King Fahd University of Petroleum & Minerals
Electrical Engineering Department
EE571: Digital Communications I (111)

Major Exam I

Serial # 0

October 22nd, 2011
5:15 PM-6:30 PM
Building 59-2022

Name: KEY

ID:

Question	Mark
1	/7
2	/7
3	/11
4	/9
Total	**/34**

Instructions:

1. This is a closed-books/notes exam.
2. Read the questions carefully. Plan which question to start with.
3. CLEARLY LABEL ALL SIGNIFICANT VALUES ON BOTH AXIES OF ANY SKETCH
4. Work in your own.
5. Strictly no mobile phones are allowed.

$$\sin u \sin v = \frac{1}{2}[\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2}[\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2}[\sin(u+v) - \sin(u-v)]$$

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Good luck

Dr. Ali Muqaibel

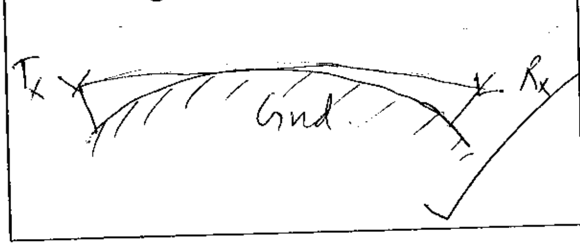
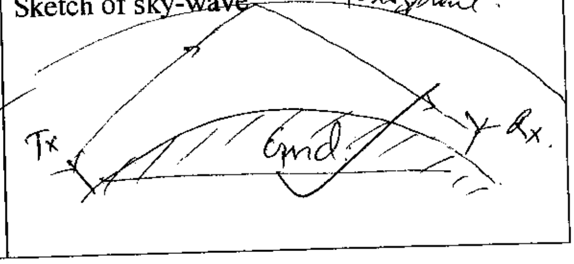
Problem 1: **(1+1+1+1+1+2=7 points)**

a. What is the minimum practical length for a GSM antenna (f_c=900 MHz, $L > \lambda/10$)?

Since $\lambda = \frac{c}{f_c} = \frac{3\times10^8}{9\times10^8} = \frac{1}{3}$; $\lambda/10 = 1/30 \Rightarrow L > \frac{1}{30}$ m.

or $L > 33.33\times10^{-3}$

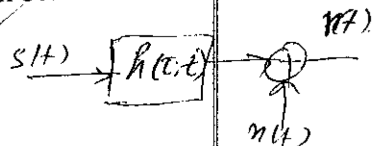
b. Using sketch, clearly explain the difference between ground wave propagation and sky wave propagation.

Sketch for ground-wave	Sketch of sky-wave
	

c. In wave propagation we define skin depth. Explain the relation between skin depth and frequency. *(no need for exact formula)*

They are inversely related Skindepth $\propto 1/f$.

d. For the linear **time-variant** filter with additive noise, write the **integral** relation between the input, $s(t)$, the output $r(t)$, the noise $n(t)$ and the impulse response of the filter.



$r(t) = s(t) * h(\tau;t) + n(t) =$

$r(t) = \int_{-\infty}^{\infty} h(\tau;t)\, s(t-\tau)\, d\tau + n(t)$.

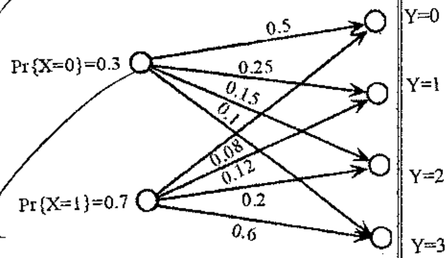
A communication channel with binary input and quaternary output alphabet is shown in the Figure. The input probabilities and the transition probabilities are shown on the figure.

a) What is the probability of Y=0.

$P(Y=0) = P(Y=0/X=0)\,P(X=0) + P(Y=0/X=1)\,P(X=1) = 0.5\times0.3 + 0.08\times0.7$

$= 0.206$ Ans

b) If Y=0 is received what is the probability that X=0 was transmitted? What is the probability the X=1 was transmitted.



This is a posteriori prob.

