Quantization
& The T1 Carrier System

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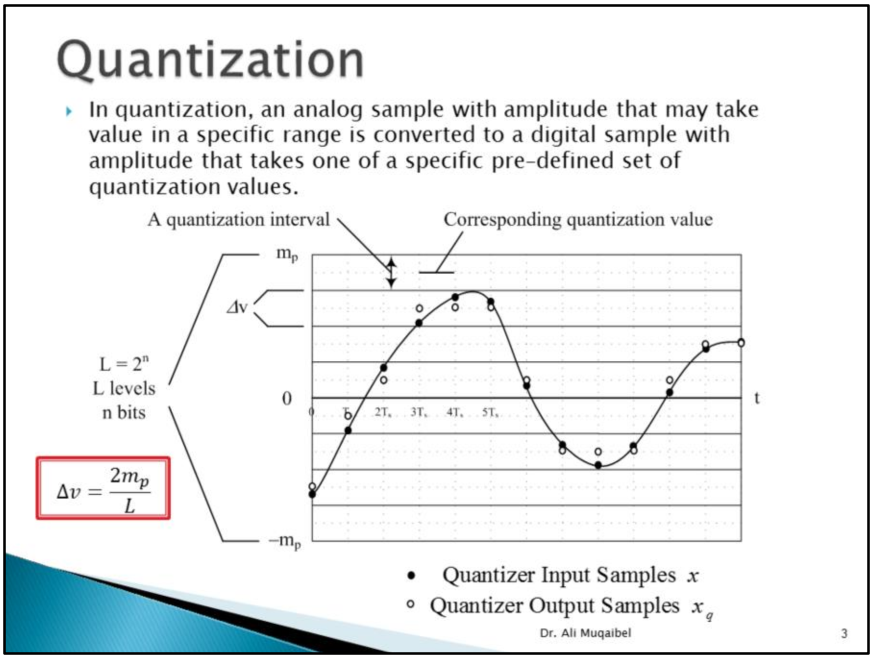
Outline (Class Objective)

- Introduction
- Quantization Noise Power
- Generation of the PCM Signal
  - Companding: Non-uniform Quantization
    - For μ-law
    - For A-law
- SNR impact
- The T1 Carrier System
  - Framing
  - Signaling

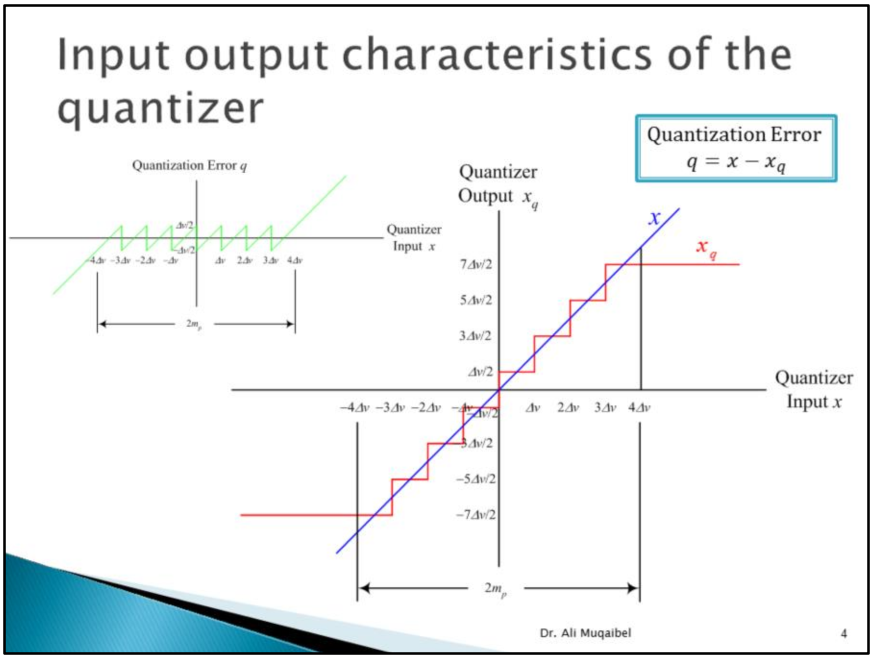


The process of quantizing a signal is the first part of converting a sequence of analog samples to a PCM code.

