Example:

Consider the syndrome circuit for (*7, 4*) cyclic code generated by

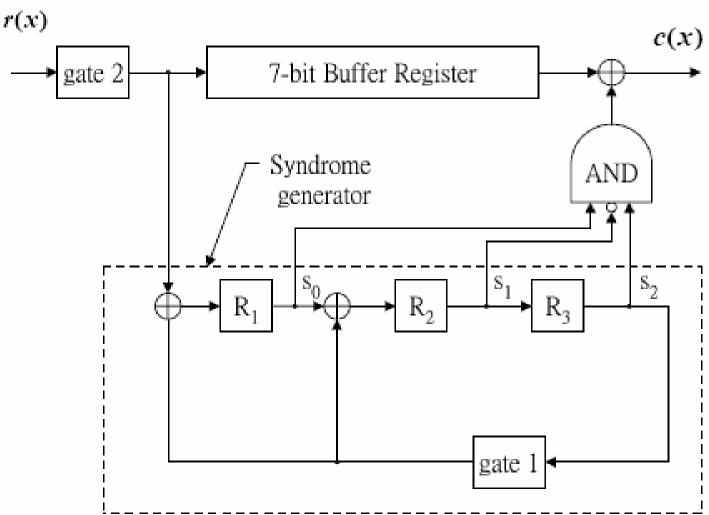
$g(x) = 1 + x + x^3$.

The code has $d_{min} = 3$ and is capable of correcting any single error in $e(x)$.

Since the error patterns consist of the cyclic shifts of (*0 0 0 0 0 0 1*), the decoder needs only to be able to recognize one of the seven nonzero syndromes to be able to correct all of the nonzero error patterns.

The syndrome s = (*1 0 1*), corresponding to the error pattern e = (*0 0 0 0 0 0 1*), is the best choice since it allows one to release the corrected codeword bits before the error location actually is identified.

Fig. 4.6. (page 160)



Decoding begins by first setting all of the shift-register cells to zero. The received word $\bar{r}$ is then shifted bit-by-bit into the *7*-bit received word buffer and the syndrome-computation circuit, simultaneously.

Once the received word is completely shifted into the buffer, the shift-registers of the syndrome computation circuit contain the syndrome for the received word. As one continues to shift cyclically the contents of the received-word buffer and the syndrome computation circuit, the syndrome computation circuit computes the syndrome for the cyclically shifted versions of the received word.

If at any point the computed syndrome is $s = (1\ \ 0\ \ 1)$, it is "detected" by the AND gate when its output goes to *1*. This value is then used to complement and correct the error in the rightmost bit in the buffer as it leaves the buffer.

- **A general decoder (Meggitt decoder) for an (n, k) cyclic code is shown below. (Lin / Costello p. 105)**

The decoding operation is described as follows:

1. **Shift the received polynomial $r(x)$ into a buffer and the syndrome registers simultaneously. (That is, syndrome is formed.)**
2. **Check whether the syndrome $s(x)$ corresponds to a correctable error pattern $e(x) = e_0 + e_1 x + \cdots + e_{n-1} x^{n-1}$ with an error at the highest-order position x^{n-1}. (i.e. $e_{n-1} = 1$?)**
3. **Correct r_{n-1} if $e_{n-1} = 1$.**
4. **Cyclically shift the buffer and syndrome registers once simultaneously. Now the buffer register contains $r^{(1)}(x)$ and the syndrome register contains the syndrome $s^{(1)}(x)$ of $r^{(1)}(x)$.**
5. **Check whether $s^{(1)}(x)$ corresponds to a correctable error pattern $e^{(1)}(x)$ with an error at the highest-order position x^{n-1}.**
6. **Correct r_{n-2} if it is erroneous.**
7. **Repeat the same process until n shifts.**

8. **If the error pattern is correctable, the buffer register contains the transmitted codeword and the syndrome register contains zeros.**
9. **If the syndrome register does not contain all zero at the end of the decoding process, an uncorrectable error pattern has been detected.**

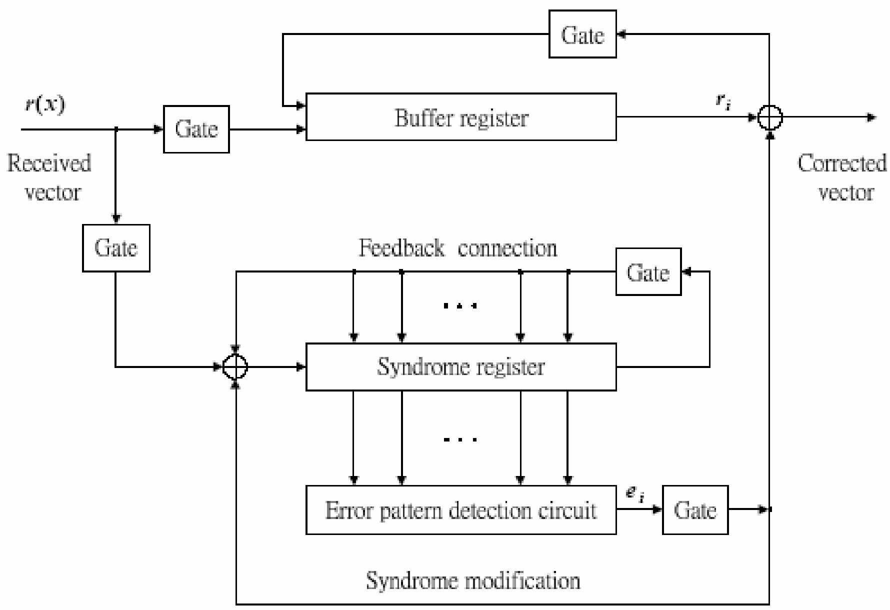


Figure 4.8 General cyclic code decoder with received polynomial $r(x)$ shifted into the syndrome register from left end.