(a) $P(X=0/Y=0) = \frac{P(Y=0/X=0)\,P(X=0)}{P(Y=0)}$

$= \frac{0.5\times0.3}{0.206} = 0.728$ Ans

Again Aposteriori prob.

(b) $P(X=1/Y=0) = \frac{P(Y=0/X=1)\,P(X=1)}{P(Y=0) = 0.206}$

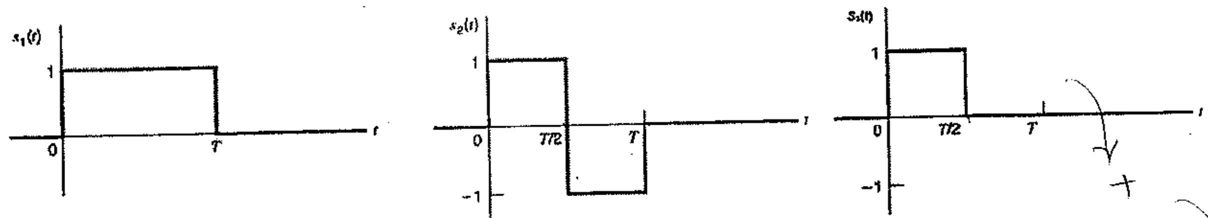
$= \frac{0.08\times0.7}{0.206} = 0.272$ Ans

Problem 2: **(7 points)**

The figure below displays the waveforms of three signals.

a) Using Gram Schmidt orthogonalization procedure, **find** and **sketch** an orthonormal basis for this set of signals. (*use the same order as given*) **(5 points)**

b) Represent the three signals in terms of the basis and show them as vectors in the signal-space diagram. **(2 points)**



(a)

$\rightarrow \phi_1(t) = \dfrac{s_1(t)}{\sqrt{E_1}} = \boxed{\dfrac{s_1(t)}{\sqrt{T}}}$

$E_1 = \int_0^T (1)^2 dt = T.$

$\rightarrow \phi_2'(t) = s_2(t) - \underbrace{\langle s_2(t), \phi_1(t)\rangle}_{c_{21}} \phi_1(t).$

$c_{21} = \int_0^T s_2(t) \dfrac{s_1(t)}{\sqrt{T}} dt = 0$

$\Rightarrow \phi_2'(t) = s_2(t) \rightarrow \phi_2(t) = \dfrac{s_2(t)}{\sqrt{E_2}}$

$E_2 = \int_0^T (1)^2 dt = T.$

$\boxed{\phi_2(t) = \dfrac{s_2(t)}{\sqrt{T}}}$

$\rightarrow \phi_3'(t) = s_3(t) - c_{31}\phi_1(t) - c_{32}\phi_2(t).$

$c_{31} = \int_0^T s_3(t)\phi_1(t) dt = \int_0^T s_3(t) \dfrac{s_1(t)}{\sqrt{T}} dt$

$= \dfrac{1}{\sqrt{T}} \int_0^{T/2} (1) dt = T/2 \times \dfrac{1}{\sqrt{T}} = \sqrt{T}/2$

$c_{32} = \int_0^T s_3(t)\phi_2(t) dt = \int_0^T s_3(t) \dfrac{s_2(t)}{\sqrt{T}} dt$

$= \dfrac{1}{\sqrt{T}} \int_0^{T/2} 1 dt = T/2 \times \dfrac{1}{\sqrt{T}} = \dfrac{\sqrt{T}}{2}.$

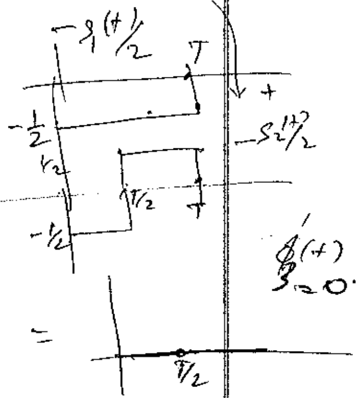
$\therefore \phi_3'(t) = s_3(t) - \dfrac{\sqrt{T}}{2}\phi_1(t) - \dfrac{\sqrt{T}}{2}\phi_2(t)$