In quantization, an analog sample with amplitude that may take value in a specific range is converted to a digital sample with amplitude that takes one of a specific pre–defined set of quantization values. This is performed by dividing the range of possible values of the analog samples into L different levels, and assigning the center value of each level to any sample that falls in that quantization interval. The problem with this process is that it approximates the value of an analog sample with the nearest of the quantization values. So, for almost all samples, the quantized samples will differ from the original samples by a small amount. This amount is called the quantization error. To get some idea on the effect of this quantization error, quantizing audio signals results in a hissing noise similar to what you would hear when play a random signal. Assume that a signal with power P_s is to be quantized using a quantizer with $L = 2^n$ levels ranging in voltage from $-m_p$ to m_p as shown in the figure.

We can define the variable Δv to be the height of the each of the L levels of the quantizer as shown above. This gives a value of Δv equal to $\Delta v = \frac{2m_p}{L}$

Therefore, for a set of quantizers with the same m_p, the larger the number of levels of a quantizer, the smaller the size of each quantization interval, and for a set of quantizers with the same number of quantization intervals, the larger m_p is the larger the quantization interval length to accommodate all the quantization range.

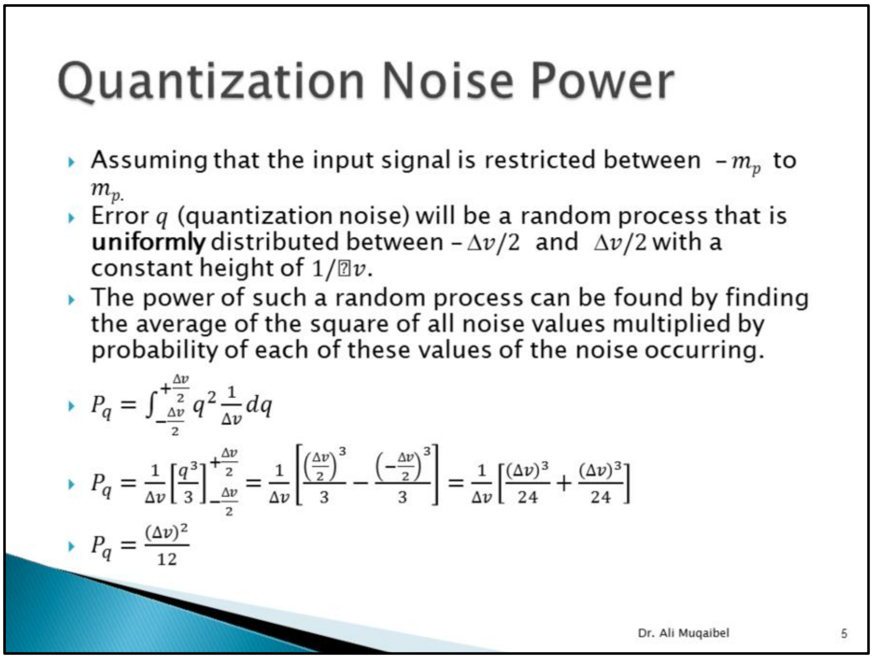


Now if we look at the input output characteristics of the quantizer, it will be similar to the red line in the following figure. Note that as long as the input is within the quantization range of the quantizer, the output of the quantizer represented by the red line follows the input of the quantizer. When the input of the quantizer exceeds the range of $-m_p$ to m_p, the output of the quantizer starts to deviate from the input and the quantization error (difference between an input and the corresponding output sample) increases significantly.

