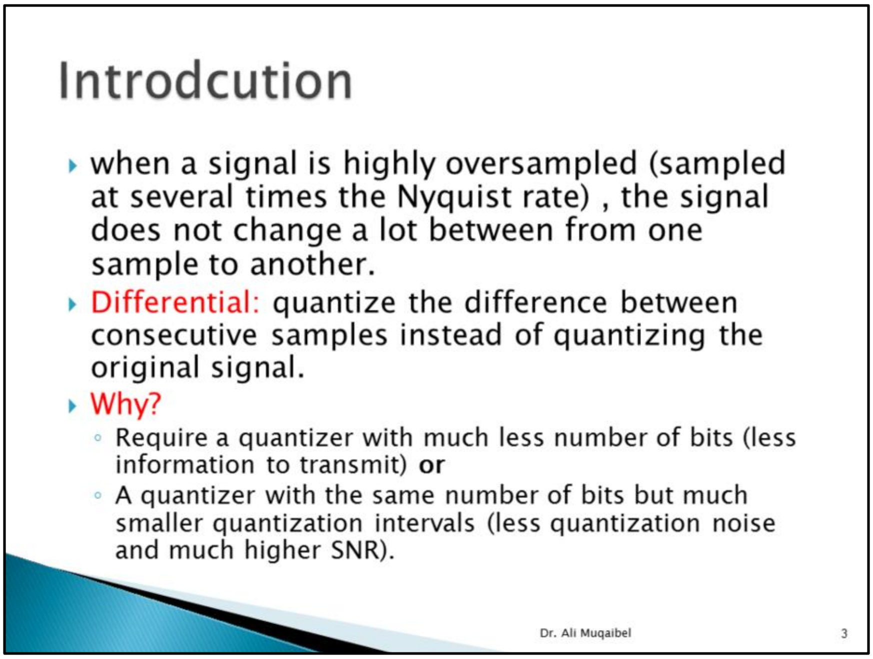
Differential Pulse Code Modulation (DPCM)

& Delta Modulation (DM)

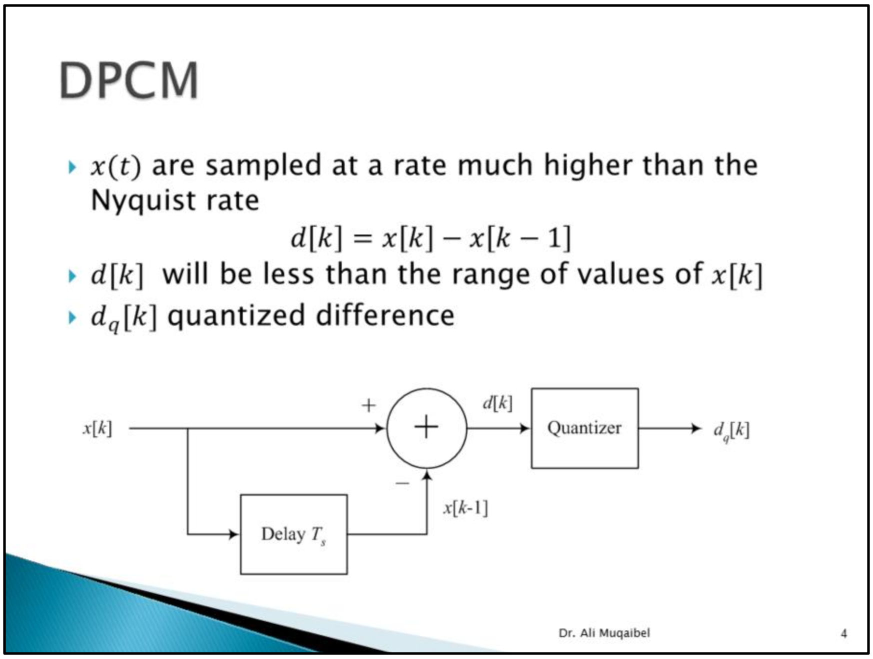
Dr. Ali Muqaibel

Outline (Class Objective)

- Introduction
- DPCM & Motivation
  - Predictive DPCM
  - How to predict?
  - SNR Improvement
  - Adaptive DPCM (ADPCM)
- Delta Modulation (DM) also ADM
- Bandwidth for PCM