$\phi_3'(t) = s_3(t) - \dfrac{s_1(t)}{2} - \dfrac{s_2(t)}{2} = 0.$

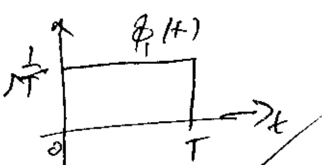
~~$c_3 = \int_0^T \phi_3(t) dt$~~

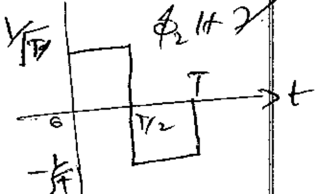
$\phi_3'(t) = 0$

so we have just 2 basis



$\phi_1(t) = \dfrac{s_1(t)}{\sqrt{T}}, \quad \phi_2(t) = \dfrac{s_2(t)}{\sqrt{T}}$





(b) $s_1(t) = \sqrt{T}\phi_1(t).$ $\quad \underline{s_1} = [\sqrt{T}, 0].$

$s_2(t) = \sqrt{T}\phi_2(t)$ $\quad \underline{s_2} = [0, \sqrt{T}].$

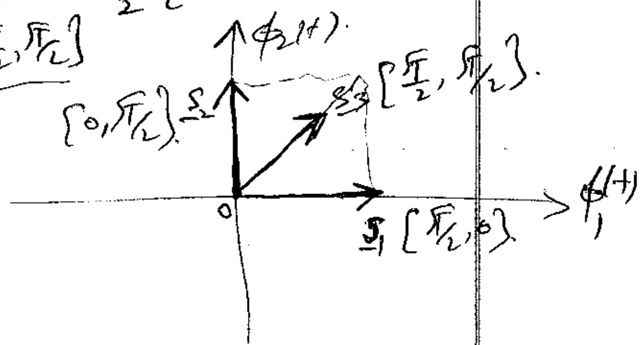
$s_3(t) = s_{31}\phi_1(t) + s_{32}\phi_2(t).$

$s_{31} = \langle s_3(t), \phi_1(t)\rangle = \int_0^{T/2} \frac{1}{\sqrt{T}} dt = \frac{T}{2} \times \frac{1}{\sqrt{T}} = \sqrt{T}/2$

$s_{32} = \langle s_3(t), \phi_2(t)\rangle = \int_0^{T/2} \frac{1}{\sqrt{T}} dt = \sqrt{T}/2$

$\Rightarrow s_3(t) = \frac{\sqrt{T}}{2}[\phi_1(t)] + \frac{\sqrt{T}}{2}\phi_2(t)$

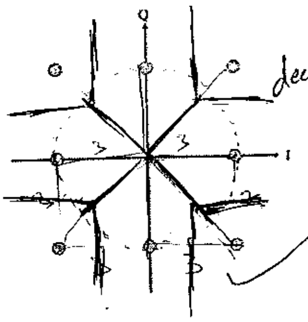
$\underline{s_3} = [\frac{\sqrt{T}}{2}, \frac{\sqrt{T}}{2}]$



Problem 3: **(1+1+3+2+4=11 points)**

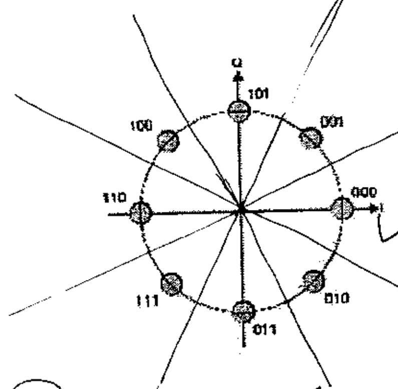
Two signal constellation diagrams are shown in the figure. Let the basis functions be as follows:

$\sqrt{\frac{2}{T}}\cos(2\pi f_c t), \quad 0\le t\le T$, $\sqrt{\frac{2}{T}}\sin(2\pi f_c t), \quad 0\le t\le T$ where $f_c=2/T$



decision boundaries

$\sqrt{3^2+3^2} = 3\sqrt{2}$ (1)



decision boundaries (1)

a) What type of modulation is used in each case? (1)