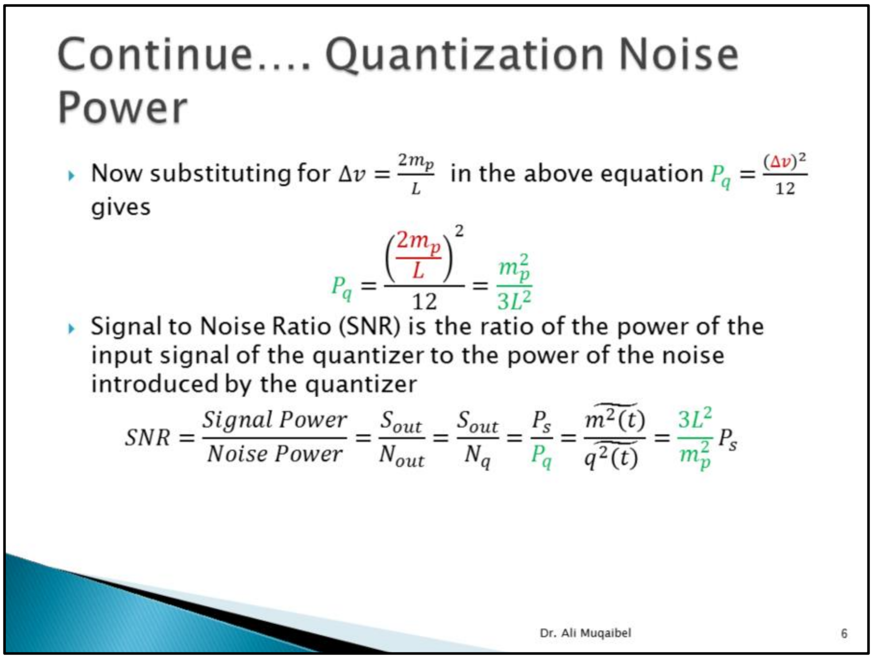
Now let us define the quantization error represented by the difference between the input sample and the corresponding output sample to be q, or

$q = x - x_q$.

Plotting this quantization error versus the input signal of a quantizer is seen next. Notice that the plot of the quantization error is obtained by taking the difference between the blow and red lines in the above figure.



To understand the following, you will need to know something about probability theory. Assuming that the input signal is restricted between $-m_p$ to m_p, the resulting quantization error q (or we can call it quantization noise) will be a random process that is uniformly distributed between $-\Delta v/2$ and $\Delta v/2$ with a constant height of $1/\Delta v$. That is, all values of quantization error in the range $-\Delta v/2$ and $\Delta v/2$ are equally probable to happen. The power of such a random process can be easily found by finding the average of the square of all noise values multiplied by probability of each of these values of the noise occurring. So,