According to the Nyquist sampling criterion, a signal must be sampled at a sampling rate that is at least twice the highest frequency in the signal to be able to reconstruct it without aliasing. The samples of a signal that is sampled at that rate or close to this rate generally have little correlation between each other (knowing a sample does not give much information about the next sample). However, when a signal is highly oversampled (sampled at several times the Nyquist rate, the signal does not change a lot between from one sample to another. Consider, for example, a sine function that is sampled at the Nyquist rate. Consecutive samples of this signal may alternate over the whole range of amplitudes from –1 and 1. However, when this signal is sampled at a rate that is 100 times the Nyquist rate (sampling period is 1/100 of the sampling period in the previous case), consecutive samples will change a little from each other. This fact can be used to improve the performance of quantizers significantly by quantizing a signal that is the difference between consecutive samples instead of quantizing the original signal. This will result in either requiring a quantizer with much less number of bits (less information to transmit) or a quantizer with the same number of bits but much smaller quantization intervals (less quantization noise and much higher SNR).



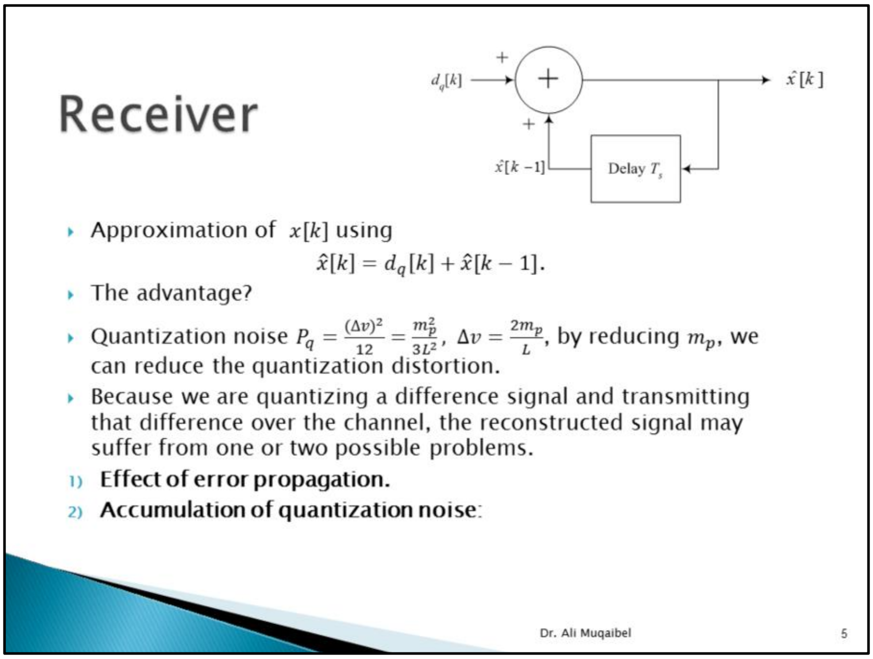
Consider a signal $x(t)$ that is sampled to obtain the samples $x(kTs)$, where T_s is the sampling period and k is an integer representing the sample number. For simplicity, the samples can be written in the form $x[k]$, where the sample period T_s is implied. Assume that the signal $x(t)$ is sampled at a very high sampling rate. We can define $d[k]$ to be the difference between the present sample of a signal and the previous sample, or

$$d[k] = x[k] - x[k-1]$$

Now this signal $d[k]$ can be quantized instead of $x[k]$ to give the quantized signal $d_q[k]$. As mentioned above, for signals $x(t)$ that are sampled at a rate much higher than the Nyquist rate, the range of values of $d[k]$ will be less than the range of values of $x[k]$.

After the transmission of the quantized signal $d_q[k]$, theoretically we can reconstruct the original signal by doing an operation that is the inverse of the above operation. So, we can obtain an approximation of $x[k]$ using
$\hat{x}[k] = d_q[k] + \hat{x}[k-1]$.

So, if $d_q[k]$ is close to $d[k]$, it appears from the above equation that obtained will be close to $d[k]$. However, this is generally not the case as will be shown later. The transmitter of the above system can be represented by the following block diagram.



The receiver that will attempt to reconstruct the original signal after transmitting it through the channel can be represented by the following block diagram.
Because we are quantizing a difference signal and transmitting that difference over the channel, the reconstructed signal may suffer from one or two possible problems.