Left diagram: 8-QAM. both amp & phase are changing **Right diagram:** 8-PSK {same amplitude. only phase is changing

b) If one of these constellations is used, determine the symbol rate if the desired rate is 90 Mbit/s

bits/symbol $= \log_2 8 = 3 \rightarrow SR = \frac{90}{3}$ M symb/sec $= 30$M symbol/sec (1)

c) For the left constellation diagram, the minimum distance between adjacent points is 3. Determine the average transmitted energy per bit, *assuming that the signal points equally probable.*

Avg. Energy per symbol $= \frac{4\times 3^2 + 4\times(3\sqrt{2})^2}{8} = \frac{36+72}{8} = 13.5$

" " " bit $= \frac{13.5}{3} = 4.5$ Ans. (3)

X/.X

d) On the two constellation diagram, draw the decision boundaries (clearly) shown in figure

The random process $V(t)$ is defined as $V(t) = X\cos(2\pi f_c t) + 2Y\sin(2\pi f_c t)$

where X and Y are random variables. What are the conditions on $E(X)$, $E(Y)$, $E(X^2)$, $E(Y^2)$, $E(XY)$ such that $V(t)$ is wide-sense stationary. For V(t) to be WSS, its mean should be constant & its ACR should depend only on time diff

(i) $E\{V(t)\} =$ (3)

$= E[X\cos 2\pi f_c t + 2Y\sin 2\pi f_c t] = E\{X\}\cos 2\pi f_c t + 2E\{Y\}\sin 2\pi f_c t$

if $E\{X\} = E\{Y\} = 0$, 1st cond. is met.

(ii) $R_V(t, t+\tau) = E\{V(t)V(t+\tau)\} = E\{(X\cos 2\pi f_c t + 2Y\sin 2\pi f_c t)\times$
$X\cos 2\pi f_c(t+\tau) + 2Y\sin 2\pi f_c(t+\tau)\}$

$= E\{X^2\}\cos 2\pi f_c t\cos 2\pi f_c(t+\tau) + 2E\{XY\}\cos 2\pi f_c t\sin 2\pi f_c(t+\tau)$
$+ 4E\{Y^2\}\sin 2\pi f_c t\sin 2\pi f_c(t+\tau)$
$+ 2E\{YX\}\sin 2\pi f_c t\cos 2\pi f_c(t+\tau)$

$= E\{X^2\}\frac{1}{2}\{\cos 2\pi f_c\tau + \cos(2\pi f_c(2t+\tau))\} + \frac{2E\{Y^2\}}{2}\{\cos(2\pi f_c\tau) - \cos 2\pi f_c(2t+\tau)\}$

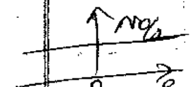
$+ 2E\{YX\}\frac{1}{2}\{\sin 2\pi f_c\tau + \cos(2\pi f_c(2t+\tau))\} + \frac{2E\{XY\}}{2}\{\sin 2\pi f_c(2t+\tau) + \sin(2\pi f_c\tau)\}$.

Problem 4: (1+1+4+1+2=9 points)

Consider a white Gaussian noise, $w(t)$ of zero mean and N_0 Watts/Hz, which is passed through an ideal band-pass filter of passband magnitude equals to 4, midband frequency f_c=1 MHz, and bandwith 10 kHz

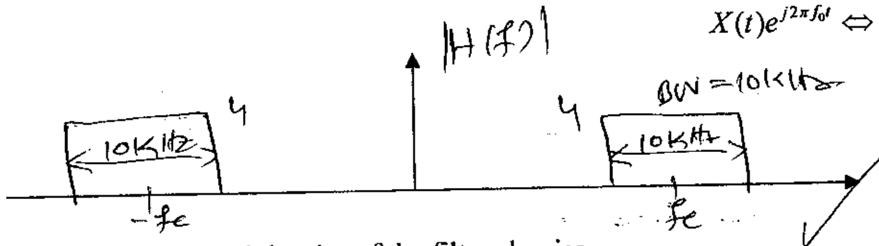
$$2W \operatorname{sinc}(2Wt) \Leftrightarrow \Pi\left(\frac{f}{2W}\right)$$

$$X(t)e^{j2\pi f_0 t} \Leftrightarrow X(f-f_0)$$



$S_w(f) = \frac{N_0}{2}$

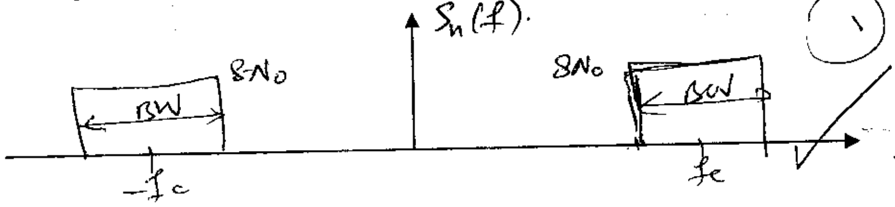
1) Sketch the transfer function of the ideal filter.