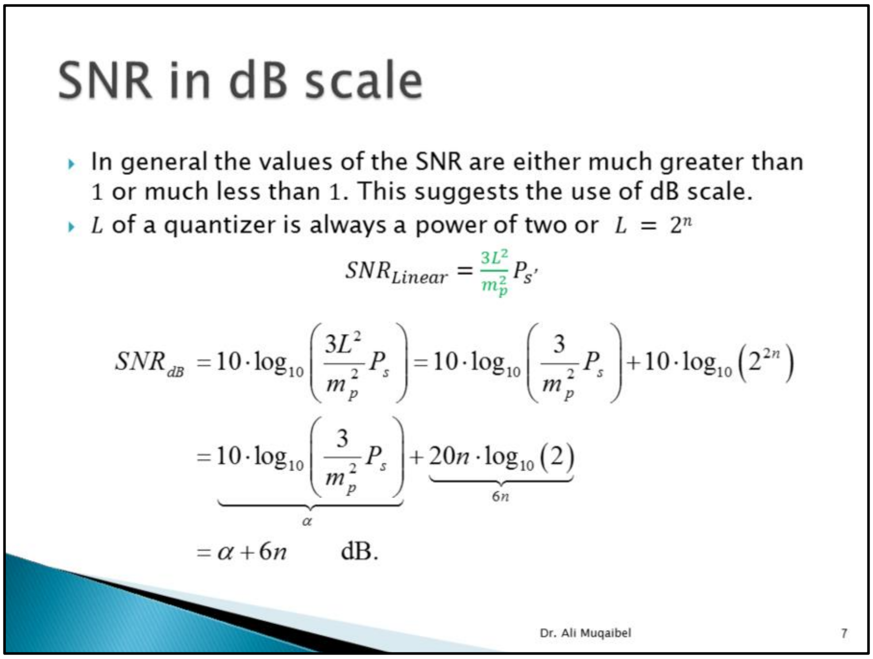


Now substituting for $\Delta v = \frac{2m_p}{L}$

in the above equation gives $P_q = \frac{\left(\frac{2m_p}{L}\right)^2}{12} = \frac{m_p^2}{3L^2}$

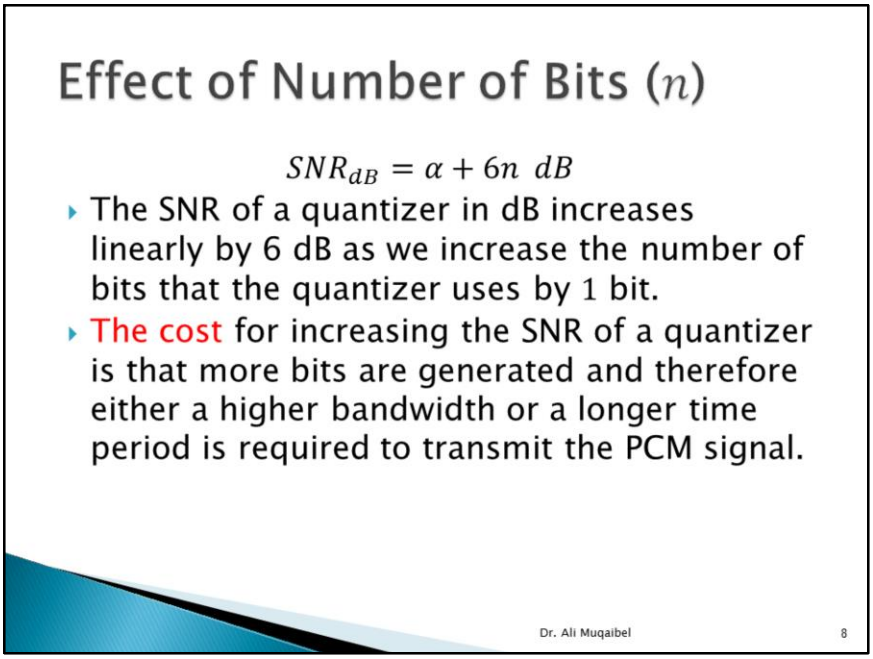
,

As predicted, the power of the noise decreases as the number of levels L increases, and increases as the edge of the quantization range m_p increases.
Now let us define the Signal to Noise Ratio (SNR) as the ratio of the power of the input signal of the quantizer to the power of the noise introduced by the quantizer (note that the SNR has many other definitions used in communication systems depending on the applications)