1. Error Propagation: in a regular PCM system, the effect of an error that happens in the transmitted signal is only limited to the sample in which the error occurs. In DPCM, an error that occurs in the transmitted signal will cause all the reconstructed samples at the receiver after that error occurs to have errors. Therefore, even if quantization error did not accumulate, an error caused by the channel will cause all successive samples to be wrong. Try this as an exercise by constructing g a table similar to the one in the coming example. Intentionally introduce an error in the reconstructed signal at a point and see what happens to the remainder of the reconstructed signal.

2. Accumulation of quantization noise: the above system suffer from the possible accumulation of the quantization noise. Unlike the quantization of a signal where quantization error in each sample of that signal is completely independent from the quantization error in other samples, the quantization error in this system may accumulate to the point that it will result in a reconstructed signal that is very different from the original signal. This is illustrated using the following table. Consider the samples of the input signal $x[k]$ given in the table. The reconstructed signal is given by $\hat{x}[k]$ shown in table. Assume the quantizer used to quantize $d[k]$ is an 8–level quantizer with quantization intervals [–4,–3), [–3,–2), [–2,–1), … , [3,4) and the output quantization levels are the center points in each interval (–3.5, –2.5, –1.5, … , 3.5).

Example

The quantizer used to quantize $d[k]$ is an 8-level quantizer with quantization intervals [-4,-3), [-3,-2), [-2,-1), … , [3,4) and the output quantization levels are the center points in each interval (-3.5, -2.5, -1.5, … , 3.5).

k	-1	0	1	2	3	4	5	6	7	8	9
$x[k]$	0	0.3	1.5	0.7	1.0	2.3	3.7	2.8	3.5	2.8	0
$x[k-1]$	0	0	0.3	1.5	0.7	1.0	2.3	3.7	2.8	3.5	2.8
$d[k]$	0	0.3	1.2	-0.8	0.3	1.3	1.4	-0.9	0.7	-0.7	-2.8
Quantization Up/Down	U (or D)	U	U	U	U	U	U	U	D	U	U
$d_q[k]$	0.5	0.5	1.5	-0.5	0.5	1.5	1.5	-0.5	0.5	-0.5	-2.5
$\hat{x}[k-1]$	0	0.5	1.0	2.5	2.0	2.5	4.0	5.5	5.0	5.5	5.0
$\hat{x}[k]$	0.5	1.0	2.5	2.0	2.5	4.0	5.5	5.0	5.5	5.0	2.5
$\hat{x}[k]-x[k]$	0.5	0.7	1.0	1.3	1.5	1.7	1.8	2.2	2.0	2.2	2.5
Err. Direction Up/Down	U	U	U	U	U	U	U	U	D	U	U

Note that the error between the original and reconstructed samples always increased except when the quantization error switched direction at $k = 7$ (the shaded box).

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So, it is clear from this table that if the quantization error for a series of samples is going in one direction, the error adds up to produce a output signal that deviates from the original signal. Note that the error between the original and reconstructed samples always increased except when the quantization error switched direction at *k* = 7 (the shaded box).

Predictive DPCM

- Estimate (predict) the value of the k^{th} sample $m[k]$ from knowledge of the previous sample values. Let this estimate be $\hat{m}[k]$, then we transmit the difference (iteratively), then we transmit the difference (perdition error)

$$d[k] = m[k] - \hat{m}[k]$$

- At the receiver, first predict and then ad the difference (iteratively)
- DPCM is a special case of predictive DPCM where

$$\hat{m}[k] = m[k-1]$$

- Example: linear
- (first order prediction $\hat{m}[k] = m[k-1] + (m[k-1] - m[k-2])$

k	-1	0	1	2	3	4	5	6	7	8	9
$x[k]$	0	0.3	1.5	1.7	1.9	2.3	2.8	3.2	3.5	3.1	2.8
$\hat{x}[k]$	0	0	0.6	2.7	1.9	2.1	2.7	3.3	3.6	3.8	2.7
$d[k]$	0	0.3	0.9	1	0	0.2	0.1	-0.1	-0.1	-0.7	+0.1

How to predict?

