T/2}\right)^2 \Rightarrow \\
\Phi_{ss}(f) &= T\left(\frac{\sin^2 \pi f T/2}{\pi f T/2}\right)^2
\end{aligned}$$

(b) We would like to have a zero in the power spectrum at f=1/T. To do that, we use a preceding scheme by introducing $\boldsymbol{b_n = a_n - k a_{n-1}}$, where k is some constant, and then transmit the $\{b_n\}$ sequence using the same $g(t)$. Is it possible to choose k to produce a frequency null at f=1/T? if yes what are the appropriate values <u>and</u> the resulting power spectrum.

$\frac{1}{T}|G(f)|^2\,\Phi_{bb}(f)$. But :

$$\begin{aligned}
\phi_{bb}(m) &= E\left[b_{n+m}b_n\right] \\
&= E\left[a_{n+m}a_n\right] + kE\left[a_{n+m-1}a_n\right] + kE\left[a_{n+m}a_{n-1}\right] + k^2E\left[a_{n+m-1}a_{n-1}\right] \\
&= \left\{\begin{array}{cc} 1+k^2, & \text{m} = 0 \\ k, & \text{m} = \pm 1 \\ 0, & \text{o.w.} \end{array}\right\}
\end{aligned}$$

Hence :

$$\Phi_{bb}(f) = \sum_{m=-\infty}^{\infty} \phi_{bb}(m)e^{-j2\pi f m T} = 1 + k^2 + 2k\cos 2\pi f T$$

We want :

$$\Phi_{ss}(1/T) = 0 \Rightarrow \Phi_{bb}(1/T) = 0 \Rightarrow 1 + k^2 + 2k = 0 \Rightarrow k = -1$$

and the resulting power spectrum is :

$$\Phi_{ss}(f) = 4T\left(\frac{\sin^2 \pi f T/2}{\pi f T/2}\right)^2 \sin^2 \pi f T$$

(c) Now assume we want to have zeros at all multiples of f_0=1/4T. It is not possible to use the previous pre-coding. What kind of pre-coding to achieve the desired results? *Justify* ***(Bonus)***

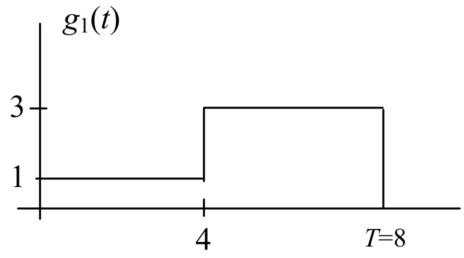
(c) The requirement for zeros at $f = l/4T,\ l = \pm1, \pm2, ...$ means : $\Phi_{bb}(l/4T) = 0 \Rightarrow 1 + k^2 + 2k\cos \pi l/2 = 0$, which cannot be satisfied for all l. We can avoid that by using precoding in the form :$b_n = a_n + k a_{n-4}$. Then :

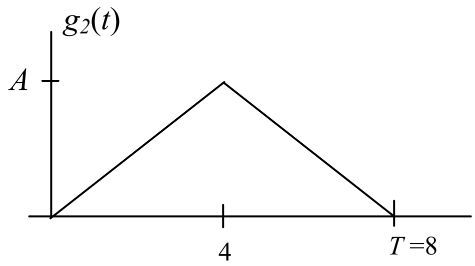
$$\phi_{bb}(m) = \left\{\begin{array}{cc} 1+k^2, & \text{m} = 0 \\ k, & \text{m} = \pm 4 \\ 0, & \text{o.w.} \end{array}\right\} \Rightarrow \Phi_{bb}(f) = 1 + k^2 + 2k\cos 2\pi f 4T$$

and , similarly to (b), a value of $k = -1$, will zero this spectrum in all multiples of $1/4T$.

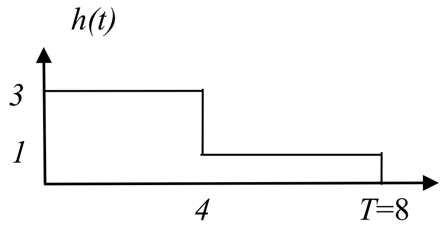
Problem 3: Matched Filter Receiver (10 points)

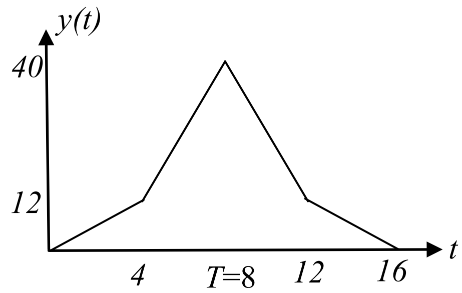
A signal $g_1(t)$ is transmitted over an AWGN channel and received using a matched filter receiver.





(a) Sketch the impulse response of the matched filter, **and** the matched filter response to $g_1(t)$. **(5 points)**





(b) If the input signal $g_1(t)$ is corrupted with zero mean white noise with variance 0.1 , what is the output SNR of the matched filter. **(2 points)**

If the variance of noise is given =0.1, it must be band limited and the answer will be function of B

Power of the zero mean filtered noise $=\sigma^2 = N_0 B = 0.1 \Rightarrow N_0 = \frac{0.1}{B}$ depends on B

SNR=$2\varepsilon/N_0$=$2(40)B/0.1$=$800B$ Note that as we increase the noise bandwidth the SNR ration improves because the noise power is fixed which mean N_0 is less.

(c) What is the advantage of using matched filter over correlator? **(1 point)**

More immune to time jitter.

(d) Another system operates by transmitting the triangle pulse $g_2(t)$ over the same channel, then feeding the received signal to a filter matched to $g_2(t)$. Determine the peak value A that will make the performance of the two systems identical with respect to (SNR) at the output of the matched filter. **(2 points)**

To get the same performance, the energy of the two signals must be the same. The energy of $g_1(t)$ is 40. The energy of $g_2(t)$ notice the symmetry.

$$= 2\int_0^4 \left(\frac{A}{4}t\right)^2 dt = \frac{2A^2}{(16)(3)}(64) = \frac{8}{3}A^2$$

Equate to 40 yields

$$\frac{8}{3}A^2\text{=}40 \ \Rightarrow \text{A=}A = \pm\sqrt{15}$$