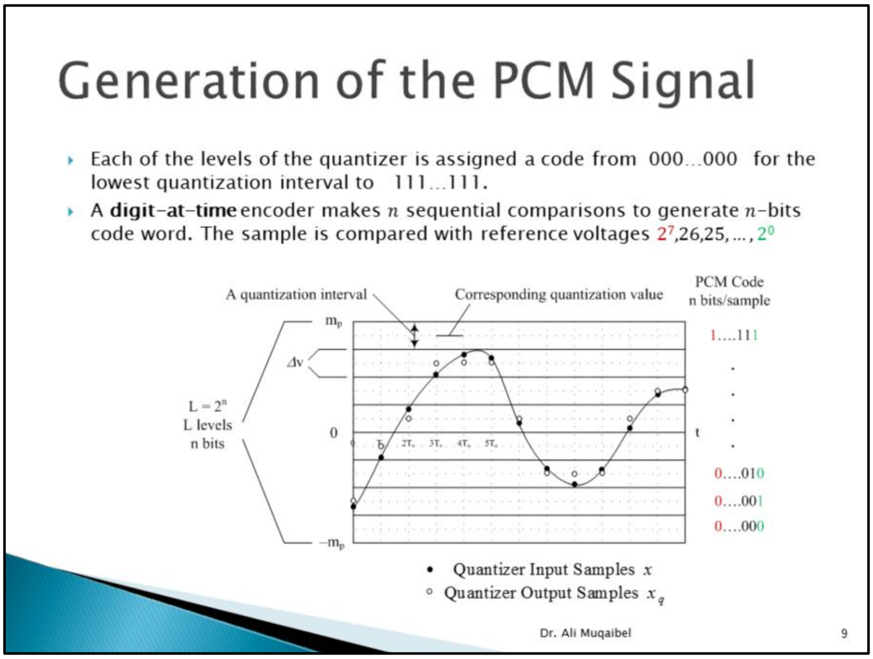


In general the values of the SNR are either much greater than 1 or much less than 1. A more useful representation of the SNR can be obtained by using logarithmic scale or dB. We know that L of a quantizer is always a power of two or $L = 2^n$. Therefore,

Note that α shown in the above representation of the SNR is a constant when quantizing a specific signal with different quantizers as long as all of these quantizers have the same value of m_p.

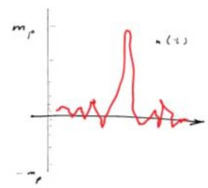


It is clear that the SNR of a quantizer in dB increases linearly by 6 dB as we increase the number of bits that the quantizer uses by 1 bit. The cost for increasing the SNR of a quantizer is that more bits are generated and therefore either a higher bandwidth or a longer time period is required to transmit the PCM signal.

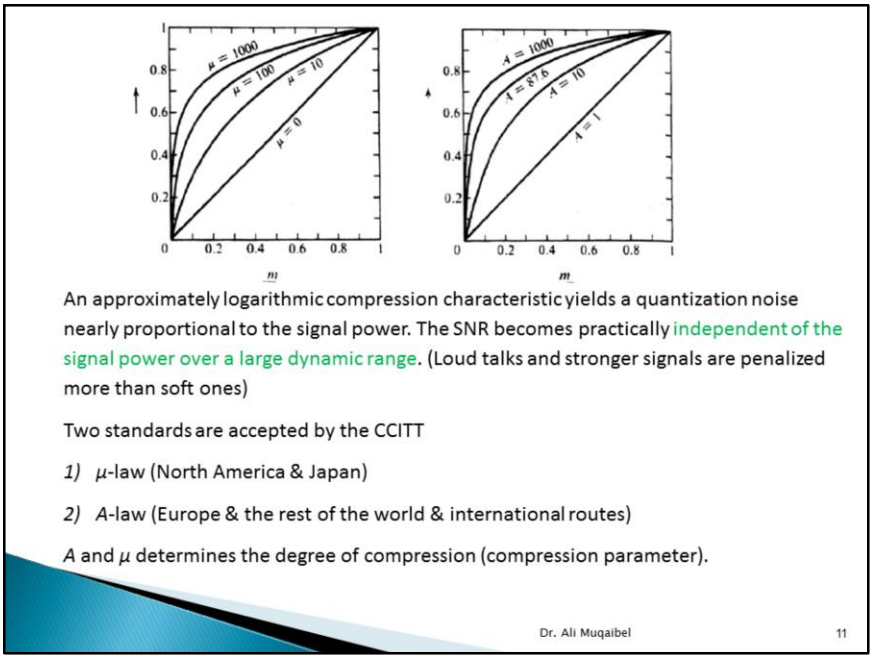


Now, once the signal has been quantized by the quantizer, the quantizer converts it to bits (1's and 0's) and outputs these bits. Looking at the figure in the previous lecture, which is shown here for convenience. We see that each of the levels of the quantizer is assigned a code from 000…000 for the lowest quantization interval to 111…111 for the highest quantization interval as shown in the column to the left of the figure. The PCM signal is obtained by outputting the bits of the different samples one bit after the other and one sample after the other.
We can use a **digit-at-time** encoder which makes n sequential comparisons to generate n-bits code word. The sample is compared with reference voltages $2^7,2^6,2^5,\ldots,2^0$.

