Continue 4.1.2

Error rates & Error Distributions for the BSC :

Example : a code word of 5 bits what is the probability that more than 3 errors are in Error (given $p = 0.1$)

$$Pr(\leq t=3) = 1 - \{ Pr\{t=4\} + Pr(t=5)\}$$

$$Pr(t=4) = \binom{5}{4} (0.1)^4 (0.9)^1 = 5 * 10^{-4} * 0.9 = 45 * 10^{-5}$$

$$Pr(t=5) = \binom{5}{5} (0.1)^5 (0.9)^0 = 1 * 10^{-5} \quad = 1 * 10^{-5}$$

- $Pr(\leq t=3) = 1 - (45+1) * 10^{-5} = 1 - 46E{-5} = 0.99954$

For Example 4.1.1

Entry (1,1) in the table

$$\bar{t} = np = 7(0.1) = 0.7$$

$$\sigma_t = \sqrt{np(1-p)} = \sqrt{0.7(0.9)} = 0.7937$$

$$t_{3\sigma} = 0.7 + 3(0.7937) = 3.081$$

- for Example 4.12

for Entry $(2,1) = 0.86 \longrightarrow 0.068$

$$Pr(t \geq t_{3\sigma}) = \sum_{j=\lceil t_{3\sigma}\rceil}^{n} \binom{n}{j} p^j (1-p)^{n-j}$$

$$= 1 - \sum_{j=0}^{\lceil t_{3\sigma}\rceil - 1} \binom{n}{j} p^j (1-p)^{n-j}$$

$$= 1 - \binom{7}{0} (0.01)^0 (0.99)^7 = 1 - (0.99)^7 = 0.068$$

4.1.3 Error Detection & Correction

Error detection (and /or) Error correction.

Example 4.1.3 Repetition Codes.

$G(0) \rightarrow 000$

$G(1) \rightarrow 111$

we cannot do both.

OR

Detect Two Errors.

Correct 1 Error.

Received Word	Decoded Word	Error Flag	Correct 1 Error
000	0	No	0
001	?	Yes	0
010	?	Yes	0
011	?	Yes	1
100	?	Yes	0
101	?	Yes	1
110	?	Yes	1
111	1	No	1

Example 4.1.4 Repetition Code with $r=3$

Can correct single-bit & detect two-bit errors.

$G(0) \rightarrow 0000$

$G(1) \rightarrow 1111$

0000	0	1000	0
0001	0	1001	Error
0010	0	1010	Error
0011	Error	1011	1
0100	0	1100	Error
0101	Error	1101	1
0110	Error	1110	1
0111	1	1111	1

Hamming Distance :

The "distance" between the received word and a legal code word is measured by counting by counting the number of bit positions in which the two code word disagree.

→ In the previous Example we based our decoding decision in minimum Hamming distance (is this correct? what are the embedded assumption). (equally likely).

- $d_H(100, 011) = 3$, $d_H(1111, 1001) = 2$

4.1.5 Hamming Distance and Code Capability.

M a set of Equally likely messages $\bar{m}$ of k bits

$2^k = |M|$

G : encoding rule unique code

$G(\bar{m}_i) \to \bar{c}_i$ of n bits $c \in C$ legal code words.

- $I(M;C) = H(M)$ given $\bar{c}_i$, $\bar{m}_i$ is uniquely identified.

for $\bar{c}_i, \bar{c}_j$ minimum Hamming distance $= d_{min}$

* what is d_{min}.

* do we want d_{min} large or small

	0001
00	1101
01	1110
10	1011
11	

d_{min} determines the error correcting and/or detecting capability of the code.

① A code can detect up to t errors if and only if $d_{min} \geq t+1$

② " " " correct up to t " " " $d_{min} \geq 2t+1$

③ " " " correct up to t_c errors & detect up to $t_d > t_c$ if and only if

$$d_{min} > 2t_c + 1 \quad \text{and} \quad d_{min} \geq t_c + t_d + 1$$

Example 4.1.5

$r = 3$ binary repetion code

$d_{min} = 4$ $\qquad 4 = 1 + 2 + 1$

Can correct 1 & detect 2

or Cannot correct 2 bits error.

Can detect three errors provided it does not attempt to correct one error $(1 + 3 + 1 = 5 > 4)$

How to find d_{min}?

depend in the code, In general it is upper bounded by

$$d_{min} \cancel{\geq} \leq r + 1 \qquad (\text{Sigleton bound})$$

① for 2^k all possibiltys (message) $r = 0$ $d_{min} = 1$

② • parity check $d_{min} \to 2$ $= r+1$ $\left(\begin{matrix}\text{Eave} & 0 \\ \text{odd.} & 1\end{matrix}\right)$

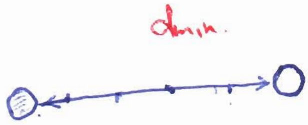
③ repeat last bit (check) $1 = d_{min} < 1 + r$

① A code can detect up to t errors. if and only if $d_{min} \geq t+1$

② " " " correct up to t " " " " $d_{min} \geq 2t+1$

③ " " " correct up to t_c errors & detect up to $t_d > t_c$ if and only if.

$d_{min} > 2t_c + 1$ and $d_{min} \geq t_c + t_d + 1$



Example 4.1.5

$r = 3$ binary repetion code

$d_{min} = 4$ $\qquad 4 = 1 + 2 + 1$

Can correct 1 & detect 2

or Cannot correct 2 bits error.

Can detect three errors provided it does not attempt to correct one error $(1+3+1 = 5 > 4)$

How to find d_{min}?

depend in the code, In general it is upper bounded by

$$d_{min} \cancel{\geq} \leq r+1$$ (Sigleton bound)

① for 2^k all possibiltys (message) $r = 0$ $\quad d_{min} = 1$

② . parity check $\quad d_{min} \to \underset{= r+1}{2}$ $\quad \begin{pmatrix} \text{even} & \dots & 0 \\ \text{odd.} & & 1 \end{pmatrix}$

③ repeat last bit (check) $\quad 1 = d_{min} < 1 + r$