1

$S_n(f) = S_w(f) \times |H(f)|^2 = \frac{N_0}{2}|H(f)|^2$

2) Sketch the power spectral density of the filtered noise,



1

3) Find the auto correlation of the filtered noise and sketch it. (*Show all important values*)

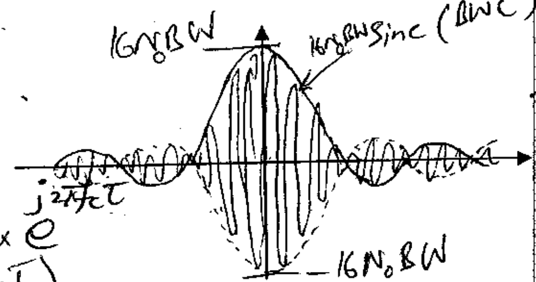
$R_n(\tau) = \mathcal{F}^{-1}\{S_n(f)\} = \mathcal{F}^{-1}\{8N_0\Pi\left(\frac{f+f_c}{BW}\right) + 8N_0\Pi\left(\frac{f-f_c}{BW}\right)\}$

$= 8N_0 BW \operatorname{Sinc}(BW\tau)\, e^{-j2\pi f_c\tau} + 8N_0 BW \operatorname{Sinc}(BW\tau) \times e^{j2\pi f_c\tau}$

$= 2 \times 8 N_0 BW \operatorname{Sinc}(BW\tau)\left\{\frac{e^{j2\pi f_c\tau} + e^{-j2\pi f_c\tau}}{2}\right\}$

$= 16 N_0 BW \operatorname{Sinc}(BW\tau) \cos 2\pi f_c\tau \longrightarrow$ This is sinc modulated signal

lowpass envelope = Sinc

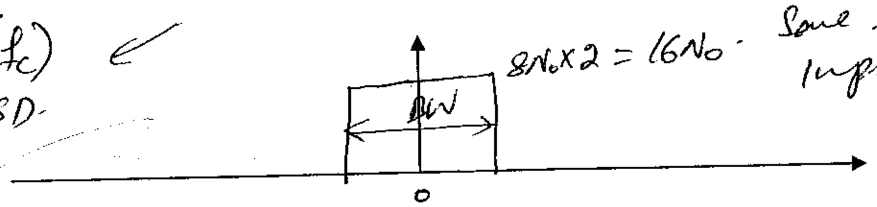


4) Sketch the power spectral density of the in-phase and quadrature components of the filtered noise.

Low pass eq:

$S_\ell(f) = 2S_+(f+f_c)$

Lpass equivalent PSD



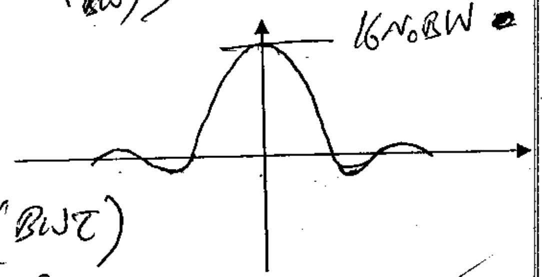
Same is case for inphase & quadrature components.

Since LP PSD is symmetric about $f=0$

$\Rightarrow S_i(f) = S_q(f) = S_\ell(f)$

5) Find the auto correlation of the in-phase and quadrature components.

$R_\ell(\tau) = \mathcal{F}\{16N_0\Pi\left(\frac{f}{BW}\right)\}$



$R_\ell(\tau) = 16N_0 BW \operatorname{Sinc}(BW\tau)$

Since PSDs are same

ACRs of inphase & quadrature compts will also be same