Companding: Non-unifrom Quantization



- Ideally we want constant SNR for all values of the message.
- Signals (voices) varies as much as 40dB (10^4 power ratio). The variation could be different due to connection lengths.
- Statistically (for voice): most of the time the signal has small amplitudes (Low SNR most of the time).
- **For** uniform quantization $\Delta v = \frac{2m_p}{L}$ and $N_q = \frac{(\Delta v)^2}{12}$
- The error depend on the **step size.** The solution is to use small steps for small amplitudes and large steps for large amplitudes (**Progressive taxation**)
- This is equivalent to first compress the signal samples & then use uniform quantizer. (Later we will have to decompress).
- Since at the transmitter/receiver we do compress/expand, we call the compensation compander =Compresser +Expander.



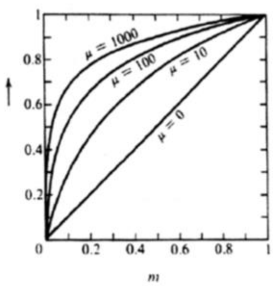
CCITT (***C****omité* ***C****onsultatif* ***I****nternational* ***T****éléphonique et* ***T****élégraphique*, an organization that sets international communications standards. CCITT, now known as ITU (the parent organization))

μ–law

$$F(x) = \mathrm{sgn}(x)\frac{\ln(1+\mu|x|)}{\ln(1+\mu)} \qquad -1 \le x \le 1$$

$$F^{-1}(y) = \mathrm{sgn}(y)(1/\mu)((1+\mu)^{|y|}-1) \qquad -1 \le y \le 1$$

- For input variation greater than $40dB,\ \mu > 100$.
- For practical telephone systems
  - $\mu = 100$ for 7bit (128 levels)
  - $\mu = 255$ for 8bit (256 levels)
- The compander with logarithmic compression can be realized by a semiconductor diode. We can also use piecewise approximation with small end-to-end inferiority.
- See Wikipedia for "μ–law algorithm"

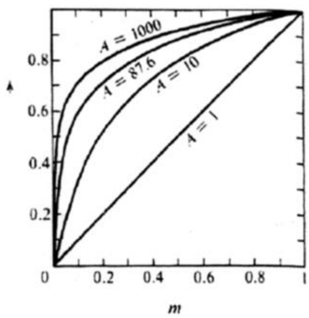


A-law

- For a given input x, the equation for A-law encoding is as $F(x)$
- A-law expansion is given by the inverse function, $F^{-1}(y)$
- In Europe, $A = 87.7$; the value 87.6 is also used.
- **Example**: how many levels will be used to represent the lowest 20% of the signal level for the case of $A = 1$ and $A = 10$?

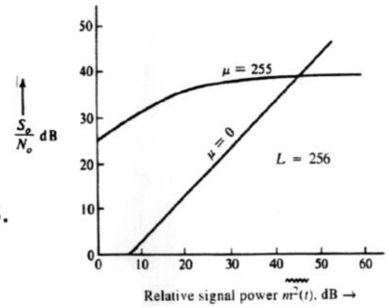
$$F(x) = \operatorname{sgn}(x)\begin{cases} \frac{A|x|}{1+\ln(A)}, & |x| < \frac{1}{A} \\ \frac{1+\ln(A|x|)}{1+\ln(A)}, & \frac{1}{A} \le |x| \le 1, \end{cases}$$

$$F^{-1}(y) = \operatorname{sgn}(y)\begin{cases} \frac{|y|(1+\ln(A))}{A}, & |y| < \frac{1}{1+\ln(A)} \\ \frac{\exp(|y|(1+\ln(A))-1)}{A}, & \frac{1}{1+\ln(A)} \le |y| < 1. \end{cases}$$



SNR impact

- When μ-law is used:
- $\frac{S_0}{N_0} = \frac{3L^2}{[\ln(1+n)]^2}$ for $\mu^2 \gg \frac{m_p^2}{\widetilde{m^2(t)}}$
- The output SNR is almost independent from the input SNR.
- Note the scale above is in dB.
- Are you familiar with the dB scale?