- We can express the next sample as (Taylor Series)

$m(t+Ts) = m(t) + T_s\ \dot{m}\ (t) + \frac{T_s^2}{2!}\ddot{m}(t) + \frac{T_s^3}{3!}\dddot{m}(t) + \cdots . \approx m(t) + T_s$
$\dot{m}\ (t)$

- A full knowledge of $m(t)$ and its derivatives allow for future prediction

$$\dot{m}\ (kT_s) \approx \frac{[m(kT_s) - m(kT_s - Ts)]}{T_s}$$

- $m[k+1] \approx m[k] + T_s\left[\frac{m[k]-m[k-1]}{T_s}\right] = 2m[k] - m[k-1]$
- For higher order prediction, we require more past samples

$$\widehat{m}[k] = a_1 m[k-1] + a_2 m[k-2] + \cdots + a_N m[k-N]$$

- N^{th} order prediction Equation
- Larger N ,better prediction

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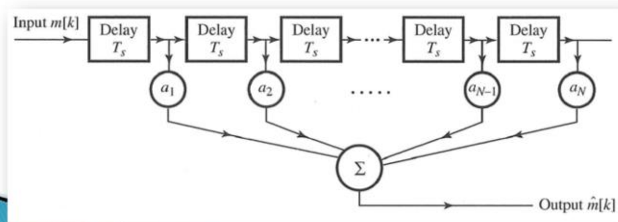
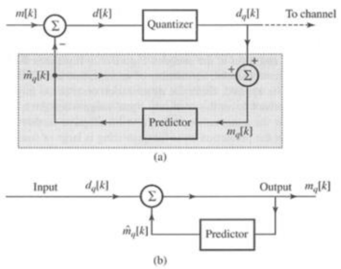
Prediction

- N^{th} order prediction Equation
  $$\widehat{m}[k] = a_1 m[k-1] + a_2 m[k-2] + \cdots + a_N m[k-N]$$
- $a_1 = 1, N = 1$: First order prediction is a simple time delay (DPCM , $\widehat{m}[k] = m[k-1]$)
- Prediction coefficient (Statistical correlation between different samples)
- Linear Prediction: transversal filter (a tapped delay line) where the tap gains=prediction coefficients.



Analysis of DPCM

- We cannot determine $\hat{m}[k]$ at the receiver because all we have is quantized samples. A solution is to find the difference with the quantized samples at the transmitter.
- $d[k] = m[k] - \hat{m}_q[k]$
- $d_q[k] = d[k] + q[k]$
- At the receiver
- $m_q[k] = \hat{m}_q[k] + d_q[k]$
- $m_q[k] = m[k] - d[k] + d_q[k] = m[k] + q[k]$



DPCM System
(a) Transmitter
(b) Receiver

SNR Prediction Improvement

- *SNR improvement,* G_p

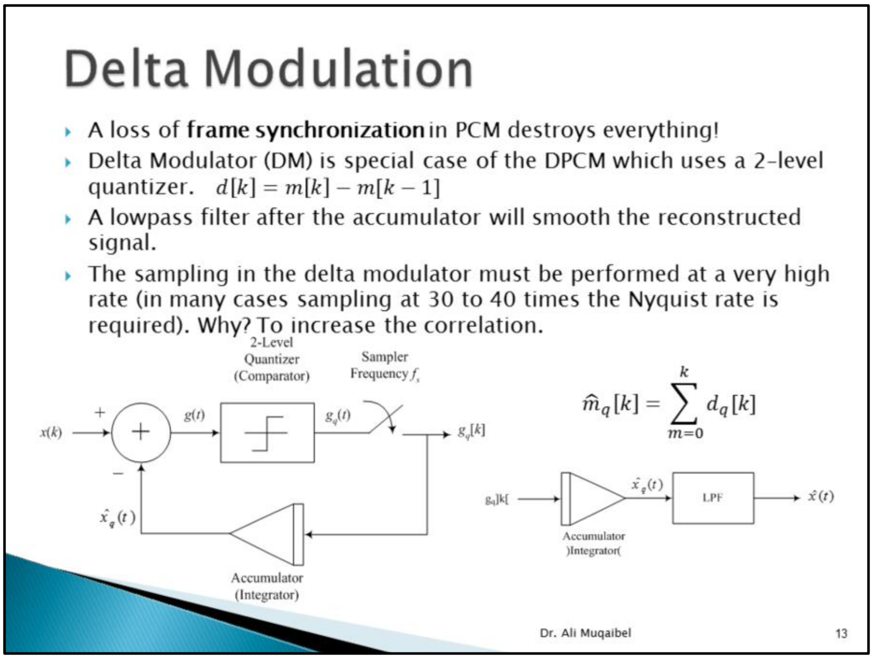
$$G_p = \frac{P_m}{P_d}$$

P_m and P_d are the powers of $m(t)$ and $d(t)$

- In practice, SNR improvements my be as high as 25dB (voice and low activity images)
- Alternatively for the same SNR, the bit rate (DPCM) can be reduced (compared with PCM) by 3 to 4 bits per sample. Thus, telephone systems using DPCM can often operate at 32 or even 24kbits/sec (instead of 64kbits/s)

