

Name:

KEY

ver. 1.0

A single-tone modulating signal  $\cos(15\pi \cdot 10^3 t)$  frequency modulates a carrier of 10 MHz and produces a frequency deviation of 75 kHz.

$$\leftarrow (2\pi (7.5 \times 10^3) t)$$

1) Find the modulation index.

$\nearrow f_m$  (2 points)

$$\beta = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m} = \frac{75 \times 10^3}{7.5 \times 10^3} = 10$$

2) Estimate the bandwidth of the FM signal.

(2 points)

$$B = 2(\Delta f + B_m) = 2(75 + 7.5) \times 10^3 = 2(82.5 \text{ K})$$

$$= 165 \text{ KHz}$$

3) Find the phase deviation produced in the FM wave.

(3 points)

$$\varphi_{FM}(t) = A \cos(\omega_c t + \frac{K_f}{\omega_m} \sin \omega_m t)$$

$K_f$  in rad/volt

instantaneous total phase  $\omega_c t + \frac{K_f}{\omega_m} \sin \omega_m t$   
 instantaneous total frequency  $\omega_c t + K_f \cos \omega_m t \Rightarrow \Delta f = K_f \Rightarrow$

$$\Delta\theta = \frac{K_f}{\omega_m} = \frac{75 \text{ K}}{7.5 \text{ K}} = 10 \frac{\text{rad}}{\text{sec}}$$

note that  $\int_{-\infty}^t \cos(\omega_m t) dt = \frac{\sin(\omega_m t)}{\omega_m}$

4) If another single-tone modulating signal produces a modulation index of 100 while maintaining the same deviation, find the frequency and amplitude of the modulating signal, assume  $k_f = 15 \text{ kHz per volt}$ .

A &  $f_m$

(3 points)

$$A \cos(\omega_m t) = A \cos(2\pi f_m t)$$

$$\beta = 100 = \frac{75 \times 10^3}{f_m} \Rightarrow$$

$$\boxed{f_m = 750 \text{ Hz}}$$

$$\beta = \frac{\Delta f}{f_m}$$

$$A \cos(\omega_c t + \frac{K_f A}{\omega_m} \sin \omega_m t)$$

$$\Delta f = (15 \text{ kHz})(A) = 75 \text{ K}$$

$$\Rightarrow A = \frac{75 \text{ K}}{15 \text{ K}} = 5 \text{ V}$$

Note: the unit of  $K_f$  is given in Hz/volt.  $K_f$  can also be represented in rad/volt