

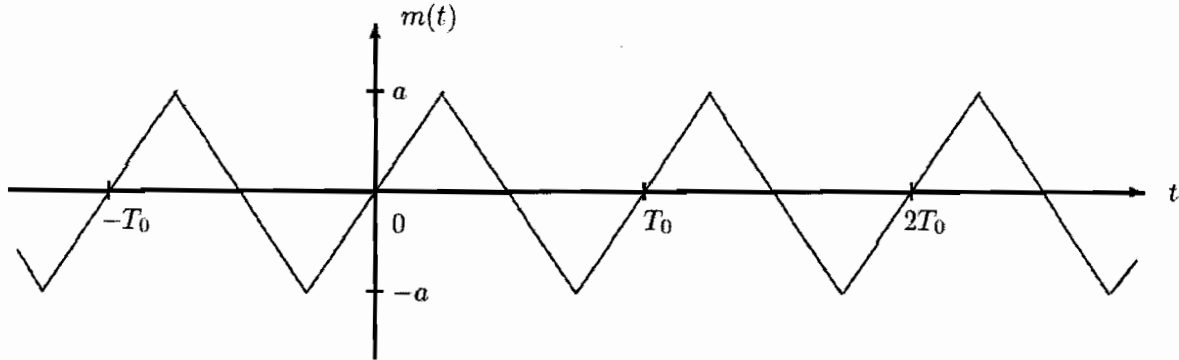
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Name: **KEY**

Section:

Signature:

Angle Modulation:



Consider the *periodic* message signal  $m(t)$  shown in the picture, and assume initially that  $a = 1$  and  $T_0 = 10^{-3}$ .

Consider the (essential) bandwidth of the signal  $W$  to be the frequency corresponding to the fifth harmonic, i.e.  $W = \frac{5}{T_0}$  Hz. Now, suppose  $m(t)$  is angle modulated on a 1 MHz carrier.

- 2 (a) Find the bandwidth of an FM signal with  $k_f = 100$ .
- 3 (b) Find the bandwidth of a PM signal with  $k_p = 2\pi \times 10^2$ .
- 4 (c) How much is the percentage of message power lost by dropping the contribution of components above the fifth harmonics.
- 1 (d) For the FM case is this narrowband FM or Wideband FM, justify your answer.

a)  $BW = 2(\Delta f + B) = 2(\Delta f + \frac{5}{T_0})$

①  $\Delta f_{FM} = \frac{K_f m_p}{2\pi} = \frac{100 a}{2\pi} = \frac{100}{2\pi}$

①  $BW_{FM} = 2(\frac{100}{2\pi} + \frac{5}{10^{-3}}) = 2(5015.92) \text{ Hz}$   
 $\approx \underline{\underline{10.032 \text{ KHz}}}$

b)  $BW = 2(\Delta f + \frac{5}{T_0}) = 2(\Delta f + 5000)$

①  $\Delta f_{PM} = \frac{K_p m_p}{2\pi} = \frac{2\pi \cdot 10^2 m_p}{2\pi}$   
 $= 100 m_p$

①  $m_p = \frac{2a}{T_0} = \frac{4(1)}{10^{-3}} = 4000 \Rightarrow \Delta f_{PM} = 4 \times 10^5$

①  $BW_{PM} = 2(4 \times 10^5 + 5000) = \underline{\underline{810 \text{ KHz}}}$

d) It is a narrow band FM (0.5) because  $BW_{FM} = 10.032 \text{ KHz}$  which is approximately  $= 2B \approx 10 \text{ KHz}$  the bandwidth for narrow band FM  $\Delta f$  is small. (0.5)

c) Total power in the signal.  $\frac{1}{T_0} \int_0^{T_0} (m(t))^2 dt = \frac{a^2}{3} = \frac{1}{3}$  see the next page for details (2)

From the book p 223  $m(t) = \sum_n C_n \cos n\omega_c t$   $\omega_c = \frac{2\pi}{1 \times 10^{-3}} = 2\pi \times 10^3$

$C_n = \begin{cases} \frac{8}{\pi^2 n^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$  (2)

$\tilde{m}(t) \approx \frac{8}{\pi^2} \cos \omega_c t + \frac{8}{\pi^2 9} \cos 3\omega_c t + \frac{8}{\pi^2 25} \cos 5\omega_c t$

power =  $\frac{(8/\pi^2)^2}{2} + \frac{(8/\pi^2 9)^2}{2} + \frac{(8/\pi^2 25)^2}{2} = 0.3330927$

% =  $\frac{1/3 - 0.3330927}{1/3} \times 100 = 0.0722 \%$

Total power  $\propto$  area under square of the curve.

$$\begin{aligned}
 \frac{1}{T_0} \int_{T_0} m^2(t) dt &= \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} m^2(t) dt \quad \leftarrow \text{symmetric } T_0/4 \\
 &= \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} \left( \frac{2a}{T_0/2} t \right)^2 dt = \frac{2}{T_0} \int_{-T_0/4}^{T_0/4} \frac{16a^2}{T_0^2} dt \\
 &= \frac{32 a^2}{T_0^3} \left. t^3 \right|_{-T_0/4}^{T_0/4} = \frac{32 a^2}{3 T_0^3} \left[ \frac{T_0^3}{4^3} - \left( -\frac{T_0^3}{4^3} \right) \right] \\
 &= \frac{32}{3} \frac{2}{4^3} a^2 = \boxed{\frac{a^2}{3}} \quad \text{for } a=1 \quad \text{power} = \frac{1}{3}
 \end{aligned}$$