Adaptive Differential PCM (ADPCM)

- Adaptivly change Δv
- $d_q[k]$ can be compared to Δv or 0 , to see if we need to increase or decrease the current Δv.
- 8bit PCM may be reduced to 4bit ADPCM at the same sampling rate.
- ITU-T standard G.726. used by Microsoft in WAVE players



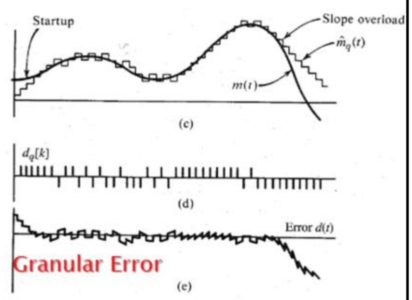
DPCM discussed in the previous lectures and PCM in general require that the receiver and transmitter be completely synchronized. Synchronization in digital systems is necessary. There are two types of synchronization in digital systems: frame (or symbol) synchronization and bit synchronization. Bit synchronization is required to make sure that the start and end of a bit are known so that a sequence of bits is counted properly. Frame synchronization is required to make sure that the different bits of a sample are known (i.e., the most significant bit and the least significant bit of a sample are properly identified to allow for the reconstruction of the sample). One bit of frame synchronization error is sufficient to completely destroy the reconstructed signal. Bit synchronization problems will eventually lead to frame synchronization problems.

A special case of the DPCM system discussed earlier can be obtained by simply using a 2–level quantizer and by placing the sampler after the quantizer. Consider the following block diagram of a DPCM system. This system is known as a Delta Modulator (DM).

To reconstruct the original signal, the received signal at the demodulator is passed through an accumulator similar to the one in the modulator. A lowpass filter after the accumulator will smooth the reconstructed signal. Therefore, the Delta Demodulator is given by

Slope overloading and Granular Errors

- $\widehat{m}_q(t)$ is a stair case approximation of $m(t)$.
- + pulse $m(t) > \widehat{m}_q(t)$.
- – pulse $m(t) < \widehat{m}_q(t)$.
- Threshold of coding and overloading.
- Variation in $m(t)$ smaller than the step value (threshold of coding) are lost is DM (Granular Error)
- If $\dot{m}(t)$ is too high, $\widehat{m}_q(t)$ cannot follow $m(t)$. (Slope overload)
- Adaptive Step , Adaptive DM (ADM)
- Detect the pattern and decide how to adapt accordingly
  - (++++, or ----) increase step
  - +-+-+-+ decrease step



Compare DM with PCM

- In DM each digit has **equal importance.**
- In DM multiplexing each channel requires its own decoder while for PCM one decoder for all (On the other hand, permit flexibility and avoid cross talk (no need for stringent multiplexing design)
- DM outperforms at low SNR while PCM outperforms at high SNR.
- DM is very simple, inexpensive, and no need for framing bits (less overhead)
- DM is suitable for voice & TV signals.

Bandwidth for PCM

- Bandwidth required for binary baseband= $R/2$.
- Bandwdith required for binary passband= R.

Exercise (Quiz 6 102)

Five television signals (video and audio) with a bandwidth of $4.5\ MHz$ each, are to be transmitted by binary PCM. The signals are sampled at 25% above the Nyquist rate. Samples are quantized into 1024 levels. In every frame, framing and synchronization requires an additional 4% extra bits. A PCM encoder is used to convert these signals before they are time-multiplexed into a single data stream.

a) Determine the sampling rate and the sampling interval for each channel.
b) At the quantizer, how many bits are used to represent each sample.
c) In every frame, how many bits are there?
d) Determine the data rate of the multiplexed signal.
e) Determine the minimum required bandwidth of baseband communication.

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Sampling rate: =1.25*Nyquist rate=1.25*2*4.5M= 11.25 M samples /sec

Sampling interval: =1/sampling rate= 1/(11.25 M)= 88.88889 *n sec*

Log_2 1024=10

5*(10)*1.04=52 bits

Date rate (bits/sec)= 52 bits/sampling interval(sec)=52 bits* sampling rate = (52)(11.25M)=585 M bits/sec

Min baseband BW= Rate/2=585 M /2 =292.5 M HZ