Signal-to-Quantization-noise ratio in PCM with and without compression

Excersize

- In a certain digital voice communication system, the error in sample amplitudes cannot be greater than 3% of the peak amplitude.
- Determine the number of bits for the quantizer, n.

- $Ans. n = 6$

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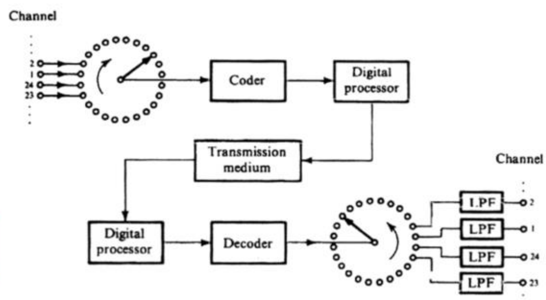
Part of Quiz 6 (072)
Ans 6

The T1 Carrier System

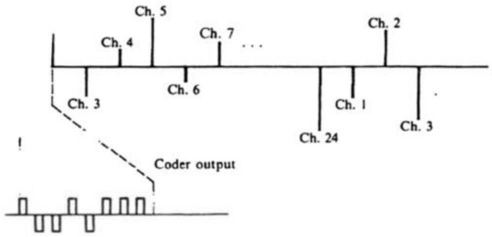
- 24 Channels (TDM-PAM).
- Encoder (Quantize and Encode) samples ,8 binary pulses (Binary Codeword).
- Regenerative repeaters /6000 feet
- @ the receiver: decode binary pulses into samples, demultiplex, LPF.



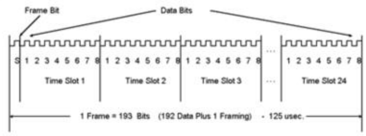
Multiplexing is not mechanical

More about T1

- The $\mathbf{1.544}\ \boldsymbol{Mbit/s}$ of the T1 system is called Digital Signal Level 1 (DS1)....DS2, DS3,DS4 also exist.
- T1(N. America & Japan)
- By CCITT a different system of 30 Channels PCM ($\mathbf{2.048}\ \boldsymbol{Mbits/s}$) is used in Europe and others



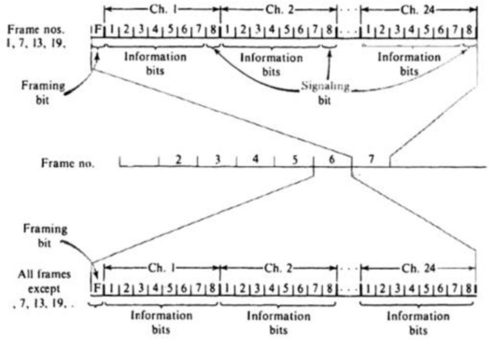
Frame Synchronization



- Frame: A segment containing one code-word (corresponding to one sample) from each of the 24 channels.
  - $\#\, of\ bits = 24 \times 8 = 192 +$ a framing bit (in order to separate information bits correctly)=193 bits/frame
  - $8000 \frac{samples}{s} => \frac{1}{8000} = 125\ \mu s$
  - $193 \frac{bits}{frame} \div 125 \frac{\mu s}{frame} = \mathbf{1.544}\ \boldsymbol{Mbit/s}$
- Framing bits: Chosen so that a sequence of framing bits, one at the beginning of each frame, forms a special pattern that is unlikely to be formed in speech signal.
- 0.4 to $0.6ms$ to detect synchronization loss, $50\ ms$ to reframe.

T1 System Signaling Format

- Signaling: bits corresponding to dialing pulses & telephone on-hook/off-hook.



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