$\frac{1}{T}|G(f)|^2\,\Phi_{bb}(f)$. But :

$$\begin{aligned}\phi_{bb}(m) &= E\left[b_{n+m}b_n\right] \\ &= E\left[a_{n+m}a_n\right] + kE\left[a_{n+m-1}a_n\right] + kE\left[a_{n+m}a_{n-1}\right] + k^2E\left[a_{n+m-1}a_{n-1}\right] \\ &= \left\{\begin{array}{rr} 1+k^2, & \mathrm{m}=0 \\ k, & \mathrm{m}=\pm 1 \\ 0, & \mathrm{o.w.}\end{array}\right\}\end{aligned}$$

Hence :

$$\Phi_{bb}(f) = \sum_{m=-\infty}^{\infty}\phi_{bb}(m)e^{-j2\pi fmT} = 1+k^2+2k\cos 2\pi fT$$

We want :

$$\Phi_{ss}(1/T)=0 \Rightarrow \Phi_{bb}(1/T)=0 \Rightarrow 1+k^2+2k=0 \Rightarrow k=-1$$

and the resulting power spectrum is :

$$\Phi_{ss}(f) = 4T\left(\frac{\sin^2\pi fT/2}{\pi fT/2}\right)^2 \sin^2\pi fT$$

(c) The requirement for zeros at $f=l/4T,\ l=\pm1,\pm2,\ldots$ means : $\Phi_{bb}(l/4T)=0 \Rightarrow 1+k^2+2k\cos\pi l/2=0$, which cannot be satisfied for all l. We can avoid that by using precoding in the form :$b_n=a_n+ka_{n-4}$. Then :

$$\phi_{bb}(m) = \left\{\begin{array}{rr} 1+k^2, & \mathrm{m}=0 \\ k, & \mathrm{m}=\pm 4 \\ 0, & \mathrm{o.w.}\end{array}\right\} \Rightarrow \Phi_{bb}(f) = 1+k^2+2k\cos 2\pi f4T$$

and , similarly to (b), a value of $k=-1$, will zero this spectrum in all multiples of $1/4T$.

Problem 4.35 :

$$\mathbf{P} = \begin{bmatrix} 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \end{bmatrix}, \quad \rho = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$

By straightforward matrix multiplication we verify that indeed :

$$\mathbf{P}^4\rho = -\frac{1}{4